5.6 Graphing Linear Inequalities in Two Variables

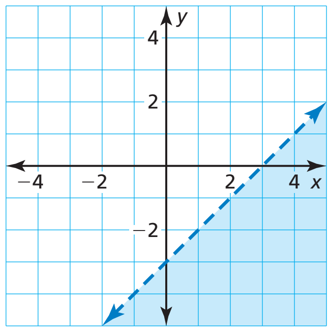
Essential Question

How can you graph a linear inequality in two variables?

A **solution of a linear inequality in two variables** is an ordered pair (x, y) that makes the inequality true. The **graph of a linear inequality** in two variables shows all the solutions of the inequality in a coordinate plane.

EXPLORATION 1 Writing a Linear Inequality in Two Variables

Work with a partner.



a. Write an equation represented by the dashed line.

b. The solutions of an inequality are represented by the shaded region. In words, describe the solutions of the inequality.

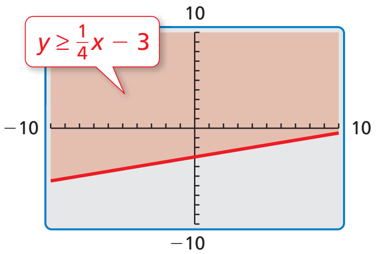
c. Write an inequality represented by the graph. Which inequality symbol did you use? Explain your reasoning.

EXPLORATION 2 Using a Graphing Calculator

Work with a partner. Use a graphing calculator to graph $y \ge \frac{1}{4}x - 3$.

a. Enter the equation $y = \frac{1}{4}x - 3$ into your calculator.

b. The inequality has the symbol $\ge$. So, the region to be shaded is above the graph of $y = \frac{1}{4}x - 3$, as shown. Verify this by testing a point in this region, such as $(0, 0)$, to make sure it is a solution of the inequality.



> **USING TOOLS STRATEGICALLY**
>
> To be proficient in math, you need to use technological tools to explore and deepen your understanding of concepts.

Because the inequality symbol is *greater than or equal to*, the line is solid and not dashed. Some graphing calculators always use a solid line when graphing inequalities. In this case, you have to determine whether the line should be solid or dashed, based on the inequality symbol used in the original inequality.

EXPLORATION 3 Graphing Linear Inequalities in Two Variables

Work with a partner. Graph each linear inequality in two variables. Explain your steps. Use a graphing calculator to check your graphs.

a. $y > x + 5$ **b.** $y \le -\frac{1}{2}x + 1$ **c.** $y \ge -x - 5$

Communicate Your Answer

4. How can you graph a linear inequality in two variables?

5. Give an example of a real-life situation that can be modeled using a linear inequality in two variables.

5.6 Lesson

Core Vocabulary

linear inequality in two variables, *p. 250*
solution of a linear inequality in two variables, *p. 250*
graph of a linear inequality, *p. 250*
half-planes, *p. 250*

Previous
ordered pair

What You Will Learn

- Check solutions of linear inequalities.
- Graph linear inequalities in two variables.
- Use linear inequalities to solve real-life problems.

Linear Inequalities

A **linear inequality in two variables**, x and y, can be written as

$$ax + by < c \qquad ax + by \leq c \qquad ax + by > c \qquad ax + by \geq c$$

where a, b, and c are real numbers. A **solution of a linear inequality in two variables** is an ordered pair (x, y) that makes the inequality true.

EXAMPLE 1 Checking Solutions

Tell whether the ordered pair is a solution of the inequality.

a. $2x + y < -3$; $(-1, 9)$ **b.** $x - 3y \geq 8$; $(2, -2)$

SOLUTION

a. $2x + y < -3$ Write the inequality.

$2(-1) + 9 \overset{?}{<} -3$ Substitute -1 for x and 9 for y.

$7 \not< -3$ ✗ Simplify. 7 is *not* less than -3.

- So, $(-1, 9)$ is *not* a solution of the inequality.

b. $x - 3y \geq 8$ Write the inequality.

$2 - 3(-2) \overset{?}{\geq} 8$ Substitute 2 for x and -2 for y.

$8 \geq 8$ ✓ Simplify. 8 is equal to 8.

- So, $(2, -2)$ is a solution of the inequality.

Monitoring Progress

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Tell whether the ordered pair is a solution of the inequality.

1. $x + y > 0$; $(-2, 2)$ **2.** $4x - y \geq 5$; $(0, 0)$

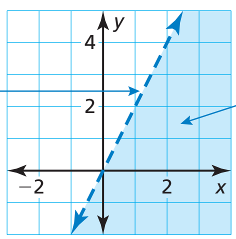
3. $5x - 2y \leq -1$; $(-4, -1)$ **4.** $-2x - 3y < 15$; $(5, -7)$

Graphing Linear Inequalities in Two Variables

The **graph of a linear inequality** in two variables shows all the solutions of the inequality in a coordinate plane.

READING
A dashed boundary line means that points on the line are *not* solutions. A solid boundary line means that points on the line are solutions.

All solutions of $y < 2x$ lie on one side of the *boundary line* $y = 2x$.



The boundary line divides the coordinate plane into two **half-planes**. The shaded half-plane is the graph of $y < 2x$.

Core Concept

Graphing a Linear Inequality in Two Variables

Step 1 Graph the boundary line for the inequality. Use a dashed line for < or >. Use a solid line for ≤ or ≥.

Step 2 Test a point that is not on the boundary line to determine whether it is a solution of the inequality.

Step 3 When the test point is a solution, shade the half-plane that contains the point. When the test point is *not* a solution, shade the half-plane that does *not* contain the point.

STUDY TIP

It is often convenient to use the origin as a test point. However, you must choose a different test point when the origin is on the boundary line.

EXAMPLE 2 Graphing a Linear Inequality in One Variable

Graph $y \leq 2$ in a coordinate plane.

SOLUTION

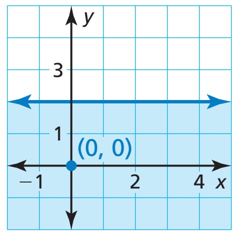
Step 1 Graph $y = 2$. Use a solid line because the inequality symbol is ≤.

Step 2 Test (0, 0).

$y \leq 2$ Write the inequality.

$0 \leq 2$ ✓ Substitute.

Step 3 Because (0, 0) is a solution, shade the half-plane that contains (0, 0).



EXAMPLE 3 Graphing a Linear Inequality in Two Variables

Graph $-x + 2y > 2$ in a coordinate plane.

SOLUTION

Step 1 Graph $-x + 2y = 2$, or $y = \frac{1}{2}x + 1$. Use a dashed line because the inequality symbol is >.

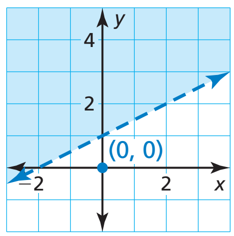
Step 2 Test (0, 0).

$-x + 2y > 2$ Write the inequality.

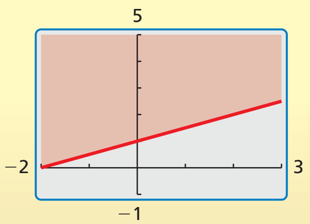
$-(0) + 2(0) \overset{?}{>} 2$ Substitute.

$0 \not> 2$ ✗ Simplify.

Step 3 Because (0, 0) is *not* a solution, shade the half-plane that does *not* contain (0, 0).



Check



Monitoring Progress

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Graph the inequality in a coordinate plane.

5. $y > -1$

6. $x \leq -4$

7. $x + y \leq -4$

8. $x - 2y < 0$

Solving Real-Life Problems

EXAMPLE 4 Modeling with Mathematics

You can spend at most \$10 on grapes and apples for a fruit salad. Grapes cost \$2.50 per pound, and apples cost \$1 per pound. Write and graph an inequality that represents the amounts of grapes and apples you can buy. Identify and interpret two solutions of the inequality.

SOLUTION

1. **Understand the Problem** You know the most that you can spend and the prices per pound for grapes and apples. You are asked to write and graph an inequality and then identify and interpret two solutions.

2. **Make a Plan** Use a verbal model to write an inequality that represents the problem. Then graph the inequality. Use the graph to identify two solutions. Then interpret the solutions.

3. **Solve the Problem**

Words	Cost per pound of grapes	•	Pounds of grapes	+	Cost per pound of apples	•	Pounds of apples	≤	Amount you can spend

Variables Let x be pounds of grapes and y be pounds of apples.

Inequality $2.50 \cdot x + 1 \cdot y \leq 10$

Step 1 Graph $2.5x + y = 10$, or $y = -2.5x + 10$. Use a solid line because the inequality symbol is $\leq$. Restrict the graph to positive values of x and y because negative values do not make sense in this real-life context.

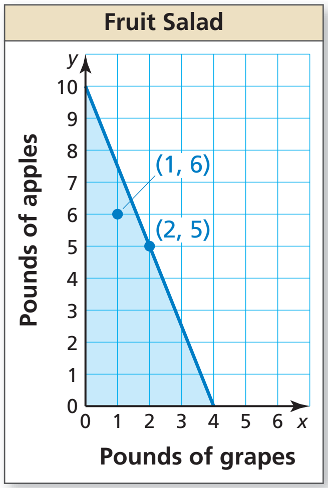
Step 2 Test (0, 0).

$$2.5x + y \leq 10 \qquad \text{Write the inequality.}$$

$$2.5(0) + 0 \overset{?}{\leq} 10 \qquad \text{Substitute.}$$

$$0 \leq 10 \checkmark \qquad \text{Simplify.}$$

Step 3 Because (0, 0) is a solution, shade the half-plane that contains (0, 0).



▶ One possible solution is (1, 6) because it lies in the shaded half-plane. Another possible solution is (2, 5) because it lies on the solid line. So, you can buy 1 pound of grapes and 6 pounds of apples, or 2 pounds of grapes and 5 pounds of apples.

4. **Look Back** Check your solutions by substituting them into the original inequality, as shown.

Check

$$2.5x + y \leq 10$$

$$2.5(1) + 6 \overset{?}{\leq} 10$$

$$8.5 \leq 10 \checkmark$$

$$2.5x + y \leq 10$$

$$2.5(2) + 5 \overset{?}{\leq} 10$$

$$10 \leq 10 \checkmark$$

Monitoring Progress

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9. You can spend at most \$12 on red peppers and tomatoes for salsa. Red peppers cost \$4 per pound, and tomatoes cost \$3 per pound. Write and graph an inequality that represents the amounts of red peppers and tomatoes you can buy. Identify and interpret two solutions of the inequality.

5.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** How can you tell whether an ordered pair is a solution of a linear inequality?

2. **WRITING** Compare the graph of a linear inequality in two variables with the graph of a linear equation in two variables.

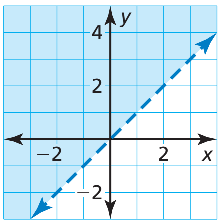
Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, tell whether the ordered pair is a solution of the inequality. *(See Example 1.)*

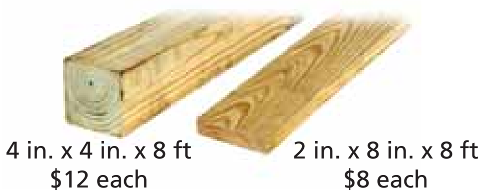
3. $x + y < 7$; $(2, 3)$
4. $x - y \leq 0$; $(5, 2)$
5. $x + 3y \geq -2$; $(-9, 2)$
6. $8x + y > -6$; $(-1, 2)$
7. $-6x + 4y \leq 6$; $(-3, -3)$
8. $3x - 5y \geq 2$; $(-1, -1)$
9. $-x - 6y > 12$; $(-8, 2)$
10. $-4x - 8y < 15$; $(-6, 3)$

In Exercises 11–16, tell whether the ordered pair is a solution of the inequality whose graph is shown.

11. $(0, -1)$
12. $(-1, 3)$
13. $(1, 4)$
14. $(0, 0)$
15. $(3, 3)$
16. $(2, 1)$



17. **MODELING WITH MATHEMATICS** A carpenter has at most \$250 to spend on lumber. The inequality $8x + 12y \leq 250$ represents the numbers x of 2-by-8 boards and the numbers y of 4-by-4 boards the carpenter can buy. Can the carpenter buy twelve 2-by-8 boards and fourteen 4-by-4 boards? Explain.



18. **MODELING WITH MATHEMATICS** The inequality $3x + 2y \geq 93$ represents the numbers x of multiple-choice questions and the numbers y of matching questions you can answer correctly to receive an A on a test. You answer 20 multiple-choice questions and 18 matching questions correctly. Do you receive an A on the test? Explain.

In Exercises 19–24, graph the inequality in a coordinate plane. *(See Example 2.)*

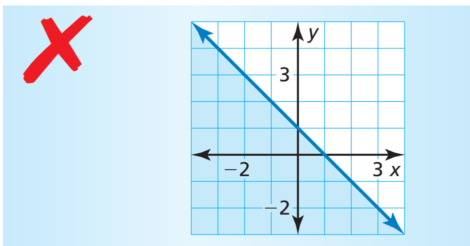
19. $y \leq 5$
20. $y > 6$
21. $x < 2$
22. $x \geq -3$
23. $y > -7$
24. $x < 9$

In Exercises 25–30, graph the inequality in a coordinate plane. *(See Example 3.)*

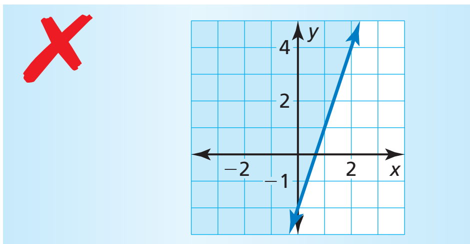
25. $y > -2x - 4$
26. $y \leq 3x - 1$
27. $-4x + y < -7$
28. $3x - y \geq 5$
29. $5x - 2y \leq 6$
30. $-x + 4y > -12$

ERROR ANALYSIS In Exercises 31 and 32, describe and correct the error in graphing the inequality.

31. $y < -x + 1$



32. $y \leq 3x - 2$

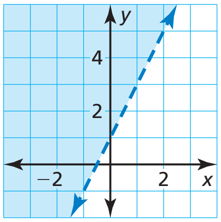


33. **MODELING WITH MATHEMATICS** You have at most \$20 to spend at an arcade. Arcade games cost \$0.75 each, and snacks cost \$2.25 each. Write and graph an inequality that represents the numbers of games you can play and snacks you can buy. Identify and interpret two solutions of the inequality. *(See Example 4.)*

34. **MODELING WITH MATHEMATICS** A drama club must sell at least \$1500 worth of tickets to cover the expenses of producing a play. Write and graph an inequality that represents how many adult and student tickets the club must sell. Identify and interpret two solutions of the inequality.

In Exercises 35–38, write an inequality that represents the graph.

35.



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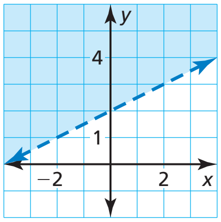

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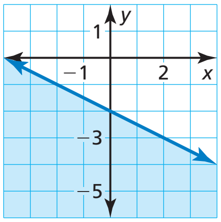

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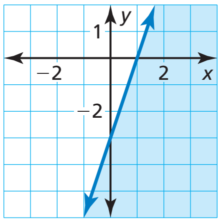


39. **PROBLEM SOLVING** Large boxes weigh 75 pounds, and small boxes weigh 40 pounds.

a. Write and graph an inequality that represents the numbers of large and small boxes a 200-pound delivery person can take on the elevator.

b. Explain why some solutions of the inequality might not be practical in real life.

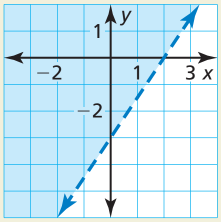
Weight limit:
2000 lb

40. **HOW DO YOU SEE IT?** Match each inequality with its graph.

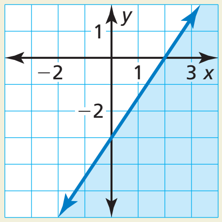
a. $3x - 2y \le 6$ **b.** $3x - 2y < 6$

c. $3x - 2y > 6$ **d.** $3x - 2y \ge 6$

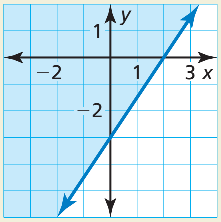
A.



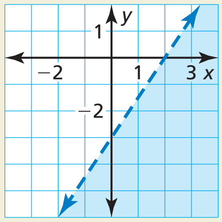
B.



C.



D.



41. **REASONING** When graphing a linear inequality in two variables, why must you choose a test point that is *not* on the boundary line?

42. **THOUGHT PROVOKING** Write a linear inequality in two variables that has the following two properties.

- $(0, 0)$, $(0, -1)$, and $(0, 1)$ are not solutions.
- $(1, 1)$, $(3, -1)$, and $(-1, 3)$ are solutions.

43. **WRITING** Can you always use $(0, 0)$ as a test point when graphing an inequality? Explain.

CRITICAL THINKING In Exercises 44 and 45, write and graph an inequality whose graph is described by the given information.

44. The points $(2, 5)$ and $(-3, -5)$ lie on the boundary line. The points $(6, 5)$ and $(-2, -3)$ are solutions of the inequality.

45. The points $(-7, -16)$ and $(1, 8)$ lie on the boundary line. The points $(-7, 0)$ and $(3, 14)$ are *not* solutions of the inequality.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the next three terms of the arithmetic sequence. *(Section 4.6)*

46. $0, 8, 16, 24, 32, \ldots$

47. $-5, -8, -11, -14, -17, \ldots$

48. $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$