2.6 Solving Absolute Value Inequalities

Essential Question

How can you solve an absolute value inequality?

EXPLORATION 1 Solving an Absolute Value Inequality Algebraically

Work with a partner. Consider the absolute value inequality

$$|x + 2| \leq 3.$$

a. Describe the values of $x + 2$ that make the inequality true. Use your description to write two linear inequalities that represent the solutions of the absolute value inequality.

b. Use the linear inequalities you wrote in part (a) to find the solutions of the absolute value inequality.

c. How can you use linear inequalities to solve an absolute value inequality?

MAKING SENSE OF PROBLEMS

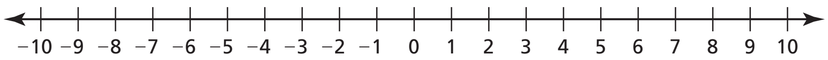
To be proficient in math, you need to explain to yourself the meaning of a problem and look for entry points to its solution.

EXPLORATION 2 Solving an Absolute Value Inequality Graphically

Work with a partner. Consider the absolute value inequality

$$|x + 2| \leq 3.$$

a. On a real number line, locate the point for which $x + 2 = 0$.



b. Locate the points that are within 3 units from the point you found in part (a). What do you notice about these points?

c. How can you use a number line to solve an absolute value inequality?

EXPLORATION 3 Solving an Absolute Value Inequality Numerically

Work with a partner. Consider the absolute value inequality

$$|x + 2| \leq 3.$$

a. Use a spreadsheet, as shown, to solve the absolute value inequality.

b. Compare the solutions you found using the spreadsheet with those you found in Explorations 1 and 2. What do you notice?

c. How can you use a spreadsheet to solve an absolute value inequality?

	A	B
1	***x***	**|x + 2|**
2	-6	4
3	-5	
4	-4	
5	-3	
6	-2	
7	-1	
8	0	
9	1	
10	2	
11		

abs(A2 + 2)

Communicate Your Answer

4. How can you solve an absolute value inequality?

5. What do you like or dislike about the algebraic, graphical, and numerical methods for solving an absolute value inequality? Give reasons for your answers.

2.6 Lesson

What You Will Learn

- Solve absolute value inequalities.
- Use absolute value inequalities to solve real-life problems.

Core Vocabulary

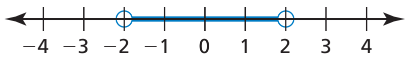
absolute value inequality, *p. 88*
absolute deviation, *p. 90*

Previous
compound inequality
mean

Solving Absolute Value Inequalities

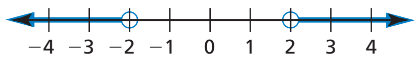
An **absolute value inequality** is an inequality that contains an absolute value expression. For example, $|x| < 2$ and $|x| > 2$ are absolute value inequalities. Recall that $|x| = 2$ means the distance between x and 0 is 2.

The inequality $|x| < 2$ means the distance between x and 0 is *less than* 2.



The graph of $|x| < 2$ is the graph of $x > -2$ *and* $x < 2$.

The inequality $|x| > 2$ means the distance between x and 0 is *greater than* 2.



The graph of $|x| > 2$ is the graph of $x < -2$ *or* $x > 2$.

You can solve these types of inequalities by solving a compound inequality.

Core Concept

Solving Absolute Value Inequalities

To solve $|ax + b| < c$ for $c > 0$, solve the compound inequality

$$ax + b > -c \qquad \text{and} \qquad ax + b < c.$$

To solve $|ax + b| > c$ for $c > 0$, solve the compound inequality

$$ax + b < -c \qquad \text{or} \qquad ax + b > c.$$

In the inequalities above, you can replace $<$ with $\leq$ and $>$ with $\geq$.

EXAMPLE 1 Solving Absolute Value Inequalities

Solve each inequality. Graph each solution, if possible.

a. $|x + 7| \leq 2$ **b.** $|8x - 11| < 0$

SOLUTION

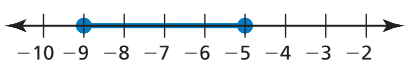
a. Use $|x + 7| \leq 2$ to write a compound inequality. Then solve.

$$x + 7 \geq -2 \quad \text{and} \quad x + 7 \leq 2 \qquad \text{Write a compound inequality.}$$

$$\underline{-7} \quad \underline{-7} \qquad\qquad \underline{-7} \quad \underline{-7} \qquad \text{Subtract 7 from each side.}$$

$$x \geq -9 \quad \text{and} \quad x \leq -5 \qquad \text{Simplify.}$$

▶ The solution is $-9 \leq x \leq -5$.



REMEMBER

A compound inequality with "and" can be written as a single inequality.

b. By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|8x - 11|$ cannot be less than 0.

▶ So, the inequality has no solution.

Monitoring Progress

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Solve the inequality. Graph the solution, if possible.

1. $|x| \leq 3.5$ **2.** $|k - 3| < -1$ **3.** $|2w - 1| < 11$

EXAMPLE 2 Solving Absolute Value Inequalities

Solve each inequality. Graph each solution.

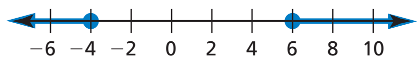
a. $|c - 1| \geq 5$ **b.** $|10 - m| \geq -2$ **c.** $4|2x - 5| + 1 > 21$

SOLUTION

a. Use $|c - 1| \geq 5$ to write a compound inequality. Then solve.

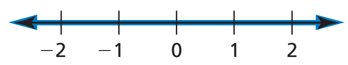
$$\begin{array}{rcl l}
c - 1 \leq -5 & \textit{or} & c - 1 \geq 5 & \text{Write a compound inequality.} \\
\underline{+1} \quad \underline{+1} & & \underline{+1} \quad \underline{+1} & \text{Add 1 to each side.} \\
c \leq -4 & \textit{or} & c \geq 6 & \text{Simplify.}
\end{array}$$

▶ The solution is $c \leq -4$ *or* $c \geq 6$.



b. By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|10 - m|$ will always be greater than -2.

▶ So, all real numbers are solutions.



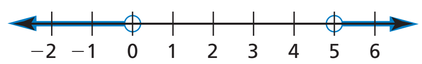
c. First isolate the absolute value expression on one side of the inequality.

$$\begin{array}{rcl l}
4|2x - 5| + 1 & > & 21 & \text{Write the inequality.} \\
\underline{-1} & & \underline{-1} & \text{Subtract 1 from each side.} \\
4|2x - 5| & > & 20 & \text{Simplify.} \\
\dfrac{4|2x - 5|}{4} & > & \dfrac{20}{4} & \text{Divide each side by 4.} \\
|2x - 5| & > & 5 & \text{Simplify.}
\end{array}$$

Then use $|2x - 5| > 5$ to write a compound inequality. Then solve.

$$\begin{array}{rcl l}
2x - 5 < -5 & \textit{or} & 2x - 5 > 5 & \text{Write a compound inequality.} \\
\underline{+5} \quad \underline{+5} & & \underline{+5} \quad \underline{+5} & \text{Add 5 to each side.} \\
2x < 0 & & 2x > 10 & \text{Simplify.} \\
\dfrac{2x}{2} < \dfrac{0}{2} & & \dfrac{2x}{2} > \dfrac{10}{2} & \text{Divide each side by 2.} \\
x < 0 & \textit{or} & x > 5 & \text{Simplify.}
\end{array}$$

▶ The solution is $x < 0$ *or* $x > 5$.



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Solve the inequality. Graph the solution.

4. $|x + 3| > 8$ **5.** $|n + 2| - 3 \geq -6$ **6.** $3|d + 1| - 7 \geq -1$

Solving Real-Life Problems

The **absolute deviation** of a number x from a given value is the absolute value of the difference of x and the given value.

$$\text{absolute deviation} = |x - \text{given value}|$$

EXAMPLE 3 Modeling with Mathematics

Computer prices	
\$890	\$750
\$650	\$370
\$660	\$670
\$450	\$650
\$725	\$825

You are buying a new computer. The table shows the prices of computers in a store advertisement. You are willing to pay the mean price with an absolute deviation of at most \$100. How many of the computer prices meet your condition?

SOLUTION

1. **Understand the Problem** You know the prices of 10 computers. You are asked to find how many computers are at most \$100 from the mean price.

2. **Make a Plan** Calculate the mean price by dividing the sum of the prices by the number of prices, 10. Use the absolute deviation and the mean price to write an absolute value inequality. Then solve the inequality and use it to answer the question.

3. **Solve the Problem**

The mean price is $\frac{6640}{10} = \$664$. Let x represent a price you are willing to pay.

$|x - 664| \leq 100$ Write the absolute value inequality.

$-100 \leq x - 664 \leq 100$ Write a compound inequality.

$564 \leq x \leq 764$ Add 664 to each expression and simplify.

▶ The prices you will consider must be at least \$564 and at most \$764. Six prices meet your condition: \$750, \$650, \$660, \$670, \$650, and \$725.

4. **Look Back** You can check that your answer is correct by graphing the computer prices and the mean on a number line. Any point within 100 of 664 represents a price that you will consider.

STUDY TIP

The absolute deviation of at most \$100 from the mean, \$664, is given by the inequality $|x - 664| \leq 100$.

Monitoring Progress

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7. **WHAT IF?** You are willing to pay the mean price with an absolute deviation of at most \$75. How many of the computer prices meet your condition?

Concept Summary

Solving Inequalities

One-Step and Multi-Step Inequalities

- Follow the steps for solving an equation. Reverse the inequality symbol when multiplying or dividing by a negative number.

Compound Inequalities

- If necessary, write the inequality as two separate inequalities. Then solve each inequality separately. Include *and* or *or* in the solution.

Absolute Value Inequalities

- If necessary, isolate the absolute value expression on one side of the inequality. Write the absolute value inequality as a compound inequality. Then solve the compound inequality.

2.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** Can you determine the solution of $|4x - 2| \geq -6$ without solving? Explain.
2. **WRITING** Describe how solving $|w - 9| \leq 2$ is different from solving $|w - 9| \geq 2$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–18, solve the inequality. Graph the solution, if possible. *(See Examples 1 and 2.)*

3. $|x| < 3$
4. $|y| \geq 4.5$
5. $|d + 9| > 3$
6. $|h - 5| \leq 10$
7. $|2s - 7| \geq -1$
8. $|4c + 5| > 7$
9. $|5p + 2| < -4$
10. $|9 - 4n| < 5$
11. $|6t - 7| - 8 \geq 3$
12. $|3j - 1| + 6 > 0$
13. $3|14 - m| > 18$
14. $-4|6b - 8| \leq 12$
15. $2|3w + 8| - 13 \leq -5$
16. $-3|2 - 4u| + 5 < -13$
17. $6|-f + 3| + 7 > 7$
18. $\frac{2}{3}|4v + 6| - 2 \leq 10$

19. **MODELING WITH MATHEMATICS** The rules for an essay contest say that entries can have 500 words with an absolute deviation of at most 30 words. Write and solve an absolute value inequality that represents the acceptable numbers of words. *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** The normal body temperature of a camel is 37°C. This temperature varies by up to 3°C throughout the day. Write and solve an absolute value inequality that represents the range of normal body temperatures (in degrees Celsius) of a camel throughout the day.

ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in solving the absolute value inequality.

21.
$$|x - 5| < 20$$
$$x - 5 < 20$$
$$x < 25$$

22.
$$|x + 4| > 13$$
$$x + 4 > -13 \quad \text{and} \quad x + 4 < 13$$
$$x > -17 \quad \text{and} \quad x < 9$$
$$-17 < x < 9$$

In Exercises 23–26, write the sentence as an absolute value inequality. Then solve the inequality.

23. A number is less than 6 units from 0.
24. A number is more than 9 units from 3.
25. Half of a number is at most 5 units from 14.
26. Twice a number is no less than 10 units from −1.

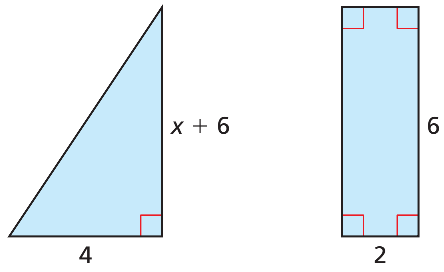
27. **PROBLEM SOLVING** An auto parts manufacturer throws out gaskets with weights that are not within 0.06 pound of the mean weight of the batch. The weights (in pounds) of the gaskets in a batch are 0.58, 0.63, 0.65, 0.53, and 0.61. Which gasket(s) should be thrown out?

28. **PROBLEM SOLVING** Six students measure the acceleration (in meters per second per second) of an object in free fall. The measured values are shown. The students want to state that the absolute deviation of each measured value x from the mean is at most d. Find the value of d.

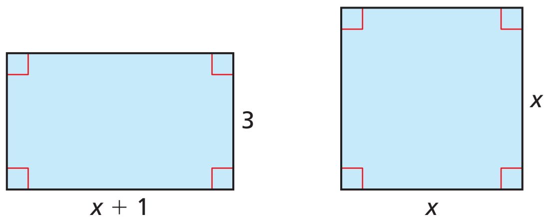
10.56, 9.52, 9.73, 9.80, 9.78, 10.91

MATHEMATICAL CONNECTIONS **In Exercises 29 and 30, write an absolute value inequality that represents the situation. Then solve the inequality.**

29. The difference between the areas of the figures is less than 2.



30. The difference between the perimeters of the figures is less than or equal to 3.



REASONING **In Exercises 31–34, tell whether the statement is true or false. If it is false, explain why.**

31. If a is a solution of $|x + 3| \leq 8$, then a is also a solution of $x + 3 \geq -8$.

32. If a is a solution of $|x + 3| > 8$, then a is also a solution of $x + 3 > 8$.

33. If a is a solution of $|x + 3| \geq 8$, then a is also a solution of $x + 3 \geq -8$.

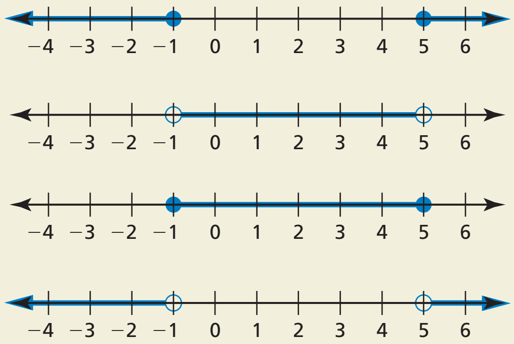
34. If a is a solution of $x + 3 \leq -8$, then a is also a solution of $|x + 3| \geq 8$.

35. **MAKING AN ARGUMENT** One of your classmates claims that the solution of $|n| > 0$ is all real numbers. Is your classmate correct? Explain your reasoning.

36. **THOUGHT PROVOKING** Draw and label a geometric figure so that the perimeter P of the figure is a solution of the inequality $|P - 60| \leq 12$.

37. **REASONING** What is the solution of the inequality $|ax + b| < c$, where $c < 0$? What is the solution of the inequality $|ax + b| > c$, where $c < 0$? Explain.

38. **HOW DO YOU SEE IT?** Write an absolute value inequality for each graph.



How did you decide which inequality symbol to use for each inequality?

39. **WRITING** Explain why the solution set of the inequality $|x| < 5$ is the *intersection* of two sets, while the solution set of the inequality $|x| > 5$ is the *union* of two sets.

40. **PROBLEM SOLVING** Solve the compound inequality below. Describe your steps.

$$|x - 3| < 4 \text{ and } |x + 2| > 8$$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Plot the ordered pair in a coordinate plane. Describe the location of the point. *(Skills Review Handbook)*

41. $A(1, 3)$ **42.** $B(0, -3)$ **43.** $C(-4, -2)$ **44.** $D(-1, 2)$

Copy and complete the table. *(Skills Review Handbook)*

45.

x	0	1	2	3	4
$5x + 1$					

46.

x	−2	−1	0	1	2
$-2x - 3$					