Selected Answers

Chapter 1

Chapter 1 Maintaining Mathematical Proficiency *(p. 1)*

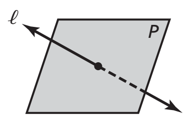
1. 4 **2.** 11 **3.** 5 **4.** 9 **5.** 8 **6.** 6
7. 1 **8.** 5 **9.** 17 **10.** 154 m^2 **11.** 84 yd^2
12. 200 in.^{2}
13. x and y can be any real number, $x \neq y$; $x = y$; no; Absolute value is never negative.

1.1 Vocabulary and Core Concept Check *(p. 8)*

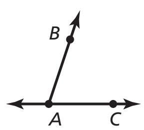
1. Collinear points lie on the same line. Coplanar points lie on the same plane.

1.1 Monitoring Progress and Modeling with Mathematics *(pp. 8–10)*

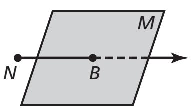
3. *Sample answer:* A, B, D, E **5.** plane S
7. $\overleftrightarrow{QW}$, line g **9.** R, Q, S; *Sample answer:* T
11. $\overrightarrow{DB}$ **13.** $\overrightarrow{AC}$ **15.** $\overrightarrow{EB}$ and $\overrightarrow{ED}$, $\overrightarrow{EA}$ and $\overrightarrow{EC}$
17. *Sample answer:*



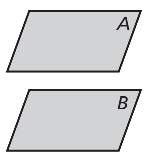
19. *Sample answer:*



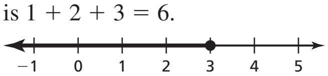
21. *Sample answer:*



23. *Sample answer:*

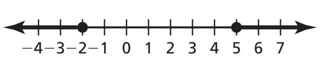


25. $\overrightarrow{AD}$ and $\overrightarrow{AC}$ are not opposite rays because A, C, and D are not collinear; $\overrightarrow{AD}$ and $\overrightarrow{AB}$ are opposite rays because A, B, and D are collinear, and A is between B and D.
27. J **29.** *Sample answer:* D **31.** *Sample answer:* C
33. $\overleftrightarrow{AE}$ **35.** point **37.** segment **39.** P, Q, R, S
41. K, L, M, N **43.** L, M, Q, R
45. yes; Use the point not on the line and two points on the line to draw the plane.
47. Three legs of the chair will meet on the floor to define a plane, but the point at the bottom of the fourth leg may not be in the same plane. When the chair tips so that this leg is on the floor, the plane defined by this leg and the two legs closest to it now lies in the plane of the floor; no; Three points define a plane, so the legs of the three-legged chair will always meet in the flat plane of the floor.
49. 6; The first two lines intersect at one point. The third line could intersect each of the first two lines. The fourth line can be drawn to intersect each of the first 3 lines. Then the total is $1 + 2 + 3 = 6$.
51.



ray

53.



rays

55. a. K, N **b.** *Sample answer:* plane JKL, plane JQN
c. J, K, L, M, N, P, Q
57. sometimes; The point may be on the line.
59. sometimes; The planes may not intersect.
61. sometimes; The points may be collinear.
63. sometimes; Lines in parallel planes do not intersect, and may not be parallel.

1.1 Maintaining Mathematical Proficiency *(p. 10)*

65. 8 **67.** 10 **69.** $x = 25$ **71.** $x = 22$

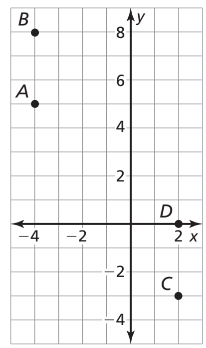
1.2 Vocabulary and Core Concept Check *(p. 16)*

1. $\overline{XY}$ represents the segment XY, while XY represents the distance between points X and Y (the length of $\overline{XY}$).

1.2 Monitoring Progress and Modeling with Mathematics *(pp. 16–18)*

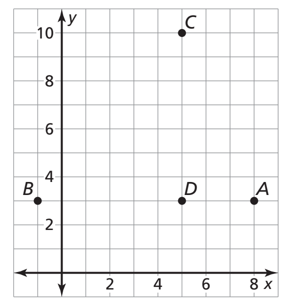
3. 3.5 cm **5.** 4.5 cm
7.

9.



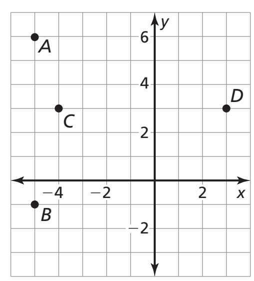
yes

11.



no

13.



yes

15. 22 **17.** 23 **19.** 24 **21.** 20
23. The absolute value should have been taken; $AB = |1 - 4.5| = 3.5$
25. $2\frac{1}{4}$ in., $1\frac{3}{4}$ in.; $\frac{1}{2}$ in.; $1\frac{2}{7}$
27. a. true; B is on $\overleftrightarrow{AC}$ between A and C.
b. false; B, C, and E are not collinear.
c. true; D is on $\overleftrightarrow{AH}$ between A and H.
d. false; C, E, and F are not collinear.
29. a. $3x + 6 = 21$; $x = 5$; $RS = 20$; $ST = 1$; $RT = 21$
b. $7x - 24 = 60$; $x = 12$; $RS = 20$; $ST = 40$; $RT = 60$
c. $2x + 3 = x + 10$; $x = 7$; $RS = 6$; $ST = 11$; $RT = 17$
d. $4x + 10 = 8x - 14$; $x = 6$; $RS = 15$; $ST = 19$; $RT = 34$

31. a. 64 ft **b.** about 0.24 min

c. You might walk slower if other people are in the hall.

33. 296.5 mi; If the round-trip distance is 647 miles, then the one-way distance is 323.5 miles. $323.5 - 27 = 296.5$

35. $|a - c| = |e - f|$; b and d are not used because when the x-values are the same, you subtract the y-values to find the length of the segment, and vice versa.

37. yes, no; $FC + CB = FB$, so $FB > CB$. $\overline{AC}$ and $\overline{DB}$ overlap but do not share an endpoint.

1.2 Maintaining Mathematical Proficiency *(p. 18)*

39. 5 **41.** $\frac{13}{2}$, or 6.5 **43.** $y = 9$ **45.** $x = 2$

1.3 Vocabulary and Core Concept Check *(p. 24)*

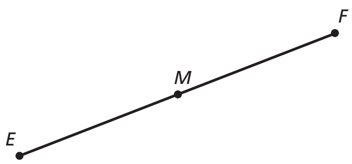
1. It bisects the segment.

1.3 Monitoring Progress and Modeling with Mathematics *(pp. 24–26)*

3. line k; 34 **5.** M; 44 **7.** M; 40 **9.** $\overrightarrow{MN}$; 32

11.

13.



15. (5, 2) **17.** $\left(1, \frac{9}{2}\right)$ **19.** (3, 12) **21.** (18, −9)

23. 10 **25.** $\sqrt{13}$, or about 3.6 **27.** $\sqrt{97}$, or about 9.8

29. 6.5

31. The square root should have been taken. $\sqrt{61} \approx 7.8$

33. about 6.7, about 6.3; no; $AB > CD$

35. a. To find the x-coordinate of the midpoint, add the x-coordinates of the endpoints, and divide by 2. To find the y-coordinate of the midpoint, add the y-coordinates of the endpoints, and divide by 2.

b. To find the x-coordinate of the other endpoint, multiply the x-coordinate of the midpoint by 2, and subtract the x-coordinate of the given endpoint. To find the y-coordinate of the other endpoint, multiply the y-coordinate of the midpoint by 2, and subtract the y-coordinate of the given endpoint.

37. a. about 10.4 m; about 9.2 m **b.** about 18.9 m

39. a. about 191 yd **b.** about 40 yd

c. about 1.5 min; $MR \approx 40$ yd, total distance $\approx 40 + 40 + 40 + 70 + 40 = 230$ yd, $\frac{230}{150} \approx 1.5$ min

41. $\left(\frac{a+b}{2}, c\right)$, $|b - a|$

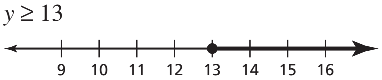
43. location D for lunch; The total distance traveled if you return home is $AM + AM + AB + AB$. The total distance traveled if you go to location D for lunch is $AB + DB + DB + AB$. Because $DB < AM$, the second option involves less traveling.

45. 13 cm

1.3 Maintaining Mathematical Proficiency *(p. 26)*

47. 26 ft, 30 ft^2 **49.** 36 yd, 60 yd^2

51. $y \geq 13$



53. $z \leq 48$

1.4 Vocabulary and Core Concept Check *(p. 34)*

1. $4s$

1.4 Monitoring Progress and Modeling with Mathematics *(pp. 34–36)*

3. quadrilateral; concave **5.** pentagon; convex

7. 22 units **9.** about 22.43 units **11.** about 16.93 units

13. 7.5 square units **15.** 9 square units

17. about 9.66 units **19.** about 12.17 units

21. 4 square units **23.** 6 square units

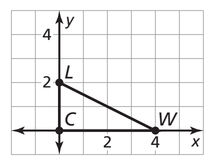
25. The length should be 5 units; $P = 2\ell + 2w = 2(5) + 2(3) = 16$; The perimeter is 16 units.

27. B

29. a. 4 square units; 16 square units; It is quadrupled.

b. yes; If you double the perimeter of the square, which is the same as doubling the side length, then the new area will be $2^2 = 4$ times as big.

31. a.



b. about 10.47 mi **c.** about 17.42 mi

33. a. y_1 and y_3 **b.** (0, 4), (4, 2), (2, −2)

c. about 15.27 units, 10 square units

35. a. 16 units, 16 square units

b. yes; The sides are all the same length because each one is the hypotenuse of a right triangle with legs that are each 2 units long. Because the slopes of the lines of each side are either 1 or −1, they are perpendicular.

c. about 11.31 units, 8 square units; It is half of the area of the larger square.

37. $x = 2$

1.4 Maintaining Mathematical Proficiency *(p. 36)*

39. $x = -1$ **41.** $x = 14$ **43.** $x = 1$

1.5 Vocabulary and Core Concept Check *(p. 43)*

1. congruent

1.5 Monitoring Progress and Modeling with Mathematics *(pp. 43–46)*

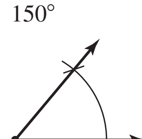
3. $\angle B$, $\angle ABC$, $\angle CBA$ **5.** $\angle 1$, $\angle K$, $\angle JKL$ (or $\angle LKJ$)

7. $\angle HMK$, $\angle KMN$, $\angle HMN$ **9.** 30°; acute

11. 85°; acute

13. The outer scale was used, but the inner scale should have been used because $\overrightarrow{OB}$ passes through 0° on the inner scale; 150°

15.

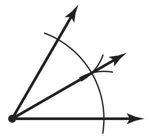


17. $\angle ADE$, $\angle BDC$, $\angle BCD$ **19.** 34°

21. 58° **23.** 42° **25.** 37°, 58° **27.** 77°, 103°

29. 32°, 58°

31.



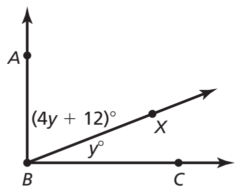
33. 63°, 126° **35.** 62°, 62° **37.** 44°, 44°, 88°

39. 65°, 65°, 130°

41. Subtract $m\angle CBD$ from $m\angle ABC$ to find $m\angle ABD$.

43. 40° **45.** 90°, 90°

47. **a.**



b. $4y + 12 + y = 92$, 76°, 16°

49. **a.** acute **b.** acute **c.** acute **d.** right

51. **a.** *Sample answer:* (1, 2) **b.** *Sample answer:* (0, 2)
c. *Sample answer:* (−2, 2) **d.** *Sample answer:* (−2, 0)

53. acute, right, or obtuse; The sum of the angles could be less than 90° (example: 30° + 20° = 50°), equal to 90° (example: 60° + 30° = 90°), or greater than 90° (example: 55° + 45° = 100°).

55. *Sample answer:* You draw a segment, ray, or line in the interior of an angle so that the two angles created are congruent to each other; Angle bisectors and segment bisectors can be segments, rays, or lines, but only a segment bisector can be a point. The two angles/segments created are congruent to each other, and their measures are each half the measure of the original angle/segment.

57. acute; It is likely that the angle with the horizontal is very small because levels are typically used when something appears to be horizontal but still needs to be checked.

1.5 Maintaining Mathematical Proficiency *(p. 46)*

59. $x = 32$ **61.** $x = 71$ **63.** $x = 12$ **65.** $x = 10$

1.6 Vocabulary and Core Concept Check *(p. 52)*

1. Adjacent angles share a common ray, and are next to each other. Vertical angles form two pairs of opposite rays, and are across from each other.

1.6 Monitoring Progress and Maintaining Mathematical Proficiency *(pp. 52–54)*

3. $\angle LJM$, $\angle MJN$ **5.** $\angle EGF$, $\angle NJP$ **7.** 67°

9. 102° **11.** $m\angle QRT = 47°$, $m\angle TRS = 133°$

13. $m\angle UVW = 12°$, $m\angle XYZ = 78°$ **15.** $\angle 1$ and $\angle 5$

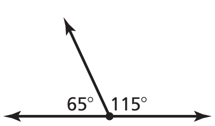
17. yes; The sides form two pairs of opposite rays.

19. 60°, 120° **21.** 9°, 81°

23. They do not share a common side, so they are not adjacent; $\angle 1$ and $\angle 2$ are adjacent.

25. 122° **27.** C

29.



31. B **33.** $x + (2x + 12) = 90$; 26° and 64°

35. $x + \left(\frac{2}{3}x - 15\right) = 180$; 117° and 63°

37. always; A linear pair forms a straight angle, which is 180°.

39. sometimes; This is possible if the lines are perpendicular.

41. always; 45 + 45 = 90

43. The measure of an obtuse angle is greater than 90°. So, you cannot add it to the measure of another angle and get 90°.

45. **a.** 50°, 40°, 140°
b. $\frac{1}{3}$; Because all 4 angles have supplements, the first paper can be any angle. Then there is a 1 in 3 chance of drawing its supplement.

47. yes; Because $m\angle KJL + x° = 90°$ and $m\angle MJN + x° = 90°$, it must be that $m\angle KJL + x° = m\angle MJN + x°$. Subtracting $x°$ from each side of the equation results in the measures being equal. So, the angles are congruent.

49. **a.** $y°$, $(180 - y)°$, $(180 - y)°$
b. They are always congruent; They are both supplementary to the same angle. So, their measures must be equal.

51. 37°, 53°; If two angles are complementary, then their sum is 90°. If x is one of the angles, then $(90 - x)$ is the complement. Write and solve the equation $90 = (x - (90 - x)) + 74$. The solution is $x = 53$.

1.6 Maintaining Mathematical Proficiency *(p. 54)*

53. never; Integers are positive or negative whole numbers. Irrational numbers are decimals that never terminate and never repeat.

55. never; The whole numbers are positive or zero.

57. always; The set of integers includes all natural numbers and their opposites (and zero).

59. sometimes; Irrational numbers can be positive or negative.

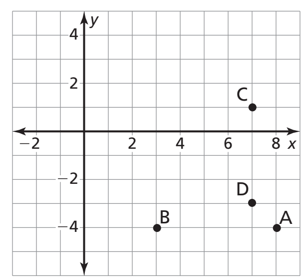
Chapter 1 Review *(pp. 56–58)*

1. *Sample answer:* plane XYN **2.** *Sample answer:* line g

3. *Sample answer:* line h **4.** *Sample answer:* $\overrightarrow{XZ}$, $\overrightarrow{YP}$

5. $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ **6.** P **7.** 41 **8.** 11

9.



no

10. $\left(\frac{1}{2}, \frac{13}{2}\right)$; about 7.1 **11.** $\left(\frac{13}{2}, -\frac{5}{2}\right)$; about 1.4

12. (−2, −3) **13.** 40 **14.** 20 units, 21 square units

15. about 23.9 units, 24.5 square units **16.** 49°, 28°

17. 88°, 23° **18.** 127° **19.** 78° **20.** 7°

21. 64° **22.** 124°

Chapter 2

Chapter 2 Maintaining Mathematical Proficiency *(p. 63)*

1. $a_n = 6n - 3$; $a_{50} = 297$ **2.** $a_n = 17n - 46$; $a_{50} = 804$

3. $a_n = 0.6n + 2.2$; $a_{50} = 32.2$

4. $a_n = \frac{1}{6}n + \frac{1}{6}$; $a_{50} = \frac{17}{2}$, or $8\frac{1}{2}$

5. $a_n = -4n + 30$; $a_{50} = -170$

6. $a_n = -6n + 14$; $a_{50} = -286$

7. $x = y - 5$ 8. $x = -4y + 3$ 9. $x = y - 3$

10. $x = \frac{y}{7}$ 11. $x = \frac{y - 6}{z + 4}$ 12. $x = \frac{z}{6y + 2}$

13. no; The sequence does not have a common difference.

2.1 Vocabulary and Core Concept Check *(p. 71)*

1. a conditional statement and its contrapositive, as well as the converse and inverse of a conditional statement

2.1 Monitoring Progress and Modeling with Mathematics *(pp. 71–74)*

3. If a polygon is a pentagon, then it has five sides.
5. If you run, then you are fast.
7. If $x = 2$, then $9x + 5 = 23$.
9. If you are in a band, then you play the drums.
11. If you are registered, then you are allowed to vote.
13. The sky is not blue. 15. The ball is pink.
17. conditional: If two angles are supplementary, then the measures of the angles sum to 180°; true

    converse: If the measures of two angles sum to 180°, then they are supplementary; true

    inverse: If the two angles are not supplementary, then their measures do not sum to 180°; true

    contrapositive: If the measures of two angles do not sum to 180°, then they are not supplementary; true
19. conditional: If you do your math homework, then you will do well on the test; false

    converse: If you do well on the test, then you did your math homework; false

    inverse: If you do not do your math homework, then you will not do well on the test; false

    contrapositive: If you do not do well on the test, then you did not do your math homework; false
21. conditional: If it does not snow, then I will run outside; false

    converse: If I run outside, then it is not snowing; true

    inverse: If it snows, then I will not run outside; true

    contrapositive: If I do not run outside, then it is snowing; false
23. conditional: If $3x - 7 = 20$, then $x = 9$; true

    converse: If $x = 9$, then $3x - 7 = 20$; true

    inverse: If $3x - 7 \neq 20$, then $x \neq 9$; true

    contrapositive: If $x \neq 9$, then $3x - 7 \neq 20$; true
25. true; By definition of right angle, the measure of the right angle shown is 90°.
27. true; If angles form a linear pair, then the sum of the measures of their angles is 180°.
29. A point is the midpoint of a segment, if and only if it is the point that divides the segment into two congruent segments.
31. Two angles are adjacent angles if and only if they share a common vertex and side, but have no common interior points.
33. A polygon has three sides if and only if it is a triangle.
35. An angle is a right angle if and only if it measures 90°.
37. Taking four English courses is a requirement regardless of how many courses the student takes total, and the courses do not have to be taken simultaneously; If students are in high school, then they will take four English courses before they graduate.
39.

p	q	$\sim p$	$\sim p \to q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

41.

p	q	$\sim p$	$\sim q$	$\sim p \to \sim q$	$\sim(\sim p \to \sim q)$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	F

43.

p	q	$\sim p$	$q \to \sim p$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	T

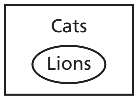
45. **a.** If a rock is igneous, then it is formed from the cooling of molten rock; If a rock is sedimentary, then it is formed from pieces of other rocks; If a rock is metamorphic, then it is formed by changing temperature, pressure, or chemistry.

    b. If a rock is formed from the cooling of molten rock, then it is igneous; true; All rocks formed from cooling molten rock are called igneous.

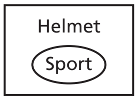
    If a rock is formed from pieces of other rocks, then it is sedimentary; true; All rocks formed from pieces or other rocks are called sedimentary.

    If a rock is formed by changing temperature, pressure, or chemistry, then it is metamorphic; true; All rocks formed by changing temperature, pressure, or chemistry are called metamorphic.

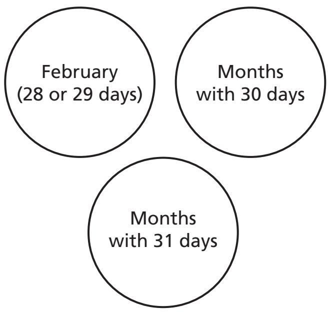
    c. *Sample answer:* If a rock is not sedimentary, then it was not formed from pieces of other rocks; This is the inverse of one of the conditional statements in part (a). So, the converse of this statement will be the contrapositive of the conditional statement. Because the contrapositive is equivalent to the conditional statement and the conditional statement was true, the contrapositive will also be true.
47. no; The contrapositive is equivalent to the original conditional statement. In order to write a conditional statement as a true biconditional statement, you must know that the converse (or inverse) is true.
49. If you tell the truth, then you don't have to remember anything.
51. If one is lucky, then a solitary fantasy can totally transform one million realities.
53. no; "If $x^2 - 10 = x + 2$, then $x = 4$" is a false statement because $x = -3$ is also possible. The converse, however, of the original conditional statement is true. In order for a biconditional statement to be true, both the conditional statement and its converse must be true.
55. A
57. If today is February 28, then tomorrow is March 1.

59. a. 

If you see a cat, then you went to the zoo to see a lion; The original statement is true, because a lion is a type of cat, but the converse is false, because you could see a cat without going to the zoo.

b. 

If you wear a helmet, then you play a sport; Both the original statement and the converse are false, because not all sports require helmets and sometimes helmets are worn for activities that are not considered a sport, such as construction work.

c. 

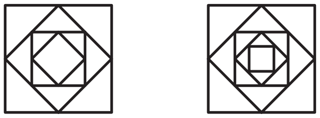
If this month is not February, then it has 31 days; The original statement is true, because February never has 31 days, but the converse is false, because a month that is not February could have 30 days.

61. *Sample answer:* If they are vegetarians, then they do not eat hamburgers.

63. *Sample answer:* Slogan: "This treadmill is a fat-burning machine!" Conditional statement: If you use this treadmill, then you will burn fat quickly.

2.1 Maintaining Mathematical Proficiency *(p. 74)*

65. add a square that connects the midpoints of the previously added square;



67. add 11; 56, 67 **69.** $1^2, 2^2, 3^2, \ldots$; 25, 36

2.2 Vocabulary and Core Concept Check *(p. 80)*

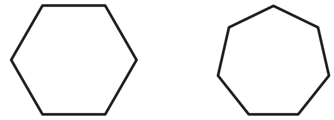
1. The prefix "Counter-" means "opposing." So, a counterexample opposes the truth of the statement.

2.2 Monitoring Progress and Modeling with Mathematics *(pp. 80–82)*

3. The absolute value of each number in the list is 1 greater than the absolute value of the previous number in the list, and the signs alternate from positive to negative; -6, 7

5. The list items are letters in backward alphabetical order; U, T

7. This is a sequence of regular polygons, each polygon having one more side than the previous polygon.



9. The product of any two even integers is an even integer. *Sample answer:* $-2(4) = -8$, $6(12) = 72$, $8(10) = 80$

11. The quotient of a number and its reciprocal is the square of that number. *Sample answer:* $9 \div \frac{1}{9} = 9 \bullet 9 = 9^2$, $\frac{2}{3} \div \frac{3}{2} = \frac{2}{3} \bullet \frac{2}{3} = \left(\frac{2}{3}\right)^2$, $\frac{1}{7} \div 7 = \frac{1}{7} \bullet \frac{1}{7} = \left(\frac{1}{7}\right)^2$

13. *Sample answer:* $1 \bullet 5 = 5$, $5 \not> 5$

15. They could both be right angles. Then, neither are acute.

17. You passed the class. **19.** not possible

21. not possible

23. If a figure is a rhombus, then the figure has two pairs of opposite sides that are parallel.

25. Law of Syllogism **27.** Law of Detachment

29. The sum of two odd integers is an even integer; Let m and n be integers. Then $(2m + 1)$ and $(2n + 1)$ are odd integers. $(2m + 1) + (2n + 1) = 2m + 2n + 2 = 2(m + n + 1)$; $2(m + n + 1)$ is divisible by 2 and is therefore an even integer.

31. inductive reasoning; The conjecture is based on the assumption that a pattern, observed in specific cases, will continue.

33. deductive reasoning; Laws of nature and the Law of Syllogism were used to draw the conclusion.

35. The Law of Detachment cannot be used because the hypothesis is not true; *Sample answer:* Using the Law of Detachment, because a square is a rectangle, you can conclude that a square has four sides.

37. Using inductive reasoning, we can make a conjecture that male tigers weigh more than female tigers because this was true in all of the specific cases listed in the table.

39. $n(n + 1)$ = the sum of first n positive even integers

41. Argument 2; This argument uses the Law of Detachment to say that when the hypothesis is met, the conclusion is true.

43. The value of y is 2 more than three times the value of x; $y = 3x + 2$; *Sample answer:* If $x = 10$, then $y = 3(10) + 2 = 32$; If $x = 72$, then $y = 3(72) + 2 = 218$.

45. a. true; Based on the Law of Syllogism, if you went camping at Yellowstone, and Yellowstone is in Wyoming, then you went camping in Wyoming.

b. false; When you go camping, you go canoeing, but even though your friend always goes camping when you do, he or she may not choose to go canoeing with you.

c. true; We know that if you go on a hike, your friend goes with you, and we know that you went on a hike. So, based on the Law of Detachment, your friend went on a hike.

d. false; We know that you and your friend went on a hike, but we do not know where. We just know that there is a 3-mile-long trail near where you are camping.

2.2 Maintaining Mathematical Proficiency *(p. 82)*

47. Segment Addition Postulate (Post. 1.2)

49. Ruler Postulate (Post. 1.1)

2.3 Vocabulary and Core Concept Check *(p. 87)*

1. three

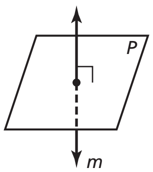
2.3 Monitoring Progress and Modeling with Mathematics *(pp. 87–88)*

3. Two Point Postulate (Post. 2.1)

5. *Sample answer:* Line q contains points J and K.

7. *Sample answer:* Through points K, H, and L, there is exactly one plane, which is plane M.

9.



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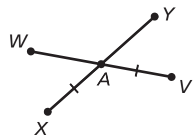


13. yes **15.** no **17.** yes **19.** yes

21. In order to determine that M is the midpoint of $\overline{AC}$ or $\overline{BD}$, the segments that would have to be marked as congruent are $\overline{AM}$ and $\overline{MC}$ or $\overline{DM}$ and $\overline{MB}$, respectively; Based on the diagram and markings, you can assume $\overline{AC}$ and $\overline{DB}$ intersect at point M, such that $\overline{AM} \cong \overline{MB}$ and $\overline{DM} \cong \overline{MC}$.

23. C, D, F, H **25.** Two-Point Postulate (Post. 2.1)

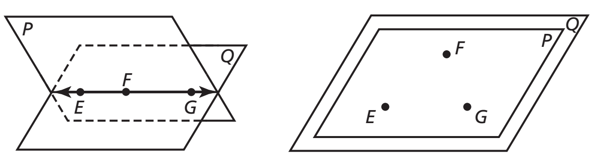
27. **a.** If there are two points, then there exists exactly one line that passes through them.

b. converse: If there exists exactly one line that passes through a given point or points, then there are two points; false; inverse: If there are not two points, then there is not exactly one line that passes through them; false; contrapositive: If there is not exactly one line that passes through a given point or points, then there are not two points; true

29. $<$

31. yes; For example, the ceiling and two walls of many rooms intersect in a point in the corner of the room.

33. Points E, F, and G must be collinear. They must be on the line that intersects plane P and plane Q; Points E, F, and G can be either collinear or not collinear.



2.3 Maintaining Mathematical Proficiency *(p. 88)*

35. $t = 2$; Addition Property of Equality

37. $x = 4$; Subtraction Property of Equality

2.4 Vocabulary and Core Concept Check *(p. 96)*

1. Reflexive Property of Equality

2.4 Monitoring Progress and Modeling with Mathematics *(p. 96–98)*

3. Subtraction Property of Equality; Addition Property of Equality; Division Property of Equality

5.

Equation	Explanation and Reason
$5x - 10 = -40$	Write the equation; Given
$5x = -30$	Add 10 to each side; Addition Property of Equality
$x = -6$	Divide each side by 5; Division Property of Equality

7.

Equation	Explanation and Reason
$2x - 8 = 6x - 20$	Write the equation; Given
$-4x - 8 = -20$	Subtract $6x$ from each side; Subtraction Property of Equality
$-4x = -12$	Add 8 to each side; Addition Property of Equality
$x = 3$	Divide each side by -4; Division Property of Equality

9.

Equation	Explanation and Reason
$5(3x - 20) = -10$	Write the equation; Given
$15x - 100 = -10$	Multiply; Distributive Property
$15x = 90$	Add 100 to each side; Addition Property of Equality
$x = 6$	Divide each side by 15; Division Property of Equality

11.

Equation	Explanation and Reason
$2(-x - 5) = 12$	Write the equation; Given
$-2x - 10 = 12$	Multiply; Distributive Property
$-2x = 22$	Add 10 to each side; Addition Property of Equality
$x = -11$	Divide each side by -2; Division Property of Equality

13.

Equation	Explanation and Reason
$4(5x - 9) = -2(x + 7)$	Write the equation; Given
$20x - 36 = -2x - 14$	Multiply on each side; Distributive Property
$22x - 36 = -14$	Add $2x$ to each side; Addition Property of Equality
$22x = 22$	Add 36 to each side; Addition Property of Equality
$x = 1$	Divide each side by 22; Division Property of Equality

15.

Equation	Explanation and Reason
$5x + y = 18$	Write the equation; Given
$y = -5x + 18$	Subtract $5x$ from each side; Subtraction Property of Equality

17.

Equation	Explanation and Reason
$2y + 0.5x = 16$	Write the equation; Given
$2y = -0.5x + 16$	Subtract $0.5x$ from each side; Subtraction Property of Equality
$y = -0.25x + 8$	Divide each side by 2; Division Property of Equality

19.

Equation	Explanation and Reason
$12 - 3y = 30x + 6$	Write the equation; Given
$-3y = 30x - 6$	Subtract 12 from each side; Subtraction Property of Equality
$y = -10x + 2$	Divide each side by -3; Division Property of Equality

21.

Equation	Explanation and Reason
$C = 2\pi r$	Write the equation; Given
$\frac{C}{2\pi} = r$	Divide each side by 2π; Division Property of Equality
$r = \frac{C}{2\pi}$	Rewrite the equation; Symmetric Property of Equality

23.

Equation	Explanation and Reason
$S = 180(n - 2)$	Write the equation; Given
$\frac{S}{180} = n - 2$	Divide each side by 180; Division Property of Equality
$\frac{S}{180} + 2 = n$	Add 2 to each side; Addition Property of Equality
$n = \frac{S}{180} + 2$	Rewrite the equation; Symmetric Property of Equality

25. Multiplication Property of Equality

27. Reflexive Property of Equality

29. Reflexive Property of Equality

31. Symmetric Property of Equality

33. $20 + CD$ **35.** $CD + EF$ **37.** $XY - GH$

39. $m\angle 1 = m\angle 3$

41. The Subtraction Property of Equality should be used to subtract x from each side of the equation in order to get the second step.

$7x = x + 24$	Given
$6x = 24$	Subtraction Property of Equality
$x = 4$	Division Property of Equality

43.

Equation	Explanation and Reason
$P = 2\ell + 2w$	Write the equation; Given
$P - 2w = 2\ell$	Subtract $2w$ from each side; Subtraction Property of Equality
$\frac{P - 2w}{2} = \ell$	Divide each side by 2; Division Property of Equality
$\ell = \frac{P - 2w}{2}$	Rewrite the equation; Symmetric Property of Equality

$\ell = 11$ m

45.

Equation	Explanation and Reason
$m\angle ABD = m\angle CBE$	Write the equation; Given
$m\angle ABD = m\angle 1 + m\angle 2$	Add measures of adjacent angles; Angle Addition Postulate (Post. 1.4)
$m\angle CBE = m\angle 2 + m\angle 3$	Add measures of adjacent angles; Angle Addition Postulate (Post. 1.4)
$m\angle ABD = m\angle 2 + m\angle 3$	Substitute $m\angle ABD$ for $m\angle CBE$; Substitution Property of Equality
$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Substitute $m\angle 1 + m\angle 2$ for $m\angle ABD$; Substitution Property of Equality
$m\angle 1 = m\angle 3$	Subtract $m\angle 2$ from each side; Subtraction Property of Equality

47. Transitive Property of Equality; Angle Addition Postulate (Post. 1.4); Transitive Property of Equality; $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$; Subtraction Property of Equality

49.

Equation	Explanation and Reason
$DC = BC, AD = AB$	Marked in diagram; Given
$AC = AC$	AC is equal to itself; Reflexive Property of Equality
$AC + AB + BC = AC + AB + BC$	Add $AB + BC$ to each side of $AC = AC$; Addition Property of Equality
$AC + AB + BC = AC + AD + DC$	Substitute AD for AB and DC for BC; Substitution Property of Equality

51. $ZY = XW = 9$ **53.** A, B, F

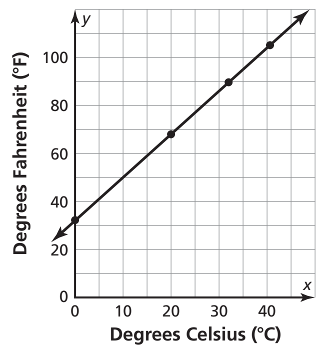
55. a.

Equation	Explanation and Reason
$C = \frac{5}{9}(F - 32)$	Write the equation; Given
$\frac{9}{5}C = F - 32$	Multiply each side by $\frac{9}{5}$; Multiplication Property of Equality
$\frac{9}{5}C + 32 = F$	Add 32 to each side; Addition Property of Equality
$F = \frac{9}{5}C + 32$	Rewrite the equation; Symmetric Property of Equality

b.

Degrees Celsius (°C)	Degrees Fahrenheit (°F)
0	32
20	68
32	89.6
41	105.8

c.



Yes, it is a linear function.

2.4 Maintaining Mathematical Proficiency *(p. 98)*

57. Segment Addition Postulate (Post. 1.2) **59.** Midpoint

2.5 Vocabulary and Core Concept Check *(p. 103)*

1. A postulate is a rule that is accepted to be true without proof, but a theorem is a statement that can be proven.

2.5 Monitoring Progress and Modeling with Mathematics *(pp. 103–104)*

3. Given; Addition Property of Equality; $PQ + QR = PR$; Transitive Property of Equality

5. Transitive Property of Segment Congruence (Thm. 2.1)

7. Symmetric Property of Angle Congruence (Thm. 2.2)

9. Symmetric Property of Segment Congruence (Thm. 2.1)

11.

STATEMENTS	REASONS
1. A segment exists with endpoints A and B.	**1.** Given
2. AB equals the length of the segment with endpoints A and B.	**2.** Ruler Postulate (Post. 1.1)
3. $AB = AB$	**3.** Reflexive Property of Equality
4. $\overline{AB} \cong \overline{AB}$	**4.** Definition of congruent segments

13.

STATEMENTS	REASONS
1. $\angle GFH \cong \angle GHF$	**1.** Given
2. $m\angle GFH = m\angle GHF$	**2.** Definition of congruent angles
3. $\angle EFG$ and $\angle GFH$ form a linear pair.	**3.** Given (diagram)
4. $\angle EFG$ and $\angle GFH$ are supplementary.	**4.** Definition of linear pair
5. $m\angle EFG + m\angle GFH = 180°$	**5.** Definition of supplementary angles
6. $m\angle EFG + m\angle GHF = 180°$	**6.** Substitution Property of Equality
7. $\angle EFG$ and $\angle GHF$ are supplementary.	**7.** Definition of supplementary angles

15. The Transitive Property of Segment Congruence (Thm. 2.1) should have been used; Because if $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$, then $\overline{MN} \cong \overline{PN}$ by the Transitive Property of Segment Congruence (Thm. 2.1).

17. equiangular; By the Transitive Property of Angle Congruence (Thm. 2.2), because $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, we know that $\angle 1 \cong \angle 3$. Because all three angles are congruent, the triangle is equiangular. (It is also equilateral and acute.)

19. The purpose of a proof is to ensure the truth of a statement with such certainty that the theorem or rule proved could be used as a justification in proving another statement or theorem. Because inductive reasoning relies on observations about patterns in specific cases, the pattern may not continue or may change. So, the ideas cannot be used to prove ideas for the general case.

21. **a.** It is a right angle.

b.

STATEMENTS	REASONS
1. $m\angle 1 + m\angle 1 + m\angle 2 + m\angle 2 = 180°$	**1.** Angle Addition Postulate (Post. 1.4)
2. $2(m\angle 1 + m\angle 2) = 180°$	**2.** Distributive Property
3. $m\angle 1 + m\angle 2 = 90°$	**3.** Division Property of Equality

23.

STATEMENTS	REASONS
1. $\overline{QR} \cong \overline{PQ}, \overline{RS} \cong \overline{PQ}$, $QR = 2x + 5$, $RS = 10 - 3x$	**1.** Given
2. $QR = PQ$, $RS = PQ$	**2.** Definition of congruent segments
3. $QR = RS$	**3.** Transitive Property of Equality
4. $2x + 5 = 10 - 3x$	**4.** Substitution Property of Equality
5. $5x + 5 = 10$	**5.** Addition Property of Equality
6. $5x = 5$	**6.** Subtraction Property of Equality
7. $x = 1$	**7.** Division Property of Equality

2.5 Maintaining Mathematical Proficiency *(p. 104)*

25. 33°

2.6 Vocabulary and Core Concept Check *(p. 111)*

1. All right angles have the same measure, 90°, and angles with the same measure are congruent.

2.6 Monitoring Progress and Modeling with Mathematics *(pp. 111–114)*

3. $\angle MSN \cong \angle PSQ$ by definition because they have the same measure; $\angle MSP \cong \angle PSR$ by the Right Angles Congruence Theorem (Thm. 2.3). They form a linear pair, which means they are supplementary by the Linear Pair Postulate (Post. 2.8), and because one is a right angle, so is the other by the Subtraction Property of Equality; $\angle NSP \cong \angle QSR$ by the Congruent Complements Theorem (Thm. 2.5) because they are complementary to congruent angles.

5. $\angle GML \cong \angle HMJ$ and $\angle GMH \cong \angle LMJ$ by the Vertical Angles Congruence Theorem (Thm. 2.6); $\angle GMK \cong \angle JMK$ by the Right Angles Congruence Theorem (Thm. 2.3). They form a linear pair, which means they are supplementary by the Linear Pair Postulate (Post. 2.8), and because one is a right angle, so is the other by the Subtraction Property of Equality.

7. $m\angle 2 = 37°$; $m\angle 3 = 143°$; $m\angle 4 = 37°$

9. $m\angle 1 = 146°$; $m\angle 3 = 146°$; $m\angle 4 = 34°$

11. $x = 11$; $y = 17$ **13.** $x = 4$; $y = 9$

15. The expressions should have been set equal to each other because they come from vertical angles;

$$(13x + 45)° = (19x + 3)°$$
$$-6x + 45 = 3$$
$$-6x = -42$$
$$x = 7$$

17. Transitive Property of Angle Congruence (Thm. 2.2); Transitive Property of Angle Congruence (Thm. 2.2)

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 3$	**1.** Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	**2.** Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\angle 2 \cong \angle 3$	**3.** Transitive Property of Angle Congruence (Thm. 2.2)
4. $\angle 2 \cong \angle 4$	**4.** Transitive Property of Angle Congruence (Thm. 2.2)

19. complementary; $m\angle 1 + m\angle 3$; Transitive Property of Equality; $m\angle 2 = m\angle 3$; congruent angles

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are complementary. $\angle 1$ and $\angle 3$ are complementary.	**1.** Given
2. $m\angle 1 + m\angle 2 = 90°$, $m\angle 1 + m\angle 3 = 90°$	**2.** Definition of complementary angles
3. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$	**3.** Transitive Property of Equality
4. $m\angle 2 = m\angle 3$	**4.** Subtraction Property of Equality
5. $\angle 2 \cong \angle 3$	**5.** Definition of congruent angles

21. Because $\angle QRS$ and $\angle PSR$ are supplementary, $m\angle QRS + m\angle PSR = 180°$ by the definition of supplementary angles. $\angle QRL$ and $\angle QRS$ form a linear pair and by definition are supplementary, which means that $m\angle QRL + m\angle QRS = 180°$. So, by the Transitive Property of Equality, $m\angle QRS + m\angle PSR = m\angle QRL + m\angle QRS$, and by the Subtraction Property of Equality, $m\angle PSR = m\angle QRL$. So, by definition of congruent angles, $\angle PSR \cong \angle QRL$, and by the Symmetric Property of Angle Congruence (Thm. 2.2), $\angle QRL \cong \angle PSR$.

23.

STATEMENTS	REASONS
1. $\angle AEB \cong \angle DEC$	**1.** Given
2. $m\angle AEB = m\angle DEC$	**2.** Definition of congruent angles
3. $m\angle DEB = m\angle DEC + m\angle BEC$	**3.** Angle Addition Postulate (Post. 1.4)
4. $m\angle DEB = m\angle AEB + m\angle BEC$	**4.** Substitution Property of Equality
5. $m\angle AEC = m\angle AEB + m\angle BEC$	**5.** Angle Addition Postulate (Post. 1.4)
6. $m\angle AEC = m\angle DEB$	**6.** Transitive Property of Equality
7. $\angle AEC \cong \angle DEB$	**7.** Definition of congruent angles

25. Your friend is correct; $\angle 1$ and $\angle 4$ are not vertical angles because they do not form two pairs of opposite rays. So, the Vertical Angles Congruence Theorem (Thm. 2.6) does not apply.

27. no; The converse would be: "If two angles are supplementary, then they are a linear pair." This is false because angles can be supplementary without being adjacent.

29. 50°; 130°; 50°; 130°

2.6 Maintaining Mathematical Proficiency *(p. 114)*

31. *Sample answer:* B, I, and C

33. *Sample answer:* plane ABC and plane BCG

35. *Sample answer:* A, B, and C

Chapter 2 Review *(pp. 116–118)*

1. conditional: If two lines intersect, then their intersection is a point.

converse: If two lines intersect in a point, then they are intersecting lines.

inverse: If two lines do not intersect, then they do not intersect in a point.

contrapositive: If two lines do not intersect in a point, then they are not intersecting lines.

biconditional: Two lines intersect if and only if their intersection is a point.

2. conditional: If $4x + 9 = 21$, then $x = 3$.

converse: If $x = 3$, then $4x + 9 = 21$.

inverse: If $4x + 9 \neq 21$, then $x \neq 3$.

contrapositive: If $x \neq 3$, then $4x + 9 \neq 21$.

biconditional: $4x + 9 = 21$ if and only if $x = 3$.

3. conditional: If angles are supplementary, then they sum to 180°.

converse: If angles sum to 180°, then they are supplementary.

inverse: If angles are not supplementary, then they do not sum to 180°.

iontrapositive: If angles do not sum to 180°, then they are not supplementary.

conditional: Angles are supplementary if and only if they sum to 180°.

4. conditional: If an angle is a right angle, then it measures 90°.

converse: If an angle measures 90°, then it is a right angle.

inverse: If an angle is not a right angle, then it does not measure 90°.

contrapositive: If an angle does not measure 90°, then it is not a right angle.

biconditional: An angle is a right angle if and only if it measures 90°.

5. The difference of any two odd integers is an even integer.

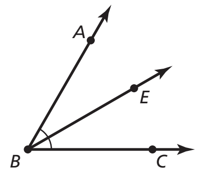
6. The product of an even and an odd integer is an even integer.

7. $m\angle B = 90°$ **8.** If $4x = 12$, then $2x = 6$. **9.** yes

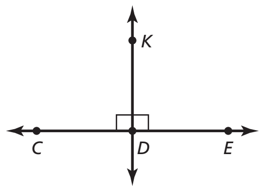
10. yes **11.** no **12.** no

Selected Answers

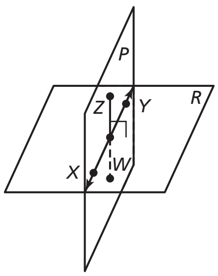
13. *Sample answer:*



14. *Sample answer:*



15. *Sample answer:*



16.

Equation	Explanation and Reason
$-9x - 21 = -20x - 87$	Write the equation; Given
$11x - 21 = -87$	Add $20x$ to each side; Addition Property of Equality
$11x = -66$	Add 21 to each side; Addition Property of Equality
$x = -6$	Divide each side by 11; Division Property of Equality

17.

Equation	Explanation and Reason
$15x + 22 = 7x + 62$	Write the equation; Given
$8x + 22 = 62$	Subtract $7x$ from each side; Subtraction Property of Equality
$8x = 40$	Subtract 22 from each side; Subtraction Property of Equality
$x = 5$	Divide each side by 8; Division Property of Equality

18.

Equation	Explanation and Reason
$3(2x + 9) = 30$	Write the equation; Given
$6x + 27 = 30$	Multiply; Distributive Property
$6x = 3$	Subtract 27 from each side; Subtraction Property of Equality
$x = \frac{1}{2}$	Divide each side by 6; Division Property of Equality

19.

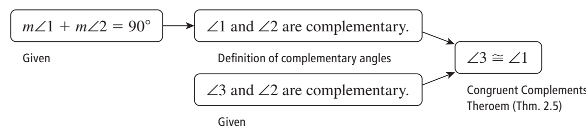
Equation	Explanation and Reason
$5x + 2(2x - 23) = -154$	Write the equation; Given
$5x + 4x - 46 = -154$	Multiply; Distributive Property
$9x - 46 = -154$	Combine like terms; Simplify.
$9x = -108$	Add 46 to each side; Addition Property of Equality
$x = -12$	Divide each side by 9; Division Property of Equality

20. Transitive Property of Equality
21. Reflexive Property of Equality
22. Symmetric Property of Angle Congruence (Thm. 2.2)
23. Reflexive Property of Angle Congruence (Thm. 2.2)
24. Transitive Property of Equality

25.

STATEMENTS	REASONS
1. An angle with vertex A exists.	1. Given
2. $m\angle A$ equals the measure of the angle with vertex A.	2. Protractor Postulate (Post. 1.3)
3. $m\angle A = m\angle A$	3. Reflexive Property of Equality
4. $\angle A \cong \angle A$	4. Definition of congruent angles

26.



Chapter 3

Chapter 3 Maintaining Mathematical Proficiency *(p. 123)*

1. $m = -\frac{3}{4}$ 2. $m = 3$ 3. $m = 0$
4. $y = -3x + 19$ 5. $y = -2x + 2$ 6. $y = 4x + 9$
7. $y = \frac{1}{2}x - 5$ 8. $y = -\frac{1}{4}x - 7$ 9. $y = \frac{2}{3}x + 9$
10. When calculating the slope of a horizontal line, the vertical change is zero. This is the numerator of the fraction, and zero divided by any number is zero. When calculating the slope of a vertical line, the horizontal change is zero. This is the denominator of the fraction, and any number divided by zero is undefined.

3.1 Vocabulary and Core Concept Check *(p. 129)*

1. skew

3.1 Monitoring Progress and Modeling with Mathematics *(pp. 129–130)*

3. $\overleftrightarrow{AB}$ 5. $\overleftrightarrow{BF}$ 7. $\overleftrightarrow{MK}$ and $\overleftrightarrow{LS}$
9. no; They are intersecting lines.
11. $\angle 1$ and $\angle 5$; $\angle 2$ and $\angle 6$; $\angle 3$ and $\angle 7$; $\angle 4$ and $\angle 8$
13. $\angle 1$ and $\angle 8$; $\angle 2$ and $\angle 7$ 15. corresponding
17. consecutive interior
19. Lines that do not intersect could also be skew; If two coplanar lines do not intersect, then they are parallel.
21. **a.** true; The floor is level with the horizontal just like the ground.
    b. false; The lines intersect the plane of the ground, so they intersect certain lines of that plane.
    c. true; The balusters appear to be vertical, and the floor of the tree house is horizontal. So, they are perpendicular.
23. yes; If the original two lines are parallel, and the transversal is perpendicular to both lines, then all eight angles are right angles.
25. $\angle HJG$, $\angle CFJ$ 27. $\angle CFD$, $\angle HJC$
29. no; They can both be in a plane that is slanted with respect to the horizontal.

3.1 Maintaining Mathematical Proficiency *(p. 130)*

31. $m\angle 1 = 21°$, $m\angle 3 = 21°$, $m\angle 4 = 159°$

3.2 Vocabulary and Core Concept Check *(p. 135)*

1. Both theorems refer to two pairs of congruent angles that are formed when two parallel lines are cut by a transversal, and the angles that are congruent are on opposite sides of the transversal. However with the Alternate Interior Angles Theorem (Thm. 3.2), the congruent angles lie between the parallel lines, and with the Alternate Exterior Angles Theorem (Thm. 3.3), the congruent angles lie outside the parallel lines.

3.2 Monitoring Progress and Modeling with Mathematics *(pp. 135–136)*

3. $m\angle 1 = 117°$ by Vertical Angles Congruence Theorem (Thm. 2.6); $m\angle 2 = 117°$ by Alternate Exterior Angles Theorem (Thm. 3.3)

5. $m\angle 1 = 122°$ by Alternate Interior Angles Theorem (Thm. 3.2); $m\angle 2 = 58°$ by Consecutive Interior Angles Theorem (Thm. 3.4)

7. 64; $2x° = 128°$

$$x = 64$$

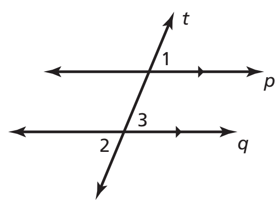
9. 12; $m\angle 5 = 65°$

$$\begin{aligned} 65° + (11x - 17)° &= 180° \\ 11x + 48 &= 180 \\ 11x &= 132 \\ x &= 12 \end{aligned}$$

11. $m\angle 1 = 100°$, $m\angle 2 = 80°$, $m\angle 3 = 100°$; Because the 80° angle is a consecutive interior angle with both $\angle 1$ and $\angle 3$, they are supplementary by the Consecutive Interior Angles Theorem (Thm. 3.4). Because $\angle 1$ and $\angle 2$ are consecutive interior angles, they are supplementary by the Consecutive Interior Angles Theorem (Thm. 3.4).

13. In order to use the Corresponding Angles Theorem (Thm. 3.1), the angles need to be formed by two parallel lines cut by a transversal, but none of the lines in this diagram appear to be parallel; $\angle 9$ and $\angle 10$ are corresponding angles.

15.



STATEMENTS	REASONS
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Corresponding Angles Theorem (Thm. 3.1)
3. $\angle 3 \cong \angle 2$	**3.** Vertical Angles Congruence Theorem (Thm. 2.6)
4. $\angle 1 \cong \angle 2$	**4.** Transitive Property of Congruence

17. $m\angle 2 = 104°$; Because the trees form parallel lines, and the rope is a transversal, the 76° angle and $\angle 2$ are consecutive interior angles. So, they are supplementary by the Consecutive Interior Angles Theorem (Thm. 3.4).

19. yes; If two parallel lines are cut by a perpendicular transversal, then the consecutive interior angles will both be right angles.

21.

$$\begin{aligned} 19x - 10 &= 180 \\ 14x + 2y - 10 &= 180; \quad x = 10, y = 25 \end{aligned}$$

23. no; In order to make the shot, you must hit the cue ball so that $m\angle 1 = 65°$. The angle that is complementary to $\angle 1$ must have a measure of 25° because this angle is alternate interior angles with the angle formed by the path of the cue ball and the vertical line drawn.

3.2 Maintaining Mathematical Proficiency *(p. 136)*

25. If two angles are congruent, then they are vertical angles; false

27. If two angles are supplementary, then they form a linear pair; false

3.3 Vocabulary and Core Concept Check *(p. 142)*

1. corresponding, alternate interior, alternate exterior

3.3 Monitoring Progress and Modeling with Mathematics *(pp. 142–144)*

3. $x = 40$; Lines m and n are parallel when the marked corresponding angles are congruent.

$$\begin{aligned} 3x° &= 120° \\ x &= 40 \end{aligned}$$

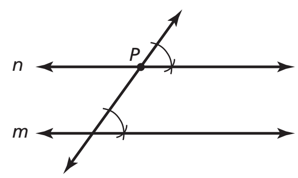
5. $x = 15$; Lines m and n are parallel when the marked consecutive interior angles are supplementary.

$$\begin{aligned} (3x - 15)° + 150° &= 180° \\ 3x + 135 &= 180 \\ 3x &= 45 \\ x &= 15 \end{aligned}$$

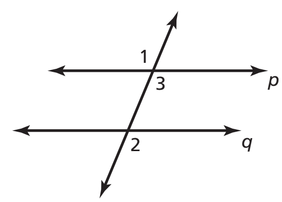
7. $x = 60$; Lines m and n are parallel when the marked consecutive interior angles are supplementary.

$$\begin{aligned} 2x° + x° &= 180° \\ 3x &= 180 \\ x &= 60 \end{aligned}$$

9.



11.



It is given that $\angle 1 \cong \angle 2$. By the Vertical Angles Congruence Theorem (Thm. 2.6), $\angle 1 \cong \angle 3$. Then by the Transitive Property of Congruence (Thm. 2.2), $\angle 2 \cong \angle 3$. So, by the Corresponding Angles Converse (Thm. 3.5), $p \parallel q$.

13. yes; Alternate Interior Angles Converse (Thm. 3.6)

15. no **17.** no

19. This diagram shows that vertical angles are always congruent. Lines a and b are not parallel unless $x = y$, and we cannot assume that they are equal.

21. yes; $m\angle DEB = 180° - 123° = 57°$ by the Linear Pair Postulate (Post. 2.8). So, by definition, a pair of corresponding angles are congruent, which means that $\overleftrightarrow{AC} \parallel \overleftrightarrow{DF}$ by the Corresponding Angles Converse (Thm. 3.5).

23. cannot be determined; The marked angles are vertical angles. You do not know anything about the angles formed by the intersection of $\overleftrightarrow{DF}$ and $\overleftrightarrow{BE}$.

25. yes; E. 20th Ave. is parallel to E. 19th Ave. by the Corresponding Angles Converse (Thm. 3.5). E. 19th Ave. is parallel to E. 18th Ave. by the Alternate Exterior Angles Converse (Thm. 3.7). E. 18th Ave. is parallel to E. 17th Ave. by the Alternate Interior Angles Converse (Thm. 3.6). So, they are all parallel to each other by the Transitive Property of Parallel Lines (Thm. 3.9).

27. The two angles marked as 108° are corresponding angles. Because they have the same measure, they are congruent to each other. So, $m \parallel n$ by the Corresponding Angles Converse (Thm. 3.5).

29. A, B, C, D; The Corresponding Angles Converse (Thm. 3.5) can be used because the angle marked at the intersection of line m and the transversal is vertical angles with, and therefore congruent to, an angle that is corresponding with the other marked angle. The Alternate Interior Angles Converse (Thm. 3.6) can be used because the angles that are marked as congruent are alternate interior angles. The Alternate Exterior Angles Converse (Thm. 3.7) can be used because the angles that are vertical with, and therefore congruent to, the marked angles are alternate exterior angles. The Consecutive Interior Angles Converse (Thm. 3.8) can be used because each of the marked angles forms a linear pair with, and is therefore supplementary to, an angle that is a consecutive interior angles with the other marked angle.

31. two; *Sample answer:* $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 7$, $\angle 3 \cong \angle 6$, $\angle 4$ and $\angle 7$ are supplementary.

33.

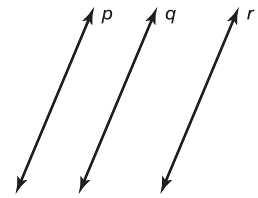
STATEMENTS	REASONS
1. $m\angle 1 = 115°$, $m\angle 2 = 65°$	**1.** Given
2. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 2$	**2.** Reflexive Property of Equality
3. $m\angle 1 + m\angle 2 = 115° + 65°$	**3.** Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = 180°$	**4.** Simplify.
5. $\angle 1$ and $\angle 2$ are supplementary.	**5.** Definition of supplementary angles
6. $m \parallel n$	**6.** Consecutive Interior Angles Converse (Thm 3.8)

35.

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	**1.** Given
2. $\angle 2 \cong \angle 3$	**2.** Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\angle 1 \cong \angle 3$	**3.** Transitive Property of Congruence
4. $\angle 1 \cong \angle 4$	**4.** Transitive Property of Congruence
5. $\overline{AB} \parallel \overline{CD}$	**5.** Alternate Interior Angles Converse (Thm. 3.6)

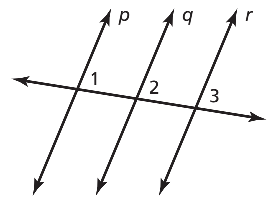
37. no; Based on the diagram $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ by the Alternate Interior Angles Converse (Thm. 3.6), but you cannot be sure that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$.

39. **a.**



b. Given: $p \parallel q$, $q \parallel r$
Prove: $p \parallel r$

c.



STATEMENTS	REASONS
1. $p \parallel q$, $q \parallel r$	**1.** Given
2. $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$	**2.** Corresponding Angles Theorem (Thm. 3.1)
3. $\angle 1 \cong \angle 3$	**3.** Transitive Property of Congruence
4. $p \parallel r$	**4.** Corresponding Angles Converse (Thm. 3.5)

3.3 Maintaining Mathematical Proficiency *(p. 144)*

41. about 6.71 **43.** 13

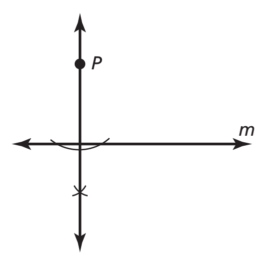
3.4 Vocabulary and Core Concept Check *(p. 152)*

1. midpoint, right

3.4 Monitoring Progress and Modeling with Mathematics *(pp. 152–154)*

3. about 3.2 units

5.



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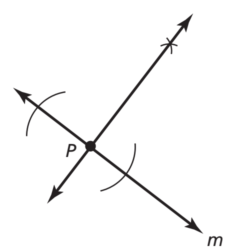
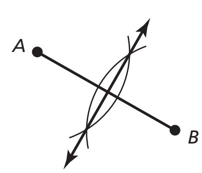
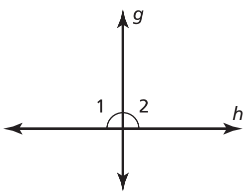


9.



11. In order to claim parallel lines by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12), *both* lines must be marked as perpendicular to the transversal; Lines x and z are perpendicular.

13. 

Because $\angle 1 \cong \angle 2$ by definition, $m\angle 1 = m\angle 2$. Also, by the Linear Pair Postulate (Post. 2.8), $m\angle 1 + m\angle 2 = 180°$. Then, by the Substitution Property of Equality, $m\angle 1 + m\angle 1 = 180°$, and $2(m\angle 1) = 180°$ by the Distributive Property. So, by the Division Property of Equality, $m\angle 1 = 90°$. Finally, $g \perp h$ by the definition of perpendicular lines.

15.

STATEMENTS	REASONS
1. $a \perp b$	**1.** Given
2. $\angle 1$ is a right angle.	**2.** Definition of perpendicular lines
3. $\angle 1 \cong \angle 4$	**3.** Vertical Angles Congruence Theorem (Thm. 2.6)
4. $m\angle 1 = 90°$	**4.** Definition of right angle
5. $m\angle 4 = 90°$	**5.** Transitive Property of Equality
6. $\angle 1$ and $\angle 2$ are a linear pair.	**6.** Definition of linear pair
7. $\angle 1$ and $\angle 2$ are supplementary.	**7.** Linear Pair Postulate (Post. 2.8)
8. $m\angle 1 + m\angle 2 = 180°$	**8.** Definition of supplementary angles
9. $90° + m\angle 2 = 180°$	**9.** Substitution Property of Equality
10. $m\angle 2 = 90°$	**10.** Subtraction Property of Equality
11. $\angle 2 \cong \angle 3$	**11.** Vertical Angles Congruence Theorem (Thm. 2.6)
12. $m\angle 3 = 90°$	**12.** Transitive Property of Equality
13. $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles.	**13.** Definition of right angle

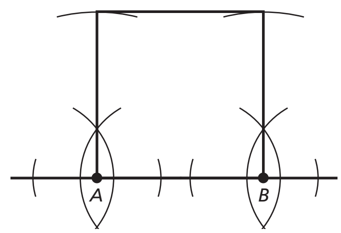
17. none; The only thing that can be concluded in this diagram is that $v \perp y$. In order to say that lines are parallel, you need to know something about both of the intersections between the transversal and the two lines.

19. $m \parallel n$, Because $m \perp q$ and $n \perp q$, lines m and n are parallel by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12). The other lines may or may not be parallel.

21. $n \parallel p$; Because $k \perp n$ and $k \perp p$, lines n and p are parallel by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12).

23. $m\angle 1 = 90°$, $m\angle 2 = 60°$, $m\angle 3 = 30°$, $m\angle 4 = 20°$, $m\angle 5 = 90°$;
$m\angle 1 = 90°$, because it is marked as a right angle.
$m\angle 2 = 90° - 30° = 60°$, because it is complementary to the 30° angle.
$m\angle 3 = 30°$, because it is vertical angles with, and therefore congruent to, the 30° angle.
$m\angle 4 = 90° - (30° + 40°) = 20°$, because it forms a right angle together with $\angle 3$ and the 40° angle.
$m\angle 5 = 90°$, because it is vertical angles with, and therefore congruent to, $\angle 1$.

25. $x = 8$ **27.** A, C, D, E

29. 

31. rectangle

33. Find the length of the segment that is perpendicular to the plane and that has one endpoint on the given point and one endpoint on the plane; You can find the distance from a line to a plane only if the line is parallel to the plane. Then you can pick any point on the line and find the distance from that point to the plane. If a line is not parallel to a plane, then the distance from the line to the plane is not defined because it would be different for each point on the line.

3.4 Maintaining Mathematical Proficiency *(p. 154)*

35. $-\frac{2}{3}$ **37.** 3 **39.** $m = -\frac{1}{2}$; $b = 7$

41. $m = -8$; $b = -6$

3.5 Vocabulary and Core Concept Check *(p. 160)*

1. directed

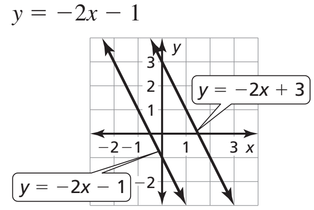
3.5 Monitoring Progress and Modeling with Mathematics *(pp. 160–162)*

3. $P(7, -0.4)$ **5.** $P(-1.5, -1.5)$ **7.** $a \parallel c$, $b \perp d$

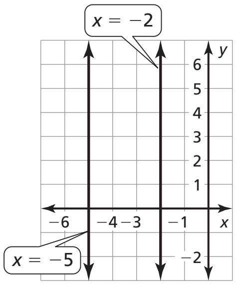
9. perpendicular; Because $m_1 \bullet m_2 = \left(\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1$, lines 1 and 2 are perpendicular by the Slopes of Perpendicular Lines Theorem (Thm. 3.14).

11. perpendicular; Because $m_1 \bullet m_2 = 1(-1) = -1$, lines 1 and 2 are perpendicular by the Slopes of Perpendicular Lines Theorem (Thm. 3.14).

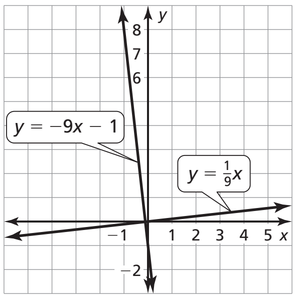
13. $y = -2x - 1$



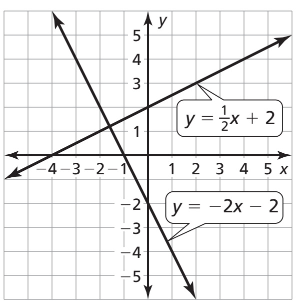
15. $x = -2$



17. $y = \frac{1}{9}x$



19. $y = \frac{1}{2}x + 2$



21. about 3.2 units **23.** about 5.4 units

25. Because the slopes are opposites but not reciprocals, their product does not equal -1. Lines 1 and 2 are neither parallel nor perpendicular.

27. $(0, 1)$; $y = 2x + 1$ **29.** $(3, 0)$; $y = \frac{3}{2}x - \frac{9}{2}$

31. $\left(-\frac{11}{5}, -\frac{6}{5}\right)$

33. no; $m_{\overline{LM}} = \frac{2}{5}$, $m_{\overline{LN}} = -\frac{7}{4}$, and $m_{\overline{MN}} = 9$. None of these can pair up to make a product of -1, so none of the segments are perpendicular.

35. $y = \frac{3}{2}x - 1$

37. $m < -1$; The slope of a line perpendicular to ℓ must be the opposite reciprocal of the slope of line ℓ. So, it must be negative, and have an absolute value greater than 1.

39. It will be the same point.

41. **a.** no solution; The lines do not intersect, so they are parallel.

b. $(7, -4)$; The lines intersect in one point.

c. infinitely many solutions; The lines are the same line.

43. $k = 4$

45. Using points $A(3, 2)$ and $B(6, 8)$ find the coordinates of point P that lies beyond point B along $\overrightarrow{AB}$ so that the ratio of AB to BP is 3 to 2. In order to keep the ratio, $\frac{AB}{BP} = \frac{3}{2}$, solve this ratio for BP to get $BP = \frac{2}{3}AB$. Next, find the rise and run from point A to point B. Leave the slope in terms of rise and run and do not simplify. $m_{\overline{AB}} = \frac{8-2}{6-3} = \frac{6}{3} = \frac{\text{rise}}{\text{run}}$. Add $\frac{2}{3}$ of the *run* to the x-coordinate of B, which is $\frac{2}{3} \cdot 3 + 6 = 8$. Add $\frac{2}{3}$ of the *rise* to the y-coordinate of B, which is $\frac{2}{3} \cdot 6 + 8 = 12$. So, the coordinates of P are $(8, 12)$.

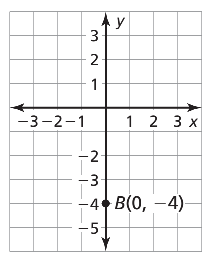
47. If lines x and y are perpendicular to line z, then by the Slopes of Perpendicular Lines Theorem (Thm. 3.14), $m_x \cdot m_z = -1$ and $m_y \cdot m_z = -1$. By the Transitive Property of Equality, $m_x \cdot m_z = m_y \cdot m_z$, and by the Division Property of Equality $m_x = m_y$. Therefore, by the Slopes of Parallel Lines Theorem (Thm. 3.13), $x \parallel y$.

49. If lines x and y are vertical lines and they are cut by any horizontal transversal, z, then $x \perp z$ and $y \perp z$ by Theorem 3.14. Therefore, $x \parallel y$ by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12).

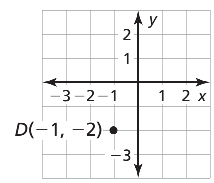
51. By definition, the x-axis is perpendicular to the y-axis. Let m be a horizontal line, and let n be a vertical line. Because any two horizontal lines are parallel, m is parallel to the x-axis. Because any two vertical lines are parallel, n is parallel to the y-axis. By the Perpendicular Transversal Theorem (Thm. 3.11), n is perpendicular to the x-axis. Then, by the Perpendicular Transversal Theorem (Thm. 3.11), n is perpendicular to m.

3.5 Maintaining Mathematical Proficiency *(p. 162)*

53.



55.



57.

x	-2	-1	0	1	2
$y = x - \frac{3}{4}$	$-\frac{11}{4}$	$-\frac{7}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{5}{4}$

Chapter 3 Review *(pp. 164–166)*

1. $\overleftrightarrow{NR}$, $\overleftrightarrow{MR}$, $\overleftrightarrow{LQ}$, $\overleftrightarrow{PQ}$ **2.** $\overleftrightarrow{LM}$, $\overleftrightarrow{JK}$, $\overleftrightarrow{NP}$

3. $\overleftrightarrow{JM}$, $\overleftrightarrow{KL}$, $\overleftrightarrow{KP}$, $\overleftrightarrow{JN}$ **4.** plane JKP **5.** $x = 145$, $y = 35$

6. $x = 13$, $y = 132$ **7.** $x = 61$, $y = 29$

8. $x = 14$, $y = 17$ **9.** $x = 107$ **10.** $x = 133$

11. $x = 32$ **12.** $x = 23$

13. $x \parallel y$; Because $x \perp z$ and $y \perp z$, lines x and y are parallel by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12).

14. none; The only thing that can be concluded in this diagram is that $x \perp z$ and $w \perp y$. In order to say that lines are parallel, you need to know something about *both* of the intersections between the two lines and a transversal.

15. $\ell \parallel m \parallel n$, $a \parallel b$; Because $a \perp n$ and $b \perp n$, lines a and b are parallel by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12). Because $m \perp a$ and $n \perp a$, lines m and n are parallel by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12). Because $\ell \perp b$ and $n \perp b$, lines ℓ and n are parallel by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12). Because $\ell \parallel n$ and $m \parallel n$, lines ℓ and m are parallel by the Transitive Property of Parallel Lines (Thm. 3.9).

16. $a \parallel b$; Because $a \perp n$ and $b \perp n$, lines a and b are parallel by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12).

17. $y = -x - 1$ 18. $y = \frac{1}{2}x + 8$ 19. $y = 3x - 6$

20. $y = \frac{1}{3}x - 2$ 21. $y = \frac{1}{2}x - 4$ 22. $y = 2x + 3$

23. $y = -\frac{1}{4}x + 4$ 24. $y = -7x - 2$

25. about 2.1 units 26. about 2.7 units

Chapter 4

Chapter 4 Maintaining Mathematical Proficiency *(p. 171)*

1. reflection 2. rotation 3. dilation

4. translation

5. no; $\frac{12}{14} = \frac{6}{7} \neq \frac{5}{7}$, The sides are not proportional.

6. yes; The corresponding angles are congruent and the corresponding side lengths are proportional.

7. yes; The corresponding angles are congruent and the corresponding side lengths are proportional.

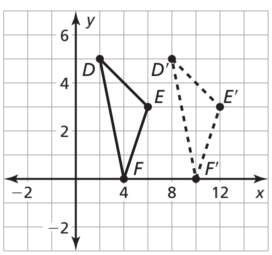
8. no; Squares have four right angles, so the corresponding angles are always congruent. Because all four sides are congruent, the corresponding sides will always be proportional.

4.1 Vocabulary and Core Concept Check *(p. 178)*

1. $\triangle ABC$ is the preimage, and $\triangle A'B'C'$ is the image.

4.1 Monitoring Progress and Modeling with Mathematics *(pp. 178–180)*

3. $\overrightarrow{CD}$, $\langle 7, -3 \rangle$

5.


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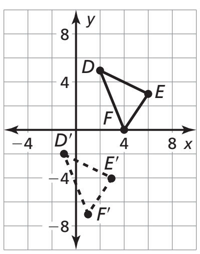
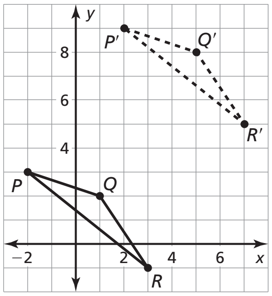


9. $\langle 3, -5 \rangle$ 11. $(x, y) \rightarrow (x - 5, y + 2)$

13. $A'(-6, 10)$ 15. $C(5, -14)$

17.


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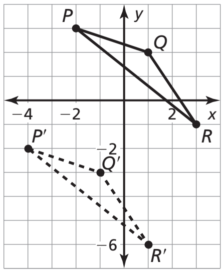
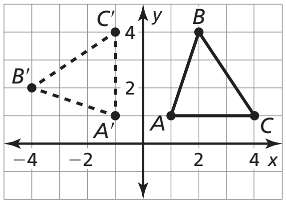
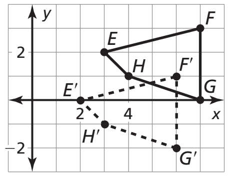


21.


23. translation: $(x, y) \rightarrow (x + 5, y + 1)$, translation: $(x, y) \rightarrow (x - 5, y - 5)$

25. The quadrilateral should have been translated left and down;



27. a. The amoeba moves right 5 squares and down 4 squares.
b. about 12.8 mm c. about 0.52 mm/sec

29. $r = 100$, $s = 8$, $t = 5$, $w = 54$

31. $E'(-3, -4)$, $F'(-2, -5)$, $G'(0, -1)$

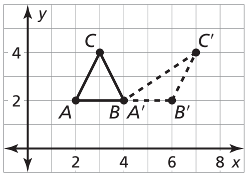
33. $(x, y) \rightarrow (x - m, y - n)$; You must go back the same number of units in the opposite direction.

35. If a rigid motion is used to transform figure A to figure A', then by definition of rigid motion, every part of figure A is congruent to its corresponding part of figure A'. If another rigid motion is used to transform figure A' to figure A'', then by definition of rigid motion, every part of figure A' is congruent to its corresponding part of figure A''. So, by the Transitive Property of Congruence, every part of figure A is congruent to its corresponding part of figure A''. So by definition of rigid motion, the composition of two (or more) rigid motions is a rigid motion.

37. Draw a rectangle. Then draw a translation of the rectangle. Next, connect each vertex of the preimage with the corresponding vertex in the image. Finally, make the hidden lines dashed.

39. yes; According to the definition of translation, the segments connecting corresponding vertices will be congruent and parallel. Also, because a translation is a rigid motion, $\overline{GH} \cong \overline{G'H'}$. So, the resulting figure is a parallelogram.

41. no; Because the value of y changes, you are not adding the same amount to each x-value.



4.1 Maintaining Mathematical Proficiency *(p. 180)*

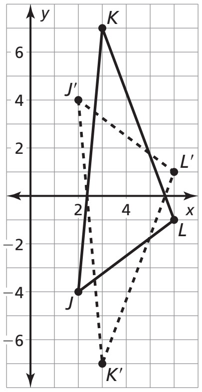
43. yes 45. no 47. x 49. $6x - 12$

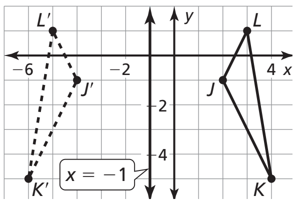
4.2 Vocabulary and Core Concept Check *(p. 186)*

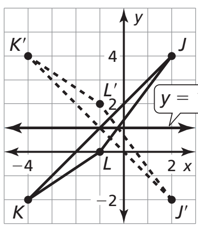
1. translation and reflection

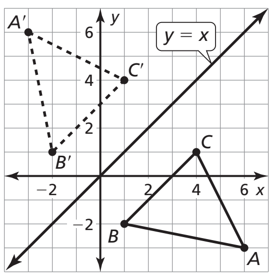
4.2 Monitoring Progress and Modeling with Mathematics *(pp. 186–188)*

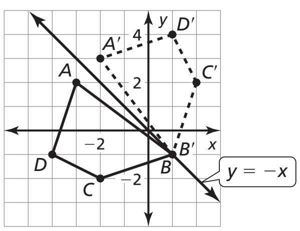
3. y-axis 5. neither

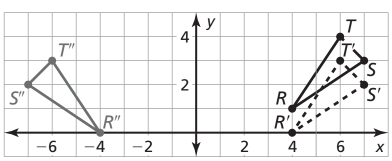
7. 

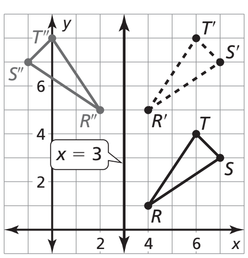
9. 

11. 

13. 

15. 

17. 

19. 

21. 1 23. 0

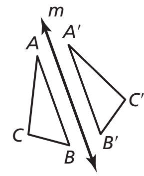
25. a. none b. 

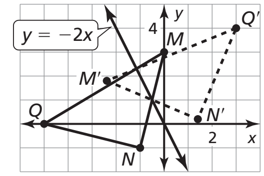
 c. 

 d. none

27. Reflect H in line n to obtain H'. Then draw $\overline{JH'}$. Label the intersection of JH' and n as K. Because JH' is the shortest distance between J and H' and $HK = H'K$, park at point K.

29. $C(5, 0)$ 31. $C(-4, 0)$ 33. $y = -3x - 4$

35. 

37. 

39. $y = x + 1$

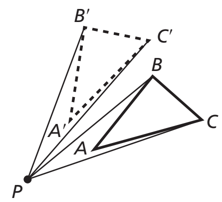
4.2 Maintaining Mathematical Proficiency *(p. 188)*

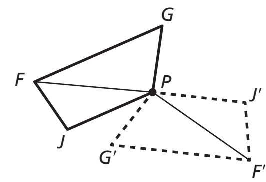
41. 130° 43. 160° 45. 30° 47. 180° 49. 50°

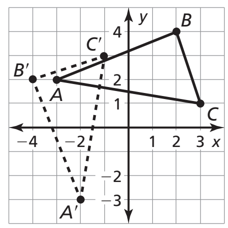
4.3 Vocabulary and Core Concept Check *(p. 194)*

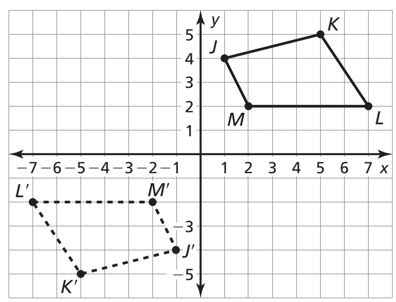
1. 270°

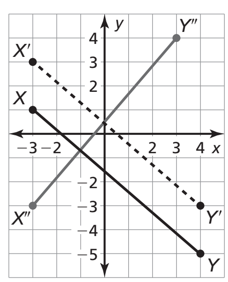
4.3 Monitoring Progress and Modeling with Mathematics *(pp. 194–196)*

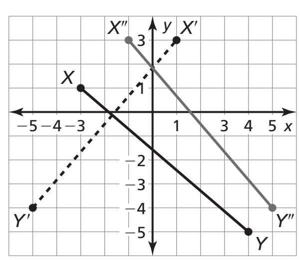
3. 

5. 

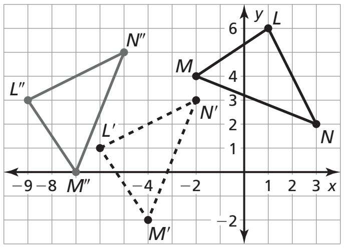
7. 

9. 

11. 

13. 

15.



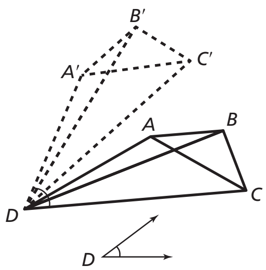
17. yes; Rotations of 90° and 180° about the center map the figure onto itself.

19. yes; Rotations of 45°, 90°, 135°, and 180° about the center map the figure onto itself.

21. F **23.** D, G

25. The rule for a 270° rotation, $(x, y) \rightarrow (y, -x)$, should have been used instead of the rule for a reflection in the x-axis; $C(-1, 1) \rightarrow C'(1, 1)$, $D(2, 3) \rightarrow D'(3, -2)$

27.



29. **a.** 90°: $y = -\frac{1}{2}x + \frac{3}{2}$, 180°: $y = 2x + 3$, 270°: $y = -\frac{1}{2}x - \frac{3}{2}$, 360°; $y = 2x - 3$; The slope of the line rotated 90° is the opposite reciprocal of the slope of the preimage, and the y-intercept is equal to the x-intercept of the preimage. The slope of the line rotated 180° is equal to the slope of the preimage, and the y-intercepts of the image and preimage are opposites. The slope of the line rotated 270° is the opposite reciprocal of the slope of the preimage, and the y-intercept is the opposite of the x-intercept of the preimage. The equation of the line rotated 360° is the same as the equation of the preimage.

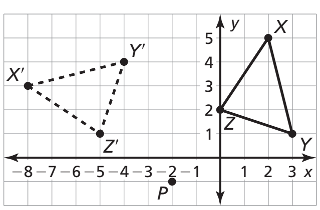
b. yes; Because the coordinates of every point change in the same way with each rotation, the relationships described will be true for an equation with any slope and y-intercept.

31. 2

33. yes; *Sample answer:* A rectangle (that is not a square) is one example of a figure that has 180° rotational symmetry, but not 90° rotational symmetry.

35. **a.** 15°, $n = 12$ **b.** 30°, $n = 6$

37.



39. (2, 120°); (2, 210°); (2, 300°); The radius remains the same. The angle increases in conjunction with the rotation.

4.3 Maintaining Mathematical Proficiency *(p. 196)*

41. $\angle A$ and $\angle J$, $\angle B$ and $\angle K$, $\angle C$ and $\angle L$, $\angle D$ and $\angle M$; $\overline{AB}$ and $\overline{JK}$, $\overline{BC}$ and $\overline{KL}$, $\overline{CD}$ and $\overline{LM}$, $\overline{DA}$ and $\overline{MJ}$

4.4 Vocabulary and Core Concept Check *(p. 204)*

1. congruent

4.4 Monitoring Progress and Modeling with Mathematics *(pp. 204–206)*

3. $\triangle HJK \cong \triangle QRS$, $\square DEFG \cong \square LMNP$; $\triangle HJK$ is a 90° rotation of $\triangle QRS$. $\square DEFG$ is a translation 7 units right and 3 units down of $\square LMNP$.

5. *Sample answer:* 180° rotation about the origin followed by a translation 5 units left and 1 unit down

7. yes; $\triangle TUV$ is a translation 4 units right of $\triangle QRS$. So, $\triangle TUV \cong \triangle QRS$.

9. no; M and N are translated 2 units right of their corresponding vertices, L and K, but P is translated only 1 unit right of its corresponding vertex, J. So, this is not a rigid motion.

11. $\triangle A''B''C''$ **13.** 5.2 in. **15.** 110°

17. A translation 5 units right and a reflection in the x-axis should have been used; $\triangle ABC$ is mapped to $\triangle A'B'C'$ by a translation 5 units right, followed by a reflection in the x-axis.

19. 42° **21.** 90°

23. Reflect the figure in two parallel lines instead of translating the figure; The third line of reflection is perpendicular to the parallel lines.

25. never; Congruence transformations are rigid motions.

27. sometimes; Reflecting in $y = x$ then $y = x$ is not a rotation. Reflecting in the y-axis then x-axis is a rotation of 180°.

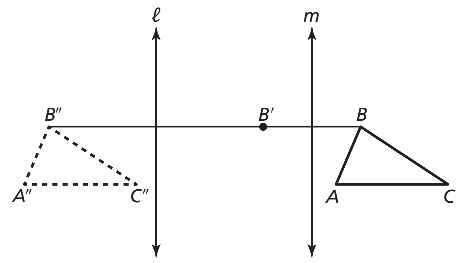
29. no; The image on the screen is larger.

31.

STATEMENTS	REASONS
1. A reflection in line ℓ maps $\overline{JK}$ to $\overline{J'K'}$, a reflection in line m maps $\overline{J'K'}$ to $\overline{J''K''}$, and $\ell \parallel m$.	**1.** Given
2. If $\overline{KK''}$ intersects line ℓ at L and line m at M, then L is the perpendicular bisector of $\overline{KK'}$, and M is the perpendicular bisector of $\overline{K'K''}$.	**2.** Definition of reflection
3. $\overline{KK'}$ is perpendicular to ℓ and m, and $KL = LK'$ and $K'M = MK''$.	**3.** Definition of perpendicular bisector
4. If d is the distance between ℓ and m, then $d = LM$.	**4.** Ruler Postulate (Post. 1.1)
5. $LM = LK' + K'M$ and $KK'' = KL + LK' + K'M + MK''$	**5.** Segment Addition Postulate (Post. 1.2)
6. $KK'' = LK' + LK' + K'M + K'M$	**6.** Substitution Property of Equality
7. $KK'' = 2(LK' + K'M)$	**7.** Distributive Property
8. $KK'' = 2(LM)$	**8.** Substitution Property of Equality
9. $KK'' = 2d$	**9.** Transitive Property of Equality

33. the second classmate that says it is a 180° rotation;
reflections: $P(1, 3) \rightarrow P'(-1, 3) \rightarrow P''(-1, -3)$ and $Q(3, 2) \rightarrow Q'(-3, 2) \rightarrow Q''(-3, -2)$
translation: $P(1, 3) \rightarrow (1 - 4, 3 - 5) \rightarrow (-3, -2)$ and $Q(3, 2) \rightarrow (3 - 4, 2 - 5) \rightarrow Q''(-1, -3)$
180° rotation $P(1, 3) \rightarrow (-1, -3)$ and $Q(3, 2) \rightarrow (-3, -2)$

35.



4.4 Maintaining Mathematical Proficiency *(p. 206)*

37. $x = -2$ **39.** $b = 6$ **41.** $n = -7.7$ **43.** 25%

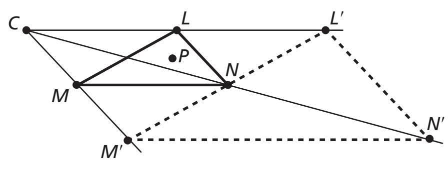
4.5 Vocabulary and Core Concept Check *(p. 212)*

1. $P'(kx, ky)$

4.5 Monitoring Progress and Modeling with Mathematics *(pp. 212–214)*

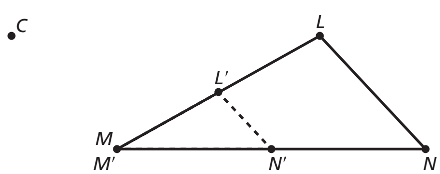
3. $\frac{3}{7}$; reduction **5.** $\frac{3}{5}$; reduction

7.



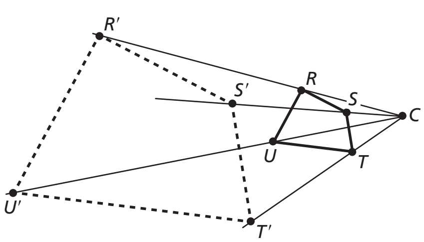
Not drawn to scale.

9.



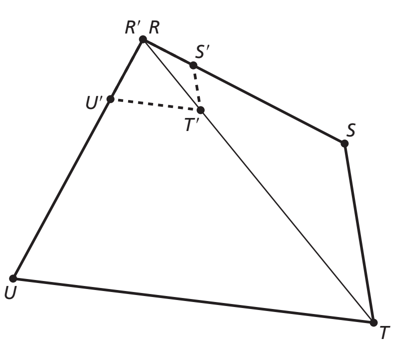
Not drawn to scale.

11.



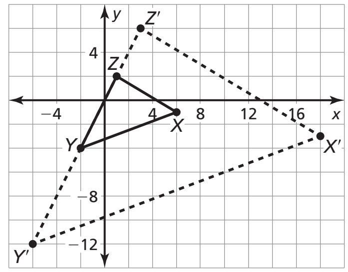
Not drawn to scale.

13.

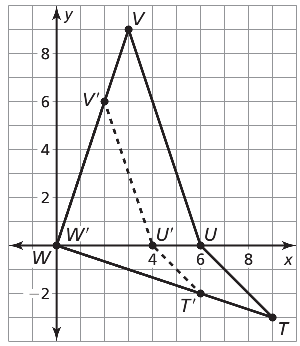


Not drawn to scale.

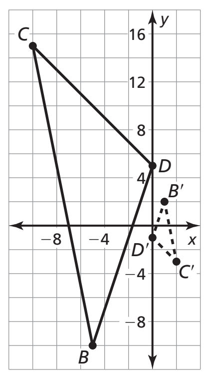
15.



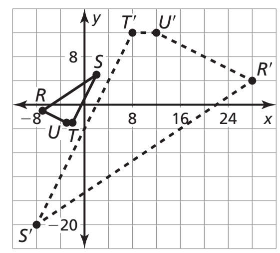
17.



19.



21.



23. The scale factor should be calculated by finding $\frac{CP'}{CP}$, not $\frac{CP}{CP'}$; $k = \frac{3}{12} = \frac{1}{4}$

25. $k = \frac{5}{3}$; $x = 21$ **27.** $k = \frac{2}{3}$; $y = 3$ **29.** $k = 2$

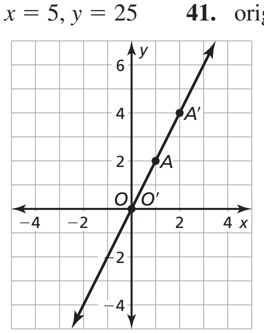
31. 300 mm **33.** 940 mm

35. grasshopper, honey bee, and monarch butterfly; The scale factor for these three is $k = \frac{15}{2}$. The scale factor for the black beetle is $k = 7$.

37. no; The scale factor for the shorter sides is $\frac{8}{4} = 2$, but the scale factor for the longer sides is $\frac{10}{6} = \frac{5}{3}$. The scale factor for both sides has to be the same or the picture will be distorted.

39. $x = 5$, $y = 25$ **41.** original **43.** original

45.

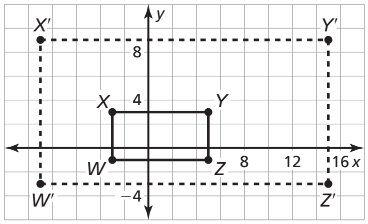


a. $O'A' = 2(OA)$ **b.** $\overleftrightarrow{O'A'}$ coincides with $\overleftrightarrow{OA}$.

47. $k = \frac{1}{16}$

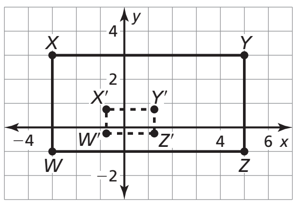
49. **a.** $P = 24$ units, $A = 32$ square units

b.



$P = 72$ units, $A = 288$ square units; The perimeter of the dilated rectangle is three times the perimeter of the original rectangle. The area of the dilated rectangle is nine times the area of the original rectangle.

c.



$P = 6$ units, $A = 2$ square units; The perimeter of the dilated rectangle is $\frac{1}{4}$ the perimeter of the original rectangle. The area of the dilated rectangle is $\frac{1}{16}$ the area of the original rectangle.

d. The perimeter changes by a factor of k. The area changes by a factor of k^2.

51. $A'(4, 4)$, $B'(4, 12)$, $C'(10, 4)$

4.5 Maintaining Mathematical Proficiency *(p. 214)*

53. $A'(1, 2)$, $B'(-1, 7)$, $C'(-4, 8)$

55. $A'(0, -1)$, $B'(-2, 4)$, $C'(-5, 5)$

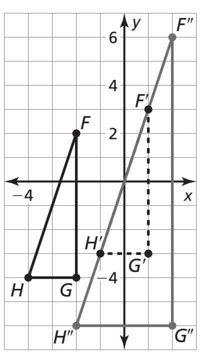
57. $A'(-1, 0)$, $B'(-3, 5)$, $C'(-6, 6)$

4.6 Vocabulary and Core Concept Check *(p. 219)*

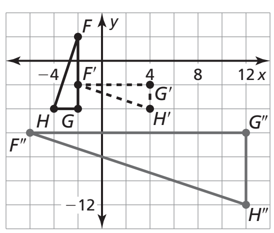
1. Congruent figures have the same size and shape. Similar figures have the same shape, but not necessarily the same size.

4.6 Monitoring Progress and Modeling with Mathematics *(pp. 219–220)*

3.



5.

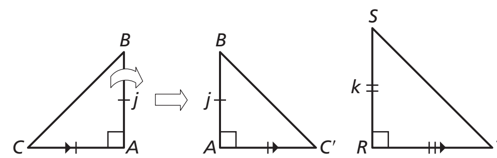


7. *Sample answer:* translation 1 unit down and 1 unit right followed by a dilation with center at $E(2, -3)$ and a scale factor of 2

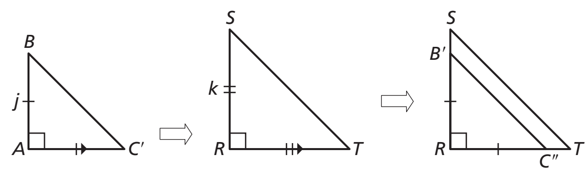
9. yes; $\triangle ABC$ can be mapped to $\triangle DEF$ by a dilation with center at the origin and a scale factor of $\frac{1}{3}$ followed by a translation of 2 units left and 3 units up.

11. no; The scale factor from $\overline{HI}$ to $\overline{JL}$ is $\frac{2}{3}$, but the scale factor from $\overline{GH}$ to $\overline{KL}$ is $\frac{5}{6}$.

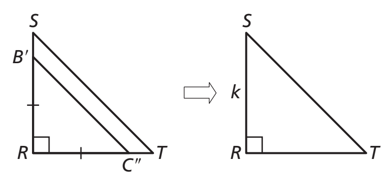
13. Reflect $\triangle ABC$ in $\overleftrightarrow{AB}$. Because reflections preserve side lengths and angle measures, the image of $\triangle ABC$, $\triangle ABC'$, is a right isosceles triangle with leg length j. Also because $\overleftrightarrow{AC} \perp \overleftrightarrow{BA}$, point C' is on $\overleftrightarrow{AC}$. So, $\overline{AC'}$ is parallel to $\overline{RT}$.



Then translate $\triangle ABC'$ so that point A maps to point R. Because translations map segments to parallel segments and $\overline{AC'} \parallel \overline{RT}$, the image of $\overline{AC'}$ lies on $\overline{RT}$.



Because translations preserve side lengths and angle measures, the image of $\triangle ABC'$, $\triangle RB'C''$, is a right isosceles triangle with leg length j. Because $\angle B'RC''$ and $\angle SRT$ are right angles, they are congruent. When $\overrightarrow{RC''}$ coincides with $\overrightarrow{RT}$, $\overrightarrow{RB'}$ coincides with $\overrightarrow{RS}$. So, $\overline{RB'}$ lies on $\overline{RS}$. Next, dilate $\triangle RB'C''$ using center of dilation R. Choose the scale factor to be the ratio of the side lengths of $\triangle RST$ and $\triangle RB'C''$, which is $\frac{k}{j}$.

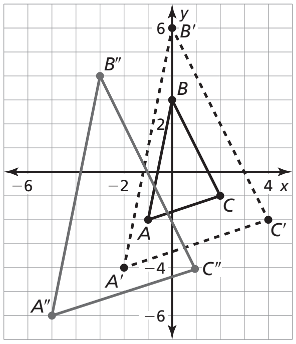


The dilation maps $\overline{RC''}$ to $\overline{RT}$ and $\overline{RB'}$ to $\overline{RS}$ because the images of $\overline{RC''}$ and $\overline{RB'}$ have side length $\frac{k}{j}(j) = k$ and the segments $\overline{RC''}$ and $\overline{RB'}$ lie on lines passing through the center of dilation. So, the dilation maps C'' to T and B' to S. A similarity transformation maps $\triangle ABC$ to $\triangle RST$. So, $\triangle ABC$ is similar to $\triangle RST$.

15. yes; The stop sign sticker can be mapped to the regular-sized stop sign by translating the sticker to the left until the centers match, and then dilating the sticker with a scale factor of 3.15. Because there is a similarity transformation that maps one stop sign to the other, the sticker is similar to the regular-sized stop sign.

17. no; The scale factor is 6 for both dimensions. So, the enlarged banner is proportional to the smaller one.

19. *Sample answer:*



$\triangle A''B''C''$ can be mapped to $\triangle ABC$ by a translation 3 units right and 2 units up, followed by a dilation with center at the origin and a scale factor of $\frac{1}{2}$.

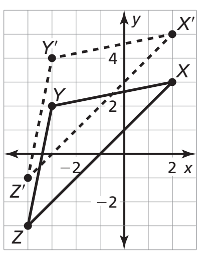
21. $J(-8, 0)$, $K(-8, 12)$, $L(-4, 12)$, $M(-4, 0)$; $J''(-9, -4)$, $K''(-9, 14)$, $L''(-3, 14)$, $M''(-3, -4)$; yes; A similarity transformation mapped quadrilateral $JKLM$ to quadrilateral $J''K''L''M''$.

4.6 Maintaining Mathematical Proficiency *(p. 220)*

23. obtuse **25.** acute

Chapter 4 Review *(pp. 222–224)*

1.



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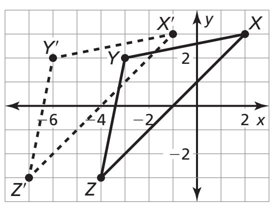

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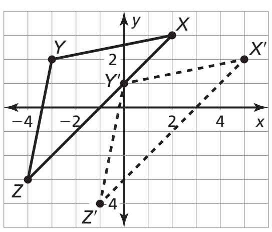

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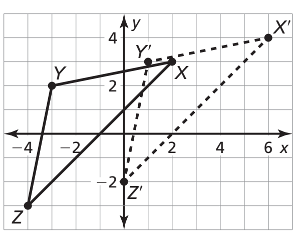

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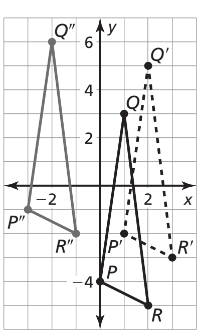

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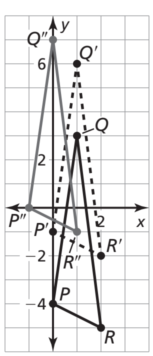

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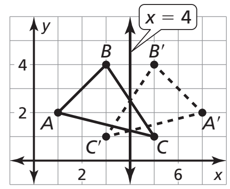

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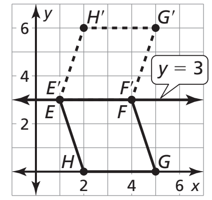
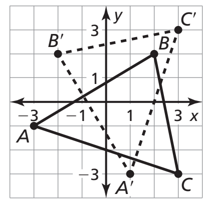


9. 2

10.



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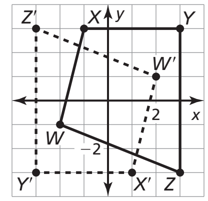

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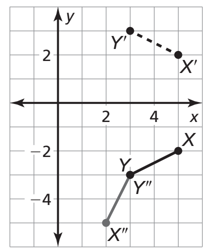


13. yes; Rotations of 60°, 120°, and 180° about the center map the figure onto itself.

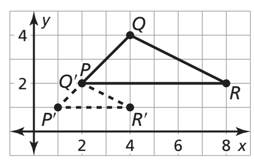
14. yes; Rotations of 72° and 144° about the center map the figure onto itself.

15. *Sample answer:* reflection in the y-axis followed by a translation 3 units down

16. *Sample answer:* 180° rotation about the origin followed by a reflection in the line $x = 2$

17. translation; rotation

18.



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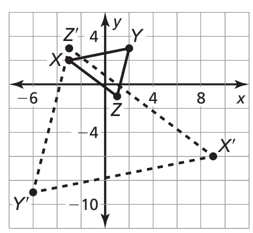


20. 1.9 cm

21. *Sample answer:* reflection in the line $x = -1$ followed by a dilation with center $(-3, 0)$ and $k = 3$

22. *Sample answer:* dilation with center at the origin and $k = \frac{1}{2}$, followed by a reflection in the line $y = x$

23. *Sample answer:* 270° rotation about the origin followed by a dilation with center at the origin and $k = 2$

Chapter 5

Chapter 5 Maintaining Mathematical Proficiency (p. 229)

1. $M(-2, 4)$; about 7.2 units **2.** $M(6, 2)$; 10 units

3. $M\left(\frac{7}{2}, -1\right)$; about 9.2 units **4.** $x = -3$ **5.** $t = 2$

6. $p = 3$ **7.** $w = 2$ **8.** $x = \frac{1}{3}$ **9.** $z = -\frac{3}{4}$

10. yes; The length can be found using the Pythagorean Theorem.

5.1 Vocabulary and Core Concept Check (p. 236)

1. no; By the Corollary to the Triangle Sum Theorem (Cor. 5.1), the acute angles of a right triangle are complementary. Because their measures have to add up to 90°, neither angle could have a measure greater than 90°.

5.1 Monitoring Progress and Modeling with Mathematics (pp. 236–238)

3. right isosceles **5.** obtuse scalene **7.** isosceles; right

9. scalene; not right **11.** 71°; acute **13.** 52°; right

15. 139° **17.** 114° **19.** 36°, 54° **21.** 37°, 53°

23. 15°, 75° **25.** 16.5°, 73.5°

27. The sum of the measures of the angles should be 180°;

$$115^\circ + 39^\circ + m\angle 1 = 180^\circ$$
$$154^\circ + m\angle 1 = 180^\circ$$
$$m\angle 1 = 26^\circ$$

29. 50° **31.** 50° **33.** 40° **35.** 90°

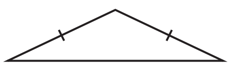
37. acute scalene

39. You could make another bend 6 inches from the first bend and leave the last side 8 inches long, or you could make another bend 7 inches from the first bend and then the last side will also be 7 inches long.

41.

STATEMENTS	REASONS
1. $\triangle ABC$ is a right triangle.	**1.** Given
2. $\angle C$ is a right angle.	**2.** Given (marked in diagram)
3. $m\angle C = 90^\circ$	**3.** Definition of a right angle
4. $m\angle A + m\angle B + m\angle C = 180^\circ$	**4.** Triangle Sum Theorem (Thm. 5.1)
5. $m\angle A + m\angle B + 90^\circ = 180^\circ$	**5.** Substitution Property of Equality
6. $m\angle A + m\angle B = 90^\circ$	**6.** Subtraction Property of Equality
7. $\angle A$ and $\angle B$ are complementary.	**7.** Definition of complementary angles

43. yes; no



An obtuse equilateral triangle is not possible, because when two sides form an obtuse angle the third side that connects them must be longer than the other two.

45. **a.** $x = 8, x = 9$ **b.** one $(x = 4)$

47. A, B, F **49.** $x = 43, y = 32$ **51.** $x = 85, y = 65$

53.

STATEMENTS	REASONS
1. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	**1.** Given (marked in diagram)
2. $\angle ACD$ and $\angle 5$ form a linear pair.	**2.** Definition of linear pair
3. $m\angle ACD + m\angle 5 = 180^\circ$	**3.** Linear Pair Postulate (Post. 2.8)
4. $m\angle 3 + m\angle 4 = m\angle ACD$	**4.** Angle Addition Postulate (Post. 1.4)
5. $m\angle 3 + m\angle 4 + m\angle 5 = 180^\circ$	**5.** Substitution Property of Equality
6. $\angle 1 \cong \angle 5$	**6.** Corresponding Angles Theorem (Thm. 3.1)
7. $\angle 2 \cong \angle 4$	**7.** Alternate Interior Angles Theorem (Thm. 3.2)
8. $m\angle 1 = m\angle 5$, $m\angle 2 = m\angle 4$	**8.** Definition of congruent angles
9. $m\angle 3 + m\angle 2 + m\angle 1 = 180^\circ$	**9.** Substitution Property of Equality

5.1 Maintaining Mathematical Proficiency (p. 238)

55. 86° **57.** 15

5.2 Vocabulary and Core Concept Check (p. 243)

1. To show that two triangles are congruent, you need to show that all corresponding parts are congruent. If two triangles have the same side lengths and angle measures, then they must be the same size and shape.

5.2 Monitoring Progress and Modeling with Mathematics (pp. 243–244)

3. corresponding angles: $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$; corresponding sides: $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \overline{AC} \cong \overline{DF}$; *Sample answer:* $\triangle BCA \cong \triangle EFD$

5. 124° **7.** 23° **9.** $x = 7, y = 8$

11. From the diagram, $\overline{WX} \cong \overline{LM}$, $\overline{XY} \cong \overline{MN}$, $\overline{YZ} \cong \overline{NJ}$, $\overline{VZ} \cong \overline{KJ}$, and $\overline{WV} \cong \overline{LK}$. Also from the diagram, $\angle V \cong \angle K$, $\angle W \cong \angle L$, $\angle X \cong \angle M$, $\angle Y \cong \angle N$, and $\angle Z \cong \angle J$. Because all corresponding parts are congruent, $VWXYZ \cong KLMNJ$.

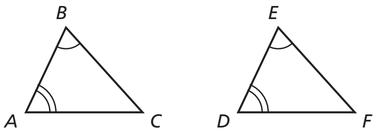
13. 20°

15.

STATEMENTS	REASONS
1. $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$, E is the midpoint of $\overline{AC}$ and $\overline{BD}$.	**1.** Given
2. $\angle AEB \cong \angle CED$	**2.** Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\angle BAE \cong \angle DCE$, $\angle ABE \cong \angle CDE$	**3.** Alternate Interior Angles Theorem (Thm. 3.2)
4. $\overline{AE} \cong \overline{CE}$, $\overline{BE} \cong \overline{DE}$	**4.** Definition of midpoint
5. $\triangle AEB \cong \triangle CED$	**5.** All corresponding parts are congruent.

17. The congruence statement should be used to ensure that corresponding parts are matched up correctly; $\angle S \cong \angle Y$; $m\angle S = m\angle Y$; $m\angle S = 90° - 42° = 48°$

19.



STATEMENTS	REASONS
1. $\angle A \cong \angle D$, $\angle B \cong \angle E$	**1.** Given
2. $m\angle A = m\angle D$, $m\angle B = m\angle E$	**2.** Definition of congruent angles
3. $m\angle A + m\angle B + m\angle C = 180°$, $m\angle D + m\angle E + m\angle F = 180°$	**3.** Triangle Sum Theorem (Thm. 5.1)
4. $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$	**4.** Transitive Property of Equality
5. $m\angle A + m\angle B + m\angle C = m\angle A + m\angle B + m\angle F$	**5.** Substitution Property of Equality
6. $m\angle C = m\angle F$	**6.** Subtraction Property of Equality
7. $\angle C \cong \angle F$	**7.** Definition of congruent angles

21. corresponding angles: $\angle J \cong \angle X$, $\angle K \cong \angle Y$, $\angle L \cong \angle Z$
corresponding sides: $\overline{JK} \cong \overline{XY}$, $\overline{KL} \cong \overline{YZ}$, $\overline{JL} \cong \overline{XZ}$

23. $\begin{cases} 17x - y = 40 \\ 2x + 4y = 50 \end{cases}$
$x = 3$, $y = 11$

25. A rigid motion maps each part of a figure to a corresponding part of its image. Because rigid motions preserve length and angle measure, corresponding parts of congruent figures are congruent, which means that the corresponding sides and corresponding angles are congruent.

5.2 Maintaining Mathematical Proficiency *(p. 244)*

27. $\overline{PQ} \cong \overline{RS}$, $\angle N \cong \angle T$

29. $\overline{DE} \cong \overline{HI}$, $\angle D \cong \angle H$, $\overline{DF} \parallel \overline{HG}$, $\angle DFE \cong \angle HGI$

5.3 Vocabulary and Core Concept Check *(p. 249)*

1. an angle formed by two sides

5.3 Monitoring Progress and Modeling with Mathematics *(pp. 249–250)*

3. $\angle JKL$ **5.** $\angle KLP$ **7.** $\angle JLK$

9. no; The congruent angles are not the included angles.

11. no; One of the congruent angles is not the included angle.

13. yes; Two pairs of sides and the included angles are congruent.

15.

STATEMENTS	REASONS
1. $\overline{SP} \cong \overline{TP}$, $\overline{PQ}$ bisects $\angle SPT$.	**1.** Given
2. $\overline{PQ} \cong \overline{PQ}$	**2.** Reflexive Property of Congruence (Thm. 2.1)
3. $\angle SPQ \cong \angle TPQ$	**3.** Definition of angle bisector
4. $\triangle SPQ \cong \triangle TPQ$	**4.** SAS Congruence Theorem (Thm. 5.5)

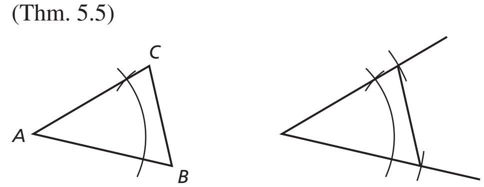
17.

STATEMENTS	REASONS
1. C is the midpoint of $\overline{AE}$ and $\overline{BD}$.	**1.** Given
2. $\angle ACB \cong \angle ECD$	**2.** Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\overline{AC} \cong \overline{EC}$, $\overline{BC} \cong \overline{DC}$	**3.** Definition of midpoint
4. $\triangle ABC \cong \triangle EDC$	**4.** SAS Congruence Theorem (Thm. 5.5)

19. $\triangle SRT \cong \triangle URT$; $\overline{RT} \cong \overline{RT}$ by the Reflexive Property of Congruence (Thm. 2.1). Also, because all points on a circle are the same distance from the center, $\overline{RS} \cong \overline{RU}$. It is given that $\angle SRT \cong \angle URT$. So, $\triangle SRT$ and $\triangle URT$ are congruent by the SAS Congruence Theorem (Thm. 5.5).

21. $\triangle STU \cong \triangle UVR$; Because the sides of the pentagon are congruent, $\overline{ST} \cong \overline{UV}$ and $\overline{TU} \cong \overline{VR}$. Also, because the angles of the pentagon are congruent, $\angle T \cong \angle V$. So, $\triangle STU$ and $\triangle UVR$ are congruent by the SAS Congruence Theorem (Thm. 5.5)

23.



25. $\triangle XYZ$ and $\triangle WYZ$ are congruent so either the expressions for $\overline{XZ}$ and $\overline{WZ}$ or the expressions for $\overline{XY}$ and $\overline{WY}$ should be set equal to each other because they are corresponding sides.

$5x - 5 = 3x + 9$
$2x - 5 = 9$
$2x = 14$
$x = 7$

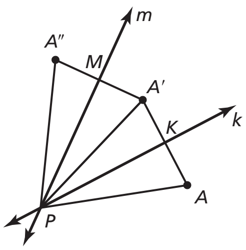
27. Because $\triangle ABC$, $\triangle BCD$, and $\triangle CDE$ are isosceles triangles, you know that $\overline{AB} \cong \overline{BC}$, $\overline{BC} \cong \overline{CD}$, and $\overline{CD} \cong \overline{DE}$. So, by the Transitive Property of Congruence (Thm. 2.1), $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DE}$. It is given that $\angle B \cong \angle D$, so $\triangle ABC \cong \triangle CDE$ by the SAS Congruence Theorem (Thm. 5.5).

29.

STATEMENTS	REASONS
1. $\overline{AC} \cong \overline{DC}$, $\overline{BC} \cong \overline{EC}$	**1.** Given
2. $\angle ACB \cong \angle DCE$	**2.** Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\triangle ABC \cong \triangle DEC$	**3.** SAS Congruence Theorem (Thm. 5.5)

$x = 4$, $y = 5$

31.



STATEMENTS	REASONS
1. A reflection in line k maps point A to A', a reflection in line m maps A' to A'', and $m\angle MPK = x°$.	**1.** Given
2. Line k is the perpendicular bisector of $\overline{AA'}$, and line m is the perpendicular bisector of $\overline{A'A''}$.	**2.** Definition of reflection
3. $\overline{AK} \cong \overline{KA'}$, $\angle AKP$ and $\angle A'KP$ are right angles, $\overline{A'M} \cong \overline{MA''}$, and $\angle A'MP$ and $\angle A''MP$ are right angles.	**3.** Definition of perpendicular bisector
4. $\angle AKP \cong \angle A'KP$, $\angle A'MP \cong \angle A''MP$	**4.** Right Angles Congruence Theorem (Thm. 2.3)
5. $\overline{KP} \cong \overline{KP}$	**5.** Reflexive Property of Congruence (Thm. 2.1)
6. $\triangle AKP \cong \triangle A'KP$, $\triangle A'MP \cong \triangle A''MP$	**6.** SAS Congruence Theorem (Thm. 5.5)
7. $\overline{AP} \cong \overline{A'P}$, $\overline{A'P} \cong \overline{A''P}$, $\angle APK \cong \angle A'PK$, $\angle A'PM \cong \angle A''PM$	**7.** Corresponding parts of congruent triangles are congruent.
8. $\overline{AP} \cong \overline{A''P}$	**8.** Transitive Property of Congruence (Thm. 2.1)
9. $m\angle APK = m\angle A'PK$, $m\angle A'PM = m\angle A''PM$	**9.** Definition of congruent angles
10. $m\angle MPK = m\angle A'PK + m\angle A'PM$, $m\angle APA'' = m\angle APK + m\angle A'PK + m\angle A'PM + m\angle A''PM$	**10.** Angle Addition Postulate (Post. 1.4)
11. $m\angle APA'' = m\angle A'PK + m\angle A'PK + m\angle A'PM + m\angle A'PM$	**11.** Substitution Property of Equality
12. $m\angle APA'' = 2(m\angle A'PK + m\angle A'PM)$	**12.** Distributive Property
13. $m\angle APA'' = 2(m\angle MPK)$	**13.** Substitution Property of Equality
14. $m\angle APA'' = 2(x°) = 2x°$	**14.** Substitution Property of Equality
15. A rotation about point P maps A to A'', and the angle of rotation is $2x°$.	**15.** Definition of rotation

5.3 Maintaining Mathematical Proficiency *(p. 250)*

33. obtuse isosceles **35.** obtuse scalene

5.4 Vocabulary and Core Concept Check *(p. 256)*

1. The vertex angle is the angle formed by the congruent sides, or legs, of an isosceles triangle.

5.4 Monitoring Progress and Modeling with Mathematics *(pp. 256–258)*

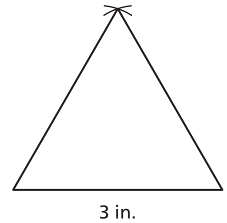
3. *A, D;* Base Angles Theorem (Thm. 5.6)

5. $\overline{CD}, \overline{CE}$; Converse of the Base Angles Theorem (Thm. 5.7)

7. $x = 12$ **9.** $x = 60$ **11.** $x = 79, y = 22$

13. $x = 60, y = 60$ **15.** $x = 30, y = 5$

17.



19. When two angles of a triangle are congruent, the sides opposite the angles are congruent; Because $\angle A \cong \angle C$, $\overline{AB} \cong \overline{BC}$. So, $BC = 5$.

21. **a.** Each edge is made out of the same number of sides of the original equilateral triangle.

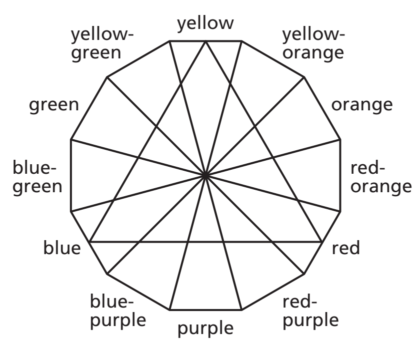
b. 1 square unit, 4 square units, 9 square units, 16 square units

c. Triangle 1 has an area of $1^2 = 1$, Triangle 2 has an area of $2^2 = 4$, Triangle 3 has an area of $3^2 = 9$, and so on. So, by inductive reasoning, you can predict that Triangle n has an area of n^2; 49 square units; $n^2 = 7^2 = 49$

23. 17 in.

25. By the Reflexive Property of Congruence (Thm. 2.1), the yellow triangle and the yellow-orange triangle share a congruent side. Because the triangles are all isosceles, by the Transitive Property of Congruence (Thm. 2.1), the yellow-orange triangle and the orange triangle share a side that is congruent to the one shared by the yellow triangle and the yellow-orange triangle. This reasoning can be continued around the wheel, so the legs of the isosceles triangles are all congruent. Because you are given that the vertex angles are all congruent, you can conclude that the yellow triangle is congruent to the purple triangle by the SAS Congruence Theorem (Thm. 5.5).

27.



equiangular equilateral

29. no; The two sides that are congruent can form an obtuse angle or a right angle.

Selected Answers

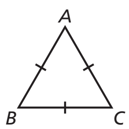
31. 6, 8, 10; If $3t = 5t - 12$, then $t = 6$. If $5t - 12 = t + 20$, then $t = 8$. If $3t = t + 20$, then $t = 10$.

33. If the base angles are $x°$, then the vertex angle is $(180 - 2x)°$, or $[2(90 - x)]°$. Because $2(90 - x)$ is divisible by 2, the vertex angle is even when the angles are whole numbers.

35. **a.** 2.1 mi; By the Exterior Angle Theorem (Thm. 5.2), $m\angle L = 70° - 35° = 35°$. Because $m\angle SRL = 35° = m\angle RLS$, by definition of congruent angles, $\angle SRL \cong \angle RLS$. So, by the Converse of the Base Angles Theorem (Thm. 5.7), $\overline{RS} \cong \overline{SL}$. So, $SL = RS = 2.1$ miles.

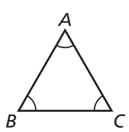
b. Find the point on the shore line that has an angle of 45° from the boat. Then, measure the distance that the boat travels until the angle is 90°. That distance is the same as the distance between the boat and the shore line because the triangle formed is an isosceles right triangle.

37.



STATEMENTS	REASONS
1. $\triangle ABC$ is equilateral.	**1.** Given
2. $\overline{AB} \cong \overline{AC}$, $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{BC}$	**2.** Definition of equilateral triangle
3. $\angle B \cong \angle C$, $\angle A \cong \angle C$, $\angle A \cong \angle B$	**3.** Base Angles Theorem (Thm. 5.6)
4. $\triangle ABC$ is equiangular.	**4.** Definition of equiangular triangle

39.



STATEMENTS	REASONS
1. $\triangle ABC$ is equiangular.	**1.** Given
2. $\angle B \cong \angle C$, $\angle A \cong \angle C$, $\angle A \cong \angle B$	**2.** Definition of equiangular triangle
3. $\overline{AB} \cong \overline{AC}$, $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{BC}$	**3.** Converse of the Base Angles Theorem (Thm. 5.7)
4. $\triangle ABC$ is equilateral.	**4.** Definition of equilateral triangle

41.

STATEMENTS	REASONS
1. $\triangle ABC$ is equilateral, $\angle CAD \cong \angle ABE \cong \angle BCF$	**1.** Given
2. $\triangle ABC$ is equiangular.	**2.** Corollary to the Base Angles Theorem (Cor. 5.2)
3. $\angle ABC \cong \angle BCA \cong \angle BAC$	**3.** Definition of equiangular triangle
4. $m\angle CAD = m\angle ABE = m\angle BCF$, $m\angle ABC = m\angle BCA = m\angle BAC$	**4.** Definition of congruent angles
5. $m\angle ABC = m\angle ABE + m\angle EBC$, $m\angle BCA = m\angle BCF + m\angle ACF$, $m\angle BAC = m\angle CAD + m\angle BAD$	**5.** Angle Addition Postulate (Post. 1.4)
6. $m\angle ABE + m\angle EBC = m\angle BCF + m\angle ACF = m\angle CAD + m\angle BAD$	**6.** Substitution Property of Equality
7. $m\angle ABE + m\angle EBC = m\angle ABE + m\angle ACF = m\angle ABE + m\angle BAD$	**7.** Substitution Property of Equality
8. $m\angle EBC = m\angle ACF = m\angle BAD$	**8.** Subtraction Property of Equality
9. $\angle EBC \cong \angle ACF \cong \angle BAD$	**9.** Definition of congruent angles
10. $\angle FEB \cong \angle DFC \cong \angle EDA$	**10.** Third Angles Theorem (Thm. 5.4)
11. $\angle FEB$ and $\angle FED$ are supplementary, $\angle DFC$ and $\angle EFD$ are supplementary, and $\angle EDA$ and $\angle FDE$ are supplementary.	**11.** Linear Pair Postulate (Post. 2.8)
12. $\angle FED \cong \angle EFD \cong \angle FDE$	**12.** Congruent Supplements Theorem (Thm. 2.4)
13. $\triangle DEF$ is equiangular.	**13.** Definition of equiangular triangle
14. $\triangle DEF$ is equilateral.	**14.** Corollary to the Converse of the Base Angles Theorem (Cor. 5.3)

5.4 Maintaining Mathematical Proficiency *(p. 258)*

43. $\overline{JK}, \overline{RS}$

5.5 Vocabulary and Core Concept Check *(p. 266)*

1. hypotenuse

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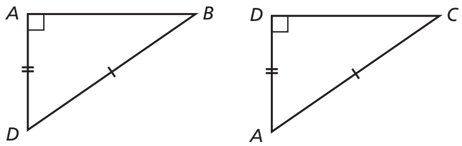
3. yes; $\overline{AB} \cong \overline{DB}$, $\overline{BC} \cong \overline{BE}$, $\overline{AC} \cong \overline{DE}$

5. yes; $\angle B$ and $\angle E$ are right angles, $\overline{AB} \cong \overline{FE}$, $\overline{AC} \cong \overline{FD}$

7. no; You are given that $\overline{RS} \cong \overline{PQ}$, $\overline{ST} \cong \overline{QT}$, and $\overline{RT} \cong \overline{PT}$. So, it should say $\triangle RST \cong \triangle PQT$ by the SSS Congruence Theorem (Thm. 5.8).

9. yes; You are given that $\overline{EF} \cong \overline{GF}$ and $\overline{DE} \cong \overline{DG}$. Also, $\overline{DF} \cong \overline{DF}$ by the Reflexive Property of Congruence (Thm. 2.1). So, $\triangle DEF \cong \triangle DGF$ by the SSS Congruence Theorem (Thm. 5.8).

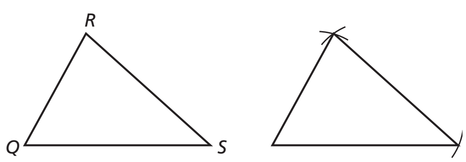
11. yes; The diagonal supports in this figure form triangles with fixed side lengths. By the SSS Congruence Theorem (Thm. 5.8), these triangles cannot change shape, so the figure is stable.

13. 

STATEMENTS	REASONS
1. $\overline{AC} \cong \overline{DB}$, $\overline{AB} \perp \overline{AD}$, $\overline{CD} \perp \overline{AD}$	1. Given
2. $\overline{AD} \cong \overline{AD}$	2. Reflexive Property of Congruence (Thm. 2.1)
3. $\angle BAD$ and $\angle CDA$ are right angles.	3. Definition of perpendicular lines
4. $\triangle BAD$ and $\triangle CDA$ are right triangles.	4. Definition of a right triangle
5. $\triangle BAD \cong \triangle CDA$	5. HL Congruence Theorem (Thm. 5.9)

15.

STATEMENTS	REASONS
1. $\overline{LM} \cong \overline{JK}$, $\overline{MJ} \cong \overline{KL}$	1. Given
2. $\overline{JL} \cong \overline{JL}$	2. Reflexive Property of Congruence (Thm. 2.1)
3. $\triangle LMJ \cong \triangle JKL$	3. SSS Congruence Theorem (Thm. 5.8)

17. 

19. The order of the points in the congruence statement should reflect the corresponding sides and angles; $\triangle TUV \cong \triangle ZYX$ by the SSS Congruence Theorem (Thm. 5.8).

21. no; The sides of a triangle do not have to be congruent to each other, but each side of one triangle must be congruent to the corresponding side of the other triangle.

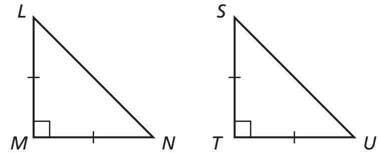
23. **a.** You need to know that the hypotenuses are congruent: $\overline{JL} \cong \overline{ML}$.

b. SAS Congruence Theorem (Thm. 5.5); By definition of midpoint, $\overline{JK} \cong \overline{MK}$. Also, $\overline{LK} \cong \overline{LK}$, by the Reflexive Property of Congruence (Thm. 2.1), and $\angle JKL \cong \angle MKL$ by the Right Angles Congruence Theorem (Thm. 2.3).

25. congruent **27.** congruent

29. yes; Use the string to compare the lengths of the corresponding sides of the two triangles to determine whether SSS Congruence Theorem (Thm. 5.8) applies.

31. both; $\overline{JL} \cong \overline{JL}$ by the Reflexive Property of Congruence (Thm. 2.1), and the other two pairs of sides are marked as congruent. So, the SSS Congruence Theorem (Thm. 5.8) can be used. Also, because $\angle M$ and $\angle K$ are right angles, they are both right triangles, and the legs and hypotenuses are congruent. So, the HL Congruence Theorem (Thm. 5.9) can be used.

33. *Sample answer:* 

35. **a.** $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Congruence (Thm. 2.1). It is given that $\overline{AB} \cong \overline{CB}$ and that $\angle ADB$ and $\angle CDB$ are right angles. So, $\triangle ABC$ and $\triangle CBD$ are right triangles and are congruent by the HL Congruence Theorem (Thm. 5.9).

b. yes; Because $\overline{AB} \cong \overline{CB} \cong \overline{CE} \cong \overline{FE}$, $\overline{BD} \cong \overline{EG}$, and they are all right triangles, it can be shown that $\triangle ABD \cong \triangle CBD \cong \triangle CEG \cong \triangle FEG$ by the HL Congruence Theorem (Thm. 5.9).

5.5 Maintaining Mathematical Proficiency *(p. 268)*

37. $\overline{DF}$ **39.** $\angle E$

5.6 Vocabulary and Core Concept Check *(p. 274)*

1. Both theorems are used to prove that two triangles are congruent, and both require two pairs of corresponding angles to be congruent. In order to use the AAS Congruence Theorem (Thm. 5.11), one pair of corresponding nonincluded sides must also be congruent. In order to use the ASA Congruence Theorem (Thm. 5.10), the pair of corresponding included sides must be congruent.

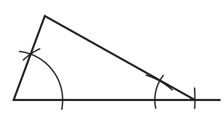
5.6 Monitoring Progress and Modeling with Mathematics *(pp. 274–276)*

3. yes; AAS Congruence Theorem (Thm. 5.11) **5.** no

7. $\angle F$; $\angle L$

9. yes; $\triangle ABC \cong \triangle DEF$ by the ASA Congruence Theorem (Thm. 5.10)

11. no; $\overline{AC}$ and $\overline{DE}$ do not correspond.

13. 

15. In the congruence statement, the vertices should be in corresponding order; $\triangle JKL \cong \triangle FGH$ by the ASA Congruence Theorem (Thm. 5.10).

17.

STATEMENTS	REASONS
1. M is the midpoint of $\overline{NL}$, $\overline{NL} \perp \overline{NQ}$, $\overline{NL} \perp \overline{MP}$, $\overline{QM} \parallel \overline{PL}$	1. Given
2. $\angle QNM$ and $\angle PML$ are right angles.	2. Definition of perpendicular lines
3. $\angle QNM \cong \angle PML$	3. Right Angles Congruence Theorem (Thm. 2.3)
4. $\angle QMN \cong \angle PLM$	4. Corresponding Angles Theorem (Thm. 3.1)
5. $\overline{NM} \cong \overline{ML}$	5. Definition of midpoint
6. $\triangle NQM \cong \triangle MPL$	6. ASA Congruence Theorem (Thm. 5.10)

Selected Answers

19.

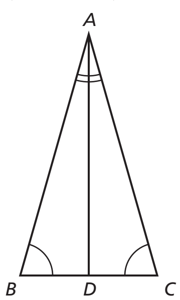
STATEMENTS	REASONS
1. $\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$	**1.** Given
2. $\angle W \cong \angle W$	**2.** Reflexive Property of Congruence (Thm. 2.2)
3. $\triangle XWV \cong \triangle ZWU$	**3.** AAS Congruence Theorem (Thm. 5.11)

21. You are given two right triangles, so the triangles have congruent right angles by the Right Angles Congruence Theorem (Thm. 2.3). Because another pair of angles and a pair of corresponding nonincluded sides (the hypotenuses) are congruent, the triangles are congruent by the AAS Congruence Theorem (Thm. 5.11).

23. You are given two right triangles, so the triangles have congruent right angles by the Right Angles Congruence Theorem (Thm. 2.3). There is also another pair of congruent corresponding angles and a pair of congruent corresponding sides. If the pair of congruent sides is the included side, then the triangles are congruent by the ASA Congruence Theorem (Thm. 5.10). If the pair of congruent sides is a nonincluded pair, then the triangles are congruent by the AAS Congruence Theorem (Thm. 5.11).

25. yes; When $x = 14$ and $y = 26$, $m\angle ABC = m\angle DBC = m\angle BCA = m\angle BCD = 80°$ and $m\angle CAB = m\angle CDB = 20°$. This satisfies the Triangle Sum Theorem (Thm. 5.1) for both triangles. Because $\overline{CB} \cong \overline{CB}$ by the Reflexive Property of Congruence (Thm. 2.1), you can conclude that $\triangle ABC \cong \triangle DBC$ by the ASA Congruence Theorem (Thm. 5.10) or the AAS Congruence Theorem (Thm. 5.11).

27.



STATEMENTS	REASONS
1. Draw $\overline{AD}$, the angle bisector of $\angle ABC$.	**1.** Construction of angle bisector
2. $\angle CAD \cong \angle BAD$	**2.** Definition of angle bisector
3. $\angle B \cong \angle C$	**3.** Given
4. $\overline{AD} \cong \overline{AD}$	**4.** Reflexive Property of Congruence (Thm. 2.1)
5. $\triangle ABD \cong \triangle ACD$	**5.** AAS Congruence Theorem (Thm. 5.11)
6. $\overline{AB} \cong \overline{AC}$	**6.** Corresponding parts of congruent triangles are congruent.

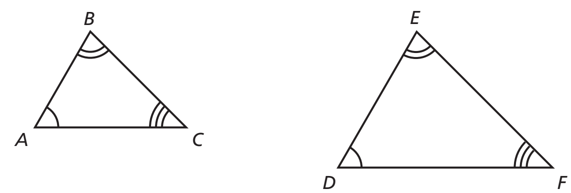
29. a.

STATEMENTS	REASONS
1. $\angle CDB \cong \angle ADB$, $\overline{DB} \perp \overline{AC}$	**1.** Given
2. $\angle ABD$ and $\angle CBD$ are right angles.	**2.** Definition of perpendicular lines
3. $\angle ABD \cong \angle CBD$	**3.** Right Angles Congruence Theorem (Thm. 2.3)
4. $\overline{BD} \cong \overline{BD}$	**4.** Reflexive Property of Congruence (Thm. 2.1)
5. $\triangle ABD \cong \triangle CBD$	**5.** ASA Congruence Theorem (Thm. 5.10)

b. Because $\triangle ABD \cong \triangle CBD$ and corresponding parts of congruent triangles are congruent, you can conclude that $\overline{AD} \cong \overline{CD}$,which means that $\triangle ACD$ is isosceles by definition.

c. no; For instance, because $\triangle ACD$ is isosceles, the girl sees her toes at the bottom of the mirror. This remains true as she moves backward, because $\triangle ACD$ remains isosceles.

31. *Sample answer:*



33. a. $\overline{TU} \cong \overline{XY}, \overline{UV} \cong \overline{YZ}, \overline{TV} \cong \overline{XZ}$;
$\overline{TU} \cong \overline{XY}, \angle U \cong \angle Y, \overline{UV} \cong \overline{YZ}$;
$\overline{UV} \cong \overline{YZ}, \angle V \cong \angle Z, \overline{TV} \cong \overline{XZ}$;
$\overline{TV} \cong \overline{XZ}, \angle T \cong \angle X, \overline{TU} \cong \overline{XY}$;
$\angle T \cong \angle X, \overline{TU} \cong \overline{XY}, \angle U \cong \angle Y$;
$\angle U \cong \angle Y, \overline{UV} \cong \overline{YZ}, \angle V \cong \angle Z$;
$\angle V \cong \angle Z, \overline{TV} \cong \overline{XZ}, \angle T \cong \angle X$;
$\angle T \cong \angle X, \angle U \cong \angle Y, \overline{UV} \cong \overline{YZ}$;
$\angle T \cong \angle X, \angle U \cong \angle Y, \overline{TV} \cong \overline{XZ}$;
$\angle U \cong \angle Y, \angle V \cong \angle Z, \overline{TV} \cong \overline{XZ}$;
$\angle U \cong \angle Y, \angle V \cong \angle Z, \overline{TU} \cong \overline{XY}$;
$\angle V \cong \angle Z, \angle T \cong \angle X, \overline{TU} \cong \overline{XY}$;
$\angle V \cong \angle Z, \angle T \cong \angle X, \overline{UV} \cong \overline{YZ}$

b. $\frac{13}{20}$, or 65%

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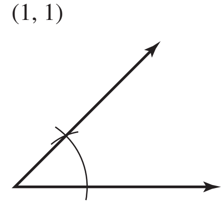
35. (1, 1)

37.



5.7 Vocabulary and Core Concept Check *(p. 281)*

1. Corresponding

5.7 Monitoring Progress and Modeling with Mathematics *(pp. 281–282)*

3. All three pairs of sides are congruent. So, by the SSS Congruence Theorem (Thm. 5.8), $\triangle ABC \cong \triangle DBC$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$.

5. The hypotenuses and one pair of legs of two right triangles are congruent. So, by the HL Congruence Theorem (Thm. 5.9), $\triangle JMK \cong \triangle LMK$. Because corresponding parts of congruent triangles are congruent, $\overline{JM} \cong \overline{LM}$.

7. From the diagram, $\angle JHN \cong \angle KGL$, $\angle N \cong \angle L$, and $\overline{JN} \cong \overline{KL}$. So, by the AAS Congruence Theorem (Thm. 5.11), $\triangle JNH \cong \triangle KLG$. Because corresponding parts of congruent triangles are congruent, $\overline{GK} \cong \overline{HJ}$.

9. Use the AAS Congruence Theorem (Thm. 5.11) to prove that $\triangle FHG \cong \triangle GKF$. Then, state that $\angle FGK \cong \angle GFH$. Use the Congruent Complements Theorem (Thm. 2.5) to prove that $\angle 1 \cong \angle 2$.

11. Use the ASA Congruence Theorem (Thm. 5.10) to prove that $\triangle STR \cong \triangle QTP$. Then, state that $\overline{PT} \cong \overline{RT}$ because corresponding parts of congruent triangles are congruent. Use the SAS Congruence Theorem (Thm. 5.5) to prove that $\triangle STP \cong \triangle QTR$. So, $\angle 1 \cong \angle 2$.

13.

STATEMENTS	REASONS
1. $\overline{AP} \cong \overline{BP}$, $\overline{AQ} \cong \overline{BQ}$	**1.** Given
2. $\overline{PQ} \cong \overline{PQ}$	**2.** Reflexive Property of Congruence (Thm. 2.1)
3. $\triangle APQ \cong \triangle BPQ$	**3.** SSS Congruence Theorem (Thm. 5.8)
4. $\angle APQ \cong \angle BPQ$	**4.** Corresponding parts of congruent triangles are congruent.
5. $\overline{PM} \cong \overline{PM}$	**5.** Reflexive Property of Congruence (Thm. 2.1)
6. $\triangle APM \cong \triangle BPM$	**6.** SAS Congruence Theorem (Thm. 5.5)
7. $\angle AMP \cong \angle BMP$	**7.** Corresponding parts of congruent triangles are congruent.
8. $\angle AMP$ and $\angle BMP$ form a linear pair.	**8.** Definition of a linear pair
9. $\overline{MP} \perp \overline{AB}$	**9.** Linear Pair Perpendicular Theorem (Thm. 3.10)
10. $\angle AMP$ and $\angle BMP$ are right angles.	**10.** Definition of perpendicular lines

15.

STATEMENTS	REASONS
1. $\overline{FG} \cong \overline{GJ} \cong \overline{HG} \cong \overline{GK}$, $\overline{JM} \cong \overline{LM} \cong \overline{KM} \cong \overline{NM}$	**1.** Given
2. $\angle FGJ \cong \angle HGK$, $\angle JML \cong \angle KMN$	**2.** Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\triangle FGJ \cong \triangle HGK$, $\triangle JML \cong \triangle KMN$	**3.** SAS Congruence Theorem (Thm. 5.5)
4. $\angle F \cong \angle H$, $\angle L \cong \angle N$	**4.** Corresponding parts of congruent triangles are congruent.
5. $FG = GJ = HG = GK$	**5.** Definition of congruent segments
6. $HJ = HG + GJ$, $FK = FG + GK$	**6.** Segment Addition Postulate (Post. 1.2)
7. $FK = HG + GJ$	**7.** Substitution Property of Equality
8. $FK = HJ$	**8.** Transitive Property of Equality
9. $\overline{FK} \cong \overline{HJ}$	**9.** Definition of congruent segments
10. $\triangle HJN \cong \triangle FKL$	**10.** AAS Congruence Theorem (Thm. 5.11)
11. $\overline{FL} \cong \overline{HN}$	**11.** Corresponding parts of congruent triangles are congruent.

17. Because $\overline{AC} \perp \overline{BC}$ and $\overline{ED} \perp \overline{BD}$, $\angle ACB$ and $\angle EDB$ are congruent right angles. Because B is the midpoint of $\overline{CD}$, $\overline{BC} \cong \overline{BD}$. The vertical angles $\angle ABC$ and $\angle EBD$ are congruent. So, $\triangle ABC \cong \triangle EBD$ by the ASA Congruence Theorem (Thm. 5.10). Then, because corresponding parts of congruent triangles are congruent, $\overline{AC} \cong \overline{ED}$. So, you can find the distance AC across the canyon by measuring $\overline{ED}$.

19.

STATEMENTS	REASONS
1. $\overline{AD} \parallel \overline{BC}$, E is the midpoint of $\overline{AC}$.	**1.** Given
2. $\overline{AE} \cong \overline{CE}$	**2.** Definition of midpoint
3. $\angle AEB \cong \angle CED$, $\angle AED \cong \angle BEC$	**3.** Vertical Angles Congruence Theorem (Thm. 2.6)
4. $\angle DAE \cong \angle BCE$	**4.** Alternate Interior Angles Theorem (Thm. 3.2)
5. $\triangle DAE \cong \triangle BCE$	**5.** ASA Congruence Theorem (Thm. 5.10)
6. $\overline{DE} \cong \overline{BE}$	**6.** Corresponding parts of congruent triangles are congruent.
7. $\triangle AEB \cong \triangle CED$	**7.** SAS Congruence Theorem (Thm. 5.5)

21. yes; You can show that $WXYZ$ is a rectangle. This means that the opposite sides are congruent. Because $\triangle WZY$ and $\triangle YXW$ share a hypotenuse, the two triangles have congruent hypotenuses and corresponding legs, which allows you to use the HL Congruence Theorem (Thm. 5.9) to prove that the triangles are congruent.

23. $\triangle GHJ$, $\triangle NPQ$

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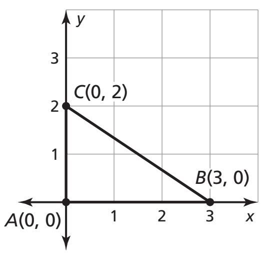
25. about 17.5 units

5.8 Vocabulary and Core Concept Check *(p. 287)*

1. In a coordinate proof, you have to assign coordinates to vertices and write expressions for the side lengths and the slopes of segments in order to show how sides are related; As with other types of proofs, you still have to use deductive reasoning and justify every conclusion with theorems, proofs, and properties of mathematics.

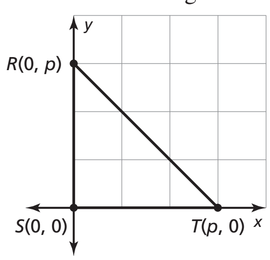
5.8 Monitoring Progress and Modeling with Mathematics *(pp. 287–288)*

3. *Sample answer:*



It is easy to find the lengths of horizontal and vertical segments and distances from the origin.

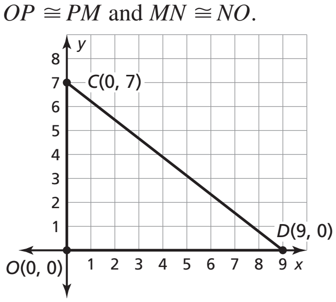
5. *Sample answer:*



It is easy to find the lengths of horizontal and vertical segments and distances from the origin.

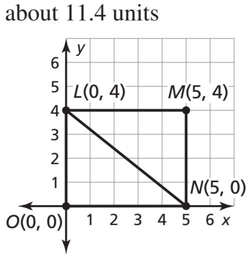
7. Find the lengths of $\overline{OP}$, $\overline{PM}$, $\overline{MN}$, and $\overline{NO}$ to show that $\overline{OP} \cong \overline{PM}$ and $\overline{MN} \cong \overline{NO}$.

9.



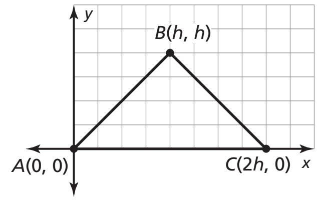
about 11.4 units

11.



about 6.4 units

13.

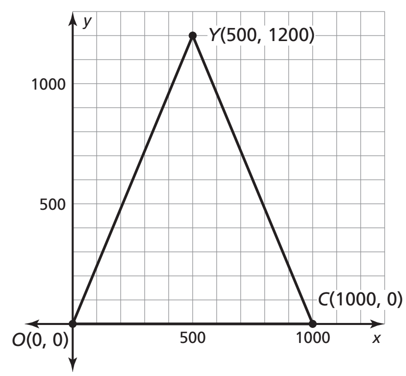


$AB = h\sqrt{2}$, $m_{\overline{AB}} = 1$, $M_{\overline{AB}}\left(\frac{h}{2}, \frac{h}{2}\right)$, $BC = h\sqrt{2}$, $m_{\overline{BC}} = -1$, $M_{\overline{BC}}\left(\frac{3h}{2}, \frac{h}{2}\right)$, $AC = 2h$, $m_{\overline{AC}} = 0$, $M_{\overline{AC}}(h, 0)$; yes; yes; Because $m_{\overline{AB}} \cdot m_{\overline{BC}} = -1$, $\overline{AB} \perp \overline{BC}$ by the Slopes of Perpendicular Lines Theorem (Thm. 3.14). So $\angle ABC$ is a right angle. $\overline{AB} \cong \overline{BC}$ because $AB = BC$. So, $\triangle ABC$ is a right isosceles triangle.

15. $N(h, k)$; $ON = \sqrt{h^2 + k^2}$, $MN = \sqrt{h^2 + k^2}$

17. $DC = k$, $BC = k$, $DE = h$, $OB = h$, $EC = \sqrt{h^2 + k^2}$, $OC = \sqrt{h^2 + k^2}$
So, $\overline{DC} \cong \overline{BC}$, $\overline{DE} \cong \overline{OB}$, and $\overline{EC} \cong \overline{OC}$. By the SSS Congruence Theorem (Thm. 5.8), $\triangle DEC \cong \triangle BOC$.

19.

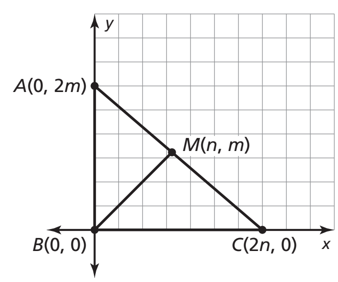


Using the Distance Formula, $OY = 1300$, and $CY = 1300$. Because $\overline{OY} \cong \overline{CY}$, $\triangle OYC$ is isosceles.

21. *Sample answer:* $(-k, -m)$ and (k, m) **23.** A

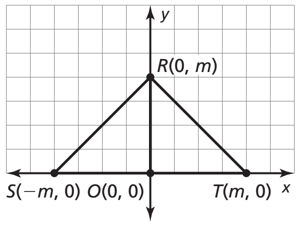
25. $(0, 0)$, $(5d, 0)$, $(0, 5d)$

27. **a.**



Because M is the midpoint of $\overline{AC}$, the coordinates of M are $M(n, m)$. Using the Distance Formula, $AM = \sqrt{n^2 + m^2}$, $BM = \sqrt{n^2 + m^2}$, and $CM = \sqrt{n^2 + m^2}$. So, the midpoint of the hypotenuse of a right triangle is the same distance from each vertex of the triangle.

b.



When any two congruent right isosceles triangles are positioned with the vertex opposite the hypotenuse on the origin and their legs on the axes as shown in the diagram, a triangle is formed and the hypotenuses of the original triangles make up two sides of the new triangle. $SR = m\sqrt{2}$ and $TR = m\sqrt{2}$ so these two sides are the same length. So, by definition, $\triangle SRT$ is isosceles.

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29. 34°

Chapter 5 Review *(pp. 290–294)*

1. acute isosceles **2.** 132° **3.** 90° **4.** 42°, 48°

5. 35°, 55°

6. corresponding sides: $\overline{GH} \cong \overline{LM}$, $\overline{HJ} \cong \overline{MN}$, $\overline{JK} \cong \overline{NP}$, and $\overline{GK} \cong \overline{LP}$; corresponding angles: $\angle G \cong \angle L$, $\angle H \cong \angle M$, $\angle J \cong \angle N$, and $\angle K \cong \angle P$; *Sample answer:* $JHGK \cong NMLP$

7. 16°

8. no; There are two pairs of congruent sides and one pair of congruent angles, but the angles are not the included angles.

9. yes;

STATEMENTS	REASONS
1. $\overline{WX} \cong \overline{YZ}$, $\angle WXZ \cong \angle YZX$	1. Given
2. $\overline{XZ} \cong \overline{XZ}$	2. Reflexive Property of Congruence (Thm. 2.1)
3. $\triangle WXZ \cong \triangle YZX$	3. SAS Congruence Theorem (Thm. 5.5)

10. P; PRQ **11.** $\overline{TR}$; $\overline{TV}$ **12.** RQS; RSQ

13. $\overline{SR}$; $\overline{SV}$ **14.** $x = 15$, $y = 5$

15. no; There is only enough information to conclude that two pairs of sides are congruent.

16. yes;

STATEMENTS	REASONS
1. $\overline{WX} \cong \overline{YZ}$, $\angle XWZ$ and $\angle ZYX$ are right angles.	1. Given
2. $\overline{XZ} \cong \overline{XZ}$	2. Reflexive Property of Congruence (Thm. 2.1)
3. $\triangle WXZ$ and $\triangle YZX$ are right triangles.	3. Definition of a right triangle
4. $\triangle WXZ \cong \triangle YZX$	4. HL Congruence Theorem (Thm. 5.9)

17. yes;

STATEMENTS	REASONS
1. $\angle E \cong \angle H$, $\angle F \cong \angle J$, $\overline{FG} \cong \overline{JK}$	1. Given
2. $\triangle EFG \cong \triangle HJK$	2. AAS Congruence Theorem (Thm. 5.11)

18. no; There is only enough information to conclude that one pair of angles and one pair of sides are congruent.

19. yes;

STATEMENTS	REASONS
1. $\angle PLN \cong \angle MLN$, $\angle PNL \cong \angle MNL$	1. Given
2. $\overline{LN} \cong \overline{LN}$	2. Reflexive Property of Congruence (Thm. 2.1)
3. $\triangle LPN \cong \triangle LMN$	3. ASA Congruence Theorem (Thm. 5.10)

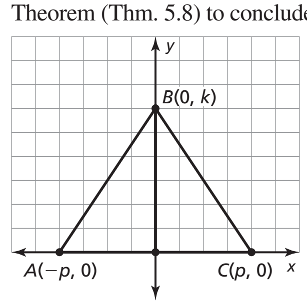
20. no; There is only enough information to conclude that one pair of angles and one pair of sides are congruent.

21. By the SAS Congruence Theorem (Thm. 5.5), $\triangle HJK \cong \triangle LMN$. Because corresponding parts of congruent triangles are congruent, $\angle K \cong \angle N$.

22. First, state that $\overline{QV} \cong \overline{QV}$. Then, use the SSS Congruence Theorem (Thm. 5.8) to prove that $\triangle QSV \cong \triangle QTV$. Because corresponding parts of congruent triangles are congruent, $\angle QSV \cong \angle QTV$. $\angle QSV \cong \angle 1$ and $\angle QTV \cong \angle 2$ by the Vertical Angles Congruence Theorem (Thm. 2.6). So, by the Transitive Property of Congruence (Thm. 2.2), $\angle 1 \cong \angle 2$.

23. Using the Distance Formula, $OP = \sqrt{h^2 + k^2}$, $QR = \sqrt{h^2 + k^2}$, $OR = j$, and $QP = j$. So, $\overline{OP} \cong \overline{QR}$ and $\overline{OR} \cong \overline{QP}$. Also, by the Reflexive Property of Congruence (Thm. 2.1), $\overline{QO} \cong \overline{QO}$. So, you can apply the SSS Congruence Theorem (Thm. 5.8) to conclude that $\triangle OPQ \cong \triangle QRO$.

24.



25. $(2k, k)$

Chapter 6

Chapter 6 Maintaining Mathematical Proficiency *(p. 299)*

1. $y = -3x + 10$ **2.** $y = x - 7$ **3.** $y = \frac{1}{4}x - \frac{7}{4}$

4. $-3 \leq w \leq 8$ **5.** $0 < m < 11$ **6.** $s \leq 5$ *or* $s > 2$

7. $d < 12$ *or* $d \geq -7$

8. yes; As with Exercises 6 and 7, if the graphs of the two inequalities overlap going in opposite directions and the variable only has to make one or the other true, then every number on the number line makes the compound inequality true.

6.1 Vocabulary and Core Concept Check *(p. 306)*

1. bisector

6.1 Monitoring Progress and Modeling with Mathematics *(pp. 306–308)*

3. 4.6; Because $GK = KJ$ and $\overleftrightarrow{HK} \perp \overleftrightarrow{GJ}$, point H is on the perpendicular bisector of $\overline{GJ}$. So, by the Perpendicular Bisector Theorem (Thm. 6.1), $GH = HJ = 4.6$.

5. 15; Because $\overleftrightarrow{DB} \perp \overline{AC}$ and point D is equidistant from A and C, point D is on the perpendicular bisector of $\overline{AC}$ by the Converse of the Perpendicular Bisector Theorem (Thm. 6.2). By definition of segment bisector, $AB = BC$. So, $5x = 4x + 3$, and the solution is $x = 3$. So, $AB = 5x = 5(3) = 15$.

7. yes; Because point N is equidistant from L and M, point N is on the perpendicular bisector of $\overline{LM}$ by the Converse of the Perpendicular Bisector Theorem (Thm. 6.2). Because only one line can be perpendicular to $\overline{LM}$ at point K, $\overleftrightarrow{NK}$ must be the perpendicular bisector of $\overline{LM}$, and P is on $\overleftrightarrow{NK}$.

9. no; You would need to know that $\overleftrightarrow{PN} \perp \overleftrightarrow{ML}$.

11. 20°; Because D is equidistant from $\overrightarrow{BC}$ and $\overrightarrow{BA}$, $\overrightarrow{BD}$ bisects $\angle ABC$ by the Converse of the Angle Bisector Theorem (Thm. 6.4). So, $m\angle ABD = m\angle CBD = 20°$.

13. 28°; Because L is equidistant from $\overrightarrow{JK}$ and $\overrightarrow{JM}$, $\overrightarrow{JL}$ bisects $\angle KJM$ by the Angle Bisector Theorem (Thm. 6.3). This means that $7x = 3x + 16$, and the solution is $x = 4$. So, $m\angle KJL = 7x = 7(4) = 28°$.

15. yes; Because H is equidistant from $\overrightarrow{EF}$ and $\overrightarrow{EG}$, $\overrightarrow{EH}$ bisects $\angle FEG$ by the Angle Bisector Theorem (Thm. 6.3).

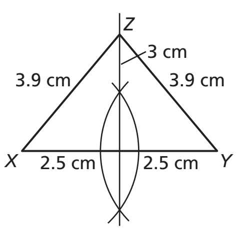
17. no; Because neither $\overline{BD}$ nor $\overline{DC}$ are marked as perpendicular to $\overrightarrow{AB}$ or $\overrightarrow{AC}$ respectively, you cannot conclude that $DB = DC$.

19. $y = x - 2$ **21.** $y = -3x + 15$

23. Because $\overline{DC}$ is not necessarily congruent to $\overline{EC}$, $\overleftrightarrow{AB}$ will not necessarily pass through point C; Because $AD = AE$, and $\overleftrightarrow{AB} \perp \overline{DE}$, $\overleftrightarrow{AB}$ is the perpendicular bisector of $\overline{DE}$.

25. Perpendicular Bisector Theorem (Thm. 6.1)

27.

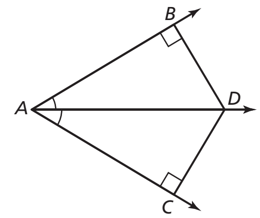


Perpendicular Bisector Theorem (Thm. 6.1)

29. B

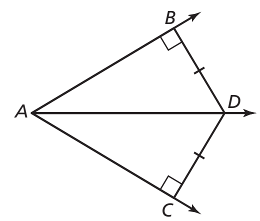
31. no; If the triangle is an isosceles triangle, then the angle bisector of the vertex angle will also be the perpendicular bisector of the base.

33. a.



If $\overrightarrow{AD}$ bisects $\angle BAC$, then by definition of angle bisector, $\angle BAD \cong \angle CAD$. Also, because $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, by definition of perpendicular lines, $\angle ABD$ and $\angle ACD$ are right angles, and congruent to each other by the Right Angles Congruence Theorem (Thm. 2.3). Also, $\overline{AD} \cong \overline{AD}$ by the Reflexive Property of Congruence (Thm. 2.1). So, by the AAS Congruence Theorem (Thm. 5.11), $\triangle ADB \cong \triangle ADC$. Because corresponding parts of congruent triangles are congruent, $DB = DC$. This means that point D is equidistant from each side of $\angle BAC$.

b.



STATEMENTS	REASONS
1. $\overline{DC} \perp \overrightarrow{AC}$, $\overline{DB} \perp \overrightarrow{AB}$, $BD = CD$	**1.** Given
2. $\angle ABD$ and $\angle ACD$ are right angles.	**2.** Definition of perpendicular lines
3. $\triangle ABD$ and $\triangle ACD$ are right triangles.	**3.** Definition of a right triangle
4. $\overline{BD} \cong \overline{CD}$	**4.** Definition of congruent segments
5. $\overline{AD} \cong \overline{AD}$	**5.** Reflexive Property of Congruence (Thm. 2.1)
6. $\triangle ABD \cong \triangle ACD$	**6.** HL Congruence Theorem (Thm. 5.9)
7. $\angle BAD \cong \angle CAD$	**7.** Corresponding parts of congruent triangles are congruent.
8. $\overrightarrow{AD}$ bisects $\angle BAC$.	**8.** Definition of angle bisector

35. a. $y = x$ **b.** $y = -x$ **c.** $y = |x|$

37. Because $\overline{AD} \cong \overline{CD}$ and $\overline{AE} \cong \overline{CE}$, by the Converse of the Perpendicular Bisector Theorem (Thm. 6.2), both points D and E are on the perpendicular bisector of $\overline{AC}$. So, $\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{AC}$. So, if $\overline{AB} \cong \overline{CB}$, then by the Converse of the Perpendicular Bisector Theorem (Thm. 6.2), point B is also on $\overleftrightarrow{DE}$. So, points D, E, and B are collinear. Conversely, if points D, E, and B are collinear, then by the Perpendicular Bisector Theorem (Thm. 6.2), point B is also on the perpendicular bisector of AC. So, $\overline{AB} \cong \overline{CB}$.

6.1 Maintaining Mathematical Proficiency *(p. 308)*

39. isosceles **41.** equilateral **43.** right

6.2 Vocabulary and Core Concept Check *(p. 315)*

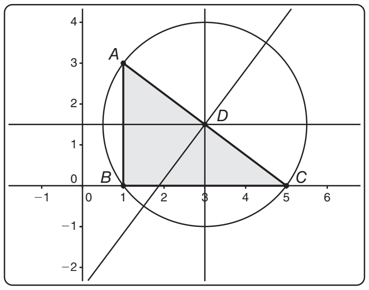
1. concurrent

6.2 Monitoring Progress and Modeling with Mathematics *(pp. 315–318)*

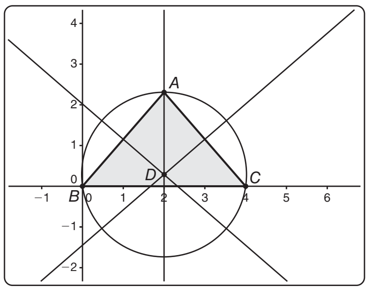
3. 9 **5.** 9 **7.** (5, 8) **9.** (−4, 9) **11.** 16

13. 6 **15.** 32

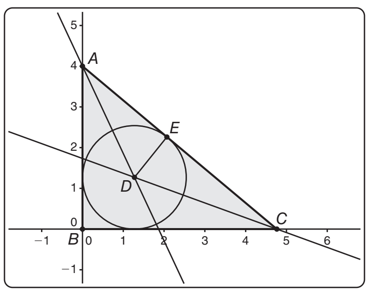
17. *Sample answer:*



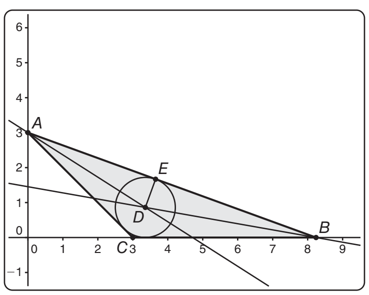
19. *Sample answer:*



21. *Sample answer:*



23. *Sample answer:*



25. Because point G is the intersection of the angle bisectors, it is the incenter. But, because $\overline{GD}$ and $\overline{GF}$ are not necessarily perpendicular to a side of the triangle, there is not sufficient evidence to conclude that $\overline{GD}$ and $\overline{GF}$ are congruent; Point G is equidistant from the sides of the triangle.

27. You could copy the positions of the three houses, and connect the points to draw a triangle. Then draw the three perpendicular bisectors of the triangle. The point where the perpendicular bisectors meet, the circumcenter, should be the location of the meeting place.

29. sometimes; If the scalene triangle is obtuse or right, then the circumcenter is outside or on the triangle, respectively. However, if the scalene triangle is acute, then the circumcenter is inside the triangle.

31. sometimes; This only happens when the triangle is equilateral.

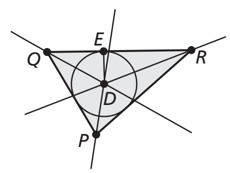
33. $\left(\frac{35}{6}, -\frac{11}{6}\right)$ **35.** $x = 6$

37. The circumcenter of any right triangle is located at the midpoint of the hypotenuse of the triangle.

Let $A(0, 2b)$, $B(0, 0)$, and $C(2a, 0)$ represent the vertices of a right triangle where $\angle B$ is the right angle. The midpoint of $\overline{AB}$ is $M_{\overline{AB}}(0, b)$. The midpoint of $\overline{BC}$ is $M_{\overline{BC}}(a, 0)$. The midpoint of $\overline{AC}$ is $M_{\overline{AC}}(a, b)$. Because $\overline{AB}$ is vertical, its perpendicular b isector is horizontal. So, the equation of the horizontal line passing through $M_{\overline{AB}}(0, b)$ is $y = b$. Because $\overline{BC}$ is horizontal, its perpendicular bisector is vertical. So, the equation of the vertical line passing through $M_{\overline{BC}}(a, 0)$ is $x = a$. The circumcenter of $\triangle ABC$ is the intersection of perpendicular bisectors, $y = b$ and $x = a$, which is (a, b). This point is also the midpoint of $\overline{AC}$.

39. The circumcenter is the point of intersection of the perpendicular bisectors of the sides of a triangle, and it is equidistant from the vertices of the triangle. In contrast, the incenter is the point of intersection of the angle bisectors of a triangle, and it is equidistant from the sides of the triangle.

41. **a.**



Because this circle is inscribed in the triangle, it is the largest circle that fits inside the triangle without extending into the boundaries.

b. yes; You would keep the center of the pool as the incenter of the triangle, but you would make the radius of the pool at least 1 foot shorter.

43. B

45. yes; In an equilateral triangle, each perpendicular bisector passes through the opposite vertex and divides the triangle into two congruent triangles. So, it is also an angle bisector.

47. **a.** equilateral; 3; In an equilateral triangle, each perpendicular bisector also bisects the opposite angle.

b. scalene; 6; In a scalene triangle, none of the perpendicular bisectors will also bisect an angle.

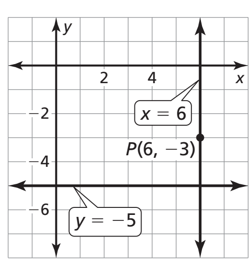
49. angle bisectors; about 2.83 in.

51. $x = \dfrac{AB + AC - BC}{2}$ or $x = \dfrac{AB \cdot AC}{AB + AC + BC}$

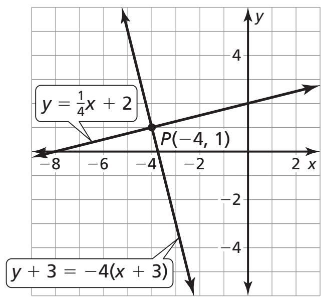
6.2 Maintaining Mathematical Proficiency *(p. 318)*

53. $M(6, 3)$; $AB \approx 11.3$ **55.** $M(-1, 7)$; $AB \approx 12.6$

57. $x = 6$



59. $y = \frac{1}{4}x + 2$



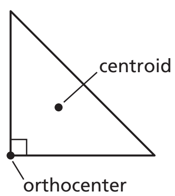
6.3 Vocabulary and Core Concept Check *(p. 324)*

1. circumcenter, incenter, centroid, orthocenter; perpendicular bisectors form the circumcenter, angle bisectors form the incenter, medians form the centroid, altitudes form the orthocenter

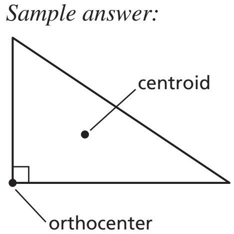
6.3 Monitoring Progress and Modeling with Mathematics *(pp. 324–326)*

3. 6, 3 **5.** 20, 10 **7.** 10, 15 **9.** 18, 27 **11.** 12
13. 10 **15.** $\left(5, \frac{11}{3}\right)$ **17.** (5, 1) **19.** outside; (0, −5)
21. inside; (−1, 2)

23.

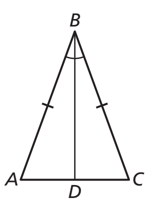


25. *Sample answer:*



27. The length of $\overline{DE}$ should be $\frac{1}{3}$ of the length of $\overline{AE}$ because it is the shorter segment from the centroid to the side;
$DE = \frac{1}{3}AE$
$DE = \frac{1}{3}(18)$
$DE = 6$

29.



Legs $\overline{AB}$ and $\overline{BC}$ of isosceles $\triangle ABC$ are congruent. $\angle ABD \cong \angle CBD$ because $\overline{BD}$ is an angle bisector of vertex angle ABC. Also, $\overline{BD} \cong \overline{BD}$ by the Reflexive Property ofCongruence (Thm. 2.1). So, $\triangle ABD \cong \triangle CBD$ by the SAS Congruence Theorem (Thm. 5.5). $\overline{AD} \cong \overline{CD}$ because corresponding parts of congruent triangles are congruent. So, $\overline{BD}$ is a median.

31. never; Because medians are always inside a triangle, and the centroid is the point of concurrency of the medians, it will always be inside the triangle.

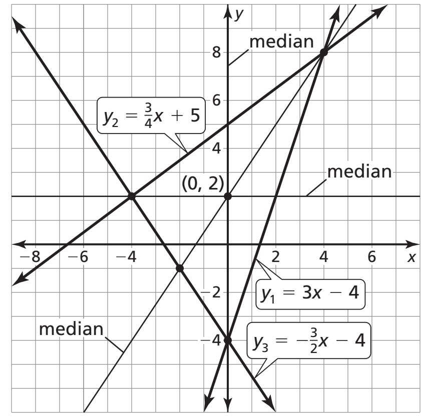
33. sometimes; A median is the same line segment as the perpendicular bisector if the triangle is equilateral or if the segment is connecting the vertex angle to the base of an isosceles triangle. Otherwise, the median and the perpendicular bisectors are not the same segment.

35. sometimes; The centroid and the orthocenter are not the same point unless the triangle is equilateral.

37. Both segments are perpendicular to a side of a triangle, and their point of intersection can fall either inside, on, or outside of the triangle. However, the altitude does not necessarily bisect the side, but the perpendicular bisector does. Also, the perpendicular bisector does not necessarily pass through the opposite vertex, but the altitude does.

39. 6.75 in.^{2}; altitude **41.** $x = 2.5$ **43.** $x = 4$

45.

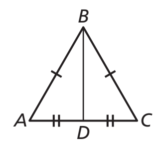


(0, 2)

47. $PE = \frac{1}{3}AE$, $PE = \frac{1}{2}AP$, $PE = AE - AP$

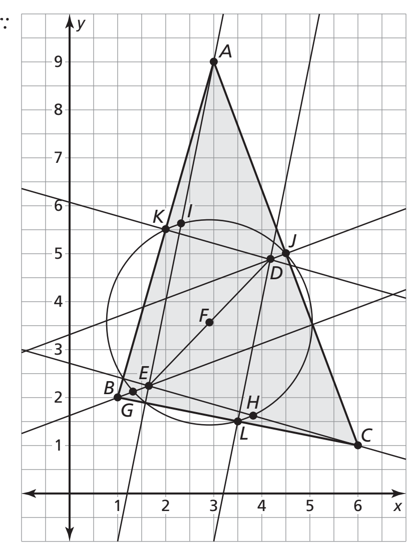
49. yes; If the triangle is equilateral, then the perpendicular bisectors, angle bisectors, medians, and altitudes will all be the same three segments.

51.



Sides $\overline{AB}$ and $\overline{BC}$ of equilateral $\triangle ABC$ are congruent. $\overline{AD} \cong \overline{CD}$ because $\overline{BD}$ is the median to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$ by the Reflexive Property of Congruence (Thm. 2.1). So, $\triangle ABD \cong \triangle CBD$ by the SSS Congruence Theorem (Thm. 5.8). $\angle ADB \cong \angle CDB$ and $\angle ABD \cong \angle CBD$ because corresponding parts of congruent triangles are congruent. Also, $\angle ADB$ and $\angle CDB$ are a linear pair. Because $\overline{BD}$ and $\overline{AC}$ intersect to form a linear pair of congruent angles, $\overline{BD} \perp \overline{AC}$. So, median $\overline{BD}$ is also an angle bisector, altitude, and perpendicular bisector of $\triangle ABC$.

53. *Sample answer:*



The circle passes through nine significant points of the triangle. They are the midpoints of the sides, the midpoints between each vertex and the orthocenter, and the points of intersection between the sides and the altitudes.

6.3 Maintaining Mathematical Proficiency *(p. 326)*

55. yes **57.** no

6.4 Vocabulary and Core Concept Check *(p. 333)*

1. midsegment

6.4 Monitoring Progress and Modeling with Mathematics *(pp. 333–334)*

3. $D(-4, -2), E(-2, 0), F(-1, -4)$

5. Because the slopes of $\overline{EF}$ and $\overline{AC}$ are the same (-4), $\overline{EF} \parallel \overline{AC}$. $EF = \sqrt{17}$ and $AC = 2\sqrt{17}$. Because $\sqrt{17} = \frac{1}{2}\left(2\sqrt{17}\right)$, $EF = \frac{1}{2}AC$.

7. $x = 13$ **9.** $x = 6$ **11.** $\overline{JK} \parallel \overline{YZ}$ **13.** $\overline{XY} \parallel \overline{KL}$

15. $\overline{JL} \cong \overline{XK} \cong \overline{KZ}$ **17.** 14 **19.** 17 **21.** 45 ft

23. An eighth segment, $\overline{FG}$, would connect the midpoints of $\overline{DL}$ and $\overline{EN}$; $\overline{DE} \parallel \overline{LN} \parallel \overline{FG}$, $DE = \frac{3}{4}LN$, and $FG = \frac{7}{8}LN$; Because you are finding quarter segments and eighth segments, use $8p$, $8q$, and $8r$: $L(0, 0)$, $M(8q, 8r)$, and $N(8p, 0)$. Find the coordinates of X, Y, D, E, F, and G. $X(4q, 4r)$, $Y(4q + 4p, 4r)$, $D(2q, 2r)$, $E(2q + 6p, 2r)$, $F(q, r)$, and $G(q + 7p, r)$.

The y-coordinates of D and E are the same, so $\overline{DE}$ has a slope of 0. The y-coordinates of F and G are also the same, so $\overline{FG}$ also has a slope of 0. $\overline{LN}$ is on the x-axis, so its slope is 0. Because their slopes are the same, $\overline{DE} \parallel \overline{LN} \parallel \overline{FG}$.

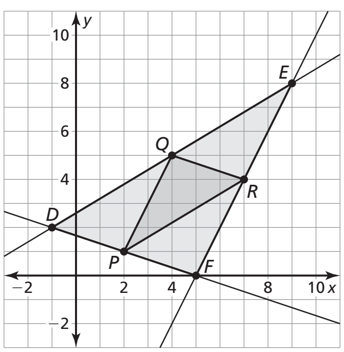
Use the Ruler Postulate (Post. 1.1) to find DE, FG, and LN.

$DE = 6p$, $FG = 7p$, and $LN = 8p$.

Because $6p = \frac{3}{4}(8p)$, $DE = \frac{3}{4}LN$. Because $7p = \frac{7}{8}(8p)$, $FG = \frac{7}{8}LN$.

25. **a.** 24 units **b.** 60 units **c.** 114 units

27. After graphing the midsegments, find the slope of each segment. Graph the line parallel to each midsegment passing through the opposite vertex. The intersections of these three lines will be the vertices of the original triangle: $(-1, 2)$, $(9, 8)$, and $(5, 0)$.



6.4 Maintaining Mathematical Proficiency *(p. 334)*

29. *Sample answer:* An isosceles triangle whose sides are 5 centimeters, 5 centimeters, and 3 centimeters is not equilateral.

6.5 Vocabulary and Core Concept Check *(p. 340)*

1. In an indirect proof, rather than proving a statement directly, you show that when the statement is false, it leads to a contradiction.

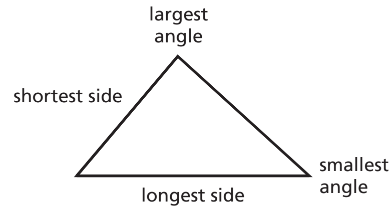
6.5 Monitoring Progress and Modeling with Mathematics *(pp. 340–342)*

3. Assume temporarily that $WV = 7$ inches.

5. Assume temporarily that $\angle B$ is a right angle.

7. A and C; The angles of an equilateral triangle are always 60°. So, an equilateral triangle cannot have a 90° angle, and cannot be a right triangle.

9.



The longest side is across from the largest angle, and the shortest side is across from the smallest angle.

11. $\angle S, \angle R, \angle T$ **13.** $\overline{AB}, \overline{BC}, \overline{AC}$ **15.** $\overline{NP}, \overline{MN}, \overline{MP}$

17. 7 in. $< x <$ 17 in. **19.** 16 in. $< x <$ 64 in. **21.** yes

23. no; $28 + 17 \ngtr 46$

25. An angle that is not obtuse could be acute or right; Assume temporarily that $\angle A$ is not obtuse.

27. Assume temporarily that the client is guilty. Then the client would have been in Los Angeles, California at the time of the crime. Because the client was in New York at the time of the crime, the assumption must be false, and the client must be innocent.

29. C

31. Assume temporarily that an odd number is divisible by 4. Let the odd number be represented by $2y + 1$ where y is a positive integer. Then, there must be a positive integer x such that $4x = 2y + 1$. However, when you divide each side of the equation by 4, you get $x = \frac{1}{2}y + \frac{1}{4}$, which is not an integer. So, the assumption must be false, and an odd number is not divisible by 4.

33. The right angle of a right triangle must always be the largest angle because the other two will have a sum of 90°. So, according to the Triangle Larger Angle Theorem (Thm. 6.10), because the right angle is larger than either of the other angles, the side opposite the right angle, which is the hypotenuse, will always have to be longer than either of the legs.

35. **a.** The width of the river must be greater than 35 yards and less than 50 yards. In $\triangle BCA$, the width of the river, $\overline{BA}$, must be less than the length of $\overline{CA}$, which is 50 yards, because the measure of the angle opposite $\overline{BA}$ is less than the measure of the angle opposite $\overline{CA}$, which must be 50°. In $\triangle BDA$, the width of the river, $\overline{BA}$, must be greater than the length of $\overline{DA}$, which is 35 yards, because the measure of the angle opposite $\overline{BA}$ is greater than the measure of the angle opposite $\overline{DA}$, which must be 40°.

b. You could measure from distances that are closer together. In order to do this, you would have to use angle measures that are closer to 45°.

37. $\angle WXY, \angle YXZ, \angle Z, \angle WYX$ and $\angle XYZ, \angle W$; In $\triangle WXY$, because $WY < WX < YX$, by the Triangle Longer Side Theorem (Thm. 6.9), $m\angle WXY < m\angle WYX < m\angle W$. Similarly, in $\triangle XYZ$, because $YZ < XY < XZ$, by the Triangle Longer Side Theorem (Thm. 6.9), $m\angle YXZ < m\angle Z < m\angle XYZ$. Because $m\angle WYX = m\angle XYZ$ and $\angle W$ is the only angle greater than either of them, we know that $\angle W$ is the largest angle. Because $\triangle WXY$ has the largest angle and one of the congruent angles, the remaining angle, $\angle WXY$, is the smallest.

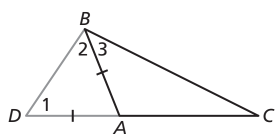
39. By the Exterior Angle Theorem (Thm. 5.2), $m\angle 1 = m\angle A + m\angle B$. Then by the Subtraction Property of Equality, $m\angle 1 - m\angle B = m\angle A$. If you assume temporarily that $m\angle 1 \leq m\angle B$, then $m\angle A \leq 0$. Because the measure of any angle in a triangle must be a positive number, the assumption must be false. So, $m\angle 1 > m\angle B$. Similarly, by the Subtraction Property of Equality, $m\angle 1 - m\angle A = m\angle B$. If you assume temporarily that $m\angle 1 \leq m\angle A$, then $m\angle B \leq 0$. Because the measure of any angle in a triangle must be a positive number, the assumption must be false. So, $m\angle 1 > m\angle A$.

41. $2\frac{1}{7} < x < 13$

43. It is given that $BC > AB$ and $BD = BA$. By the Base Angles Theorem (Thm. 5.6), $m\angle 1 = m\angle 2$. By the Angle Addition Postulate (Post. 1.4), $m\angle BAC = m\angle 1 + m\angle 3$. So, $m\angle BAC > m\angle 1$. Substituting $m\angle 2$ for $m\angle 1$ produces $m\angle BAC > m\angle 2$. By the Exterior Angle Theorem (Thm. 5.2), $m\angle 2 = m\angle 3 + m\angle C$. So, $m\angle 2 > m\angle C$. Finally, because $m\angle BAC > m\angle 2$ and $m\angle 2 > m\angle C$, you can conclude that $m\angle BAC > m\angle C$.

45. no; The sum of the other two sides would be 11 inches, which is less than 13 inches.

47.



Assume $\overline{BC}$ is longer than or the same length as each of the other sides, $\overline{AB}$ and $\overline{AC}$. Then, $AB + BC > AC$ and $AC + BC > AB$. The proof for $AB + AC > BC$ follows.

STATEMENTS	REASONS
1. $\triangle ABC$	1. Given
2. Extend $\overline{AC}$ to D so that $\overline{AB} \cong \overline{AD}$.	2. Ruler Postulate (Post. 1.1)
3. $AB = AD$	3. Definition of segment congruence
4. $AD + AC = DC$	4. Segment Addition Postulate (Post. 1.2)
5. $\angle 1 \cong \angle 2$	5. Base Angles Theorem (Thm. 5.6)
6. $m\angle 1 = m\angle 2$	6. Definition of angle congruence
7. $m\angle DBC > m\angle 2$	7. Protractor Postulate (Post. 1.3)
8. $m\angle DBC > m\angle 1$	8. Substitution Property
9. $DC > BC$	9. Triangle Larger Angle Theorem (Thm. 6.10)
10. $AD + AC > BC$	10. Substitution Property
11. $AB + AC > BC$	11. Substitution Property

49. Assume temporarily that another segment, $\overline{PA}$, where A is on plane M, is the shortest segment from P to plane M. By definition of the distance between a point and a plane, $\overline{PA} \perp$ plane M. This contradicts the given statement because there cannot be two different segments that share an endpoint and are both perpendicular to the same plane. So, the assumption is false, and because no other segment exists that is the shortest segment from P to plane M, it must be $\overline{PC}$ that is the shortest segment from P to plane M.

6.5 Maintaining Mathematical Proficiency *(p. 342)*

51. $\angle ACD$ 53. $\angle CEB$

6.6 Vocabulary and Core Concept Check *(p. 347)*

1. Theorem 6.12 refers to two angles with two pairs of sides that have the same measure, just like two hinges whose sides are the same length. Then, the angle whose measure is greater is opposite a longer side, just like the ends of a hinge are farther apart when the hinge is open wider.

6.6 Monitoring Progress and Modeling with Mathematics *(pp. 347–348)*

3. $m\angle 1 > m\angle 2$; By the Converse of the Hinge Theorem (Thm. 6.13), because $\angle 1$ is the included angle in the triangle with the longer third side, its measure is greater than that of $\angle 2$.

5. $m\angle 1 = m\angle 2$; The triangles are congruent by the SSS Congruence Theorem (Thm. 5.8). So, $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent.

7. $AD > CD$; By the Hinge Theorem (Thm. 6.12), because $\overline{AD}$ is the third side of the triangle with the larger included angle, it is longer than $\overline{CD}$.

9. $TR < UR$; By the Hinge Theorem (Thm. 6.12), because $\overline{TR}$ is the third side of the triangle with the smaller included angle, it is shorter than $\overline{UR}$.

11. $\overline{XY} \cong \overline{YZ}$ and $m\angle WYZ > m\angle WYX$ are given. By the Reflexive Property of Congruence (Thm. 2.1), $\overline{WY} \cong \overline{WY}$. So, by the Hinge Theorem (Thm. 6.12), $WZ > WX$.

13. your flight; Because $160^\circ > 150^\circ$, the distance you flew is a greater distance than the distance your friend flew by the Hinge Theorem (Thm. 6.12).

15. The measure of the included angle in $\triangle PSQ$ is greater than the measure of the included angle in $\triangle SQR$; By the Hinge Theorem (Thm. 6.12), $PQ > SR$.

17. $m\angle EGF > m\angle DGE$ by the Converse of the Hinge Theorem (Thm. 6.13).

19. Because $\overline{NR}$ is a median, $\overline{PR} \cong \overline{QR}$. $\overline{NR} \cong \overline{NR}$ by the Reflexive Property of Congruence (Thm. 2.1). So, by the Converse of the Hinge Theorem (Thm. 6.13), $\angle NRQ > \angle NRP$. Because $\angle NRQ$ and $\angle NRP$ form a linear pair, they are supplementary. So, $\angle NRQ$ must be obtuse and $\angle NRP$ must be acute.

21. $x > \frac{3}{2}$

23. $\triangle ABC$ is an obtuse triangle; If the altitudes intersect inside the triangle, then $m\angle BAC$ will always be less than $m\angle BDC$ because they both intercept the same segment, $\overline{CD}$. However, because $m\angle BAC > m\angle BDC$, $\angle A$ must be obtuse, and the altitudes must intersect outside of the triangle.

6.6 Maintaining Mathematical Proficiency *(p. 348)*

25. $x = 38$ 27. $x = 60$

Chapter 6 Review *(pp. 350–352)*

1. 20; Point B is equidistant from A and C, and $\overleftrightarrow{BD} \perp \overline{AC}$. So, by the Converse of the Perpendicular Bisector Theorem (Thm. 6.2), $DC = AD = 20$.

2. 23; $\angle PQS \cong \angle RQS$, $\overline{SR} \perp \overrightarrow{QR}$, and $\overline{SP} \perp \overrightarrow{QP}$. So, by the Angle Bisector Theorem (Thm. 6.3), $SR = SP$. This means that $6x + 5 = 9x - 4$, and the solution is $x = 3$. So, $RS = 9(3) - 4 = 23$.

3. 47°; Point J is equidistant from $\overrightarrow{FG}$ and $\overrightarrow{FH}$. So, by the Converse of the Angle Bisector Theorem (Thm. 6.4), $m\angle JFH = m\angle JFG = 47°$.

4. $(-3, -3)$ **5.** $(4, 3)$ **6.** $x = 5$ **7.** $(-6, 3)$

8. $(4, -4)$ **9.** inside; $(3, 5.2)$ **10.** outside; $(-6, -1)$

11. $(-6, 6), (-3, 6), (-3, 4)$ **12.** $(0, 3), (2, 0), (-1, -2)$

13. 4 in. $< x <$ 12 in. **14.** 3 m $< x <$ 15 m

15. 7 ft $< x <$ 29 ft

16. Assume temporarily that $YZ \not> 4$. Then, it follows that either $YZ < 4$ or $YZ = 4$. If $YZ < 4$, then $XY + YZ < XZ$ because $4 + YZ < 8$ when $YZ < 4$. If $YZ = 4$, then $XY + YZ = XZ$ because $4 + 4 = 8$. Both conclusions contradict the Triangle Inequality Theorem (Thm. 6.11), which says that $XY + YZ > XZ$. So, the temporary assumption that $YZ \not> 4$ cannot be true. This proves that in $\triangle XYZ$, if $XY = 4$ and $XZ = 8$, then $YZ > 4$.

17. $QT > ST$ **18.** $m\angle QRT > m\angle SRT$

Chapter 7

Chapter 7 Maintaining Mathematical Proficiency *(p. 357)*

1. $x = 3$ **2.** $x = 4$ **3.** $x = 7$ **4.** $a \parallel b, c \perp d$

5. $a \parallel b, c \parallel d, a \perp c, a \perp d, b \perp c, b \perp d$ **6.** $b \parallel c, b \perp d, c \perp d$

7. You can follow the order of operations with all of the other operations in the equation and treat the operations in the expression separately.

7.1 Vocabulary and Core Concept Check *(p. 364)*

1. A segment connecting consecutive vertices is a side of the polygon, not a diagonal.

7.1 Monitoring Progress and Modeling with Mathematics *(pp. 364–366)*

3. 1260° **5.** 2520° **7.** hexagon **9.** 16-gon

11. $x = 64$ **13.** $x = 89$ **15.** $x = 70$ **17.** $x = 150$

19. $m\angle X = m\angle Y = 92°$ **21.** $m\angle X = m\angle Y = 100.5°$

23. $x = 111$ **25.** $x = 32$ **27.** 108°, 72°

29. 172°, 8°

31. The measure of one interior angle of a regular pentagon was found, but the exterior angle should be found by dividing 360° by the number of angles; $\frac{360°}{5} = 72°$

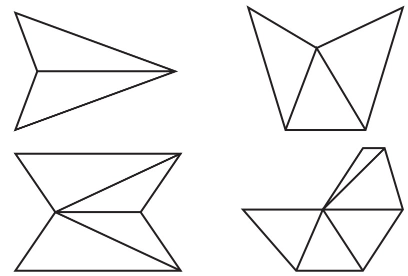
33. 120° **35.** $n = \dfrac{360}{180 - x}$ **37.** 15 **39.** 40

41. A, B; Solving the equation found in Exercise 35 for n yields a positive integer greater than or equal to 3 for A and B, but not for C and D.

43. In a quadrilateral, when all the diagonals from one vertex are drawn, the polygon is divided into two triangles. Because the sum of the measures of the interior angles of each triangle is 180°, the sum of the measures of the interior angles of the quadrilateral is $2 \cdot 180° = 360°$.

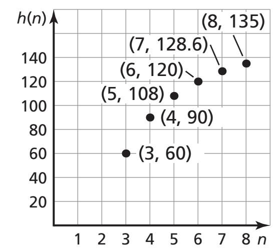
45. 21°, 21°, 21°, 21°, 138°, 138°

47. $(n - 2) \cdot 180°$; When diagonals are drawn from the vertex of the concave angle as shown, the polygon is divided into $n - 2$ triangles whose interior angle measures have the same total as the sum of the interior angle measures of the original polygon.



49. **a.** $h(n) = \dfrac{(n - 2) \cdot 180°}{n}$ **b.** $h(9) = 140°$ **c.** $n = 12$

d.



The value of $h(n)$ increases on a curve that gets less steep as n increases.

51. In a convex n-gon, the sum of the measures of the n interior angles is $(n - 2) \cdot 180°$ using the Polygon Interior Angles Theorem (Thm. 7.1). Because each of the n interior angles forms a linear pair with its corresponding exterior angle, you know that the sum of the measures of the n interior and exterior angles is $180n°$. Subtracting the sum of the interior angle measures from the sum of the measures of the linear pairs gives you $180n° - [(n - 2) \cdot 180°] = 360°$.

7.1 Maintaining Mathematical Proficiency *(p. 366)*

53. $x = 101$ **55.** $x = 16$

7.2 Vocabulary and Core Concept Check *(p. 372)*

1. In order to be a quadrilateral, a polygon must have 4 sides, and parallelograms always have 4 sides. In order to be a parallelogram, a polygon must have 4 sides with opposite sides parallel. Quadrilaterals always have 4 sides, but do not always have opposite sides parallel.

7.2 Monitoring Progress and Modeling with Mathematics *(pp. 372–374)*

3. $x = 9, y = 15$ **5.** $d = 126, z = 28$ **7.** 129°

9. 13; By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $LM = QN$.

11. 8; By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $LQ = MN$.

13. 80°; By the Parallelogram Consecutive Angles Theorem (Thm. 7.5), $\angle QLM$ and $\angle LMN$ are supplementary. So, $m\angle LMN = 180° - 100°$.

15. 100°; By the Parallelogram Opposite Angles Theorem (Thm. 7.4), $m\angle QLM = m\angle MNQ$.

17. $m = 35, n = 110$ **19.** $k = 7, m = 8$

21. In a parallelogram, consecutive angles are supplementary; Because quadrilateral *STUV* is a parallelogram, $\angle S$ and $\angle V$ are supplementary. So, $m\angle V = 180° - 50° = 130°$.

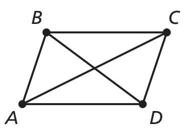
23.

STATEMENTS	REASONS
1. *ABCD* and *CEFD* are parallelograms.	**1.** Given
2. $\overline{AB} \cong \overline{DC}$, $\overline{DC} \cong \overline{FE}$	**2.** Parallelogram Opposite Sides Theorem (Thm. 7.3)
3. $\overline{AB} \cong \overline{FE}$	**3.** Transitive Property of Congruence (Thm. 2.1)

25. (1, 2.5) **27.** *F*(3, 3) **29.** *G*(2, 0) **31.** 36°, 144°

33. no; *Sample answer:* $\angle A$ and $\angle C$ are opposite angles, but $m\angle A \neq m\angle C$.

35. *Sample Answer:*

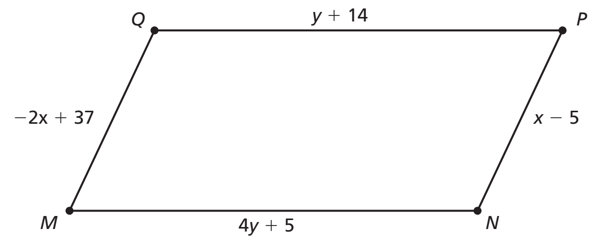


When you fold the parallelogram so that vertex *A* is on vertex *C*, the fold will pass through the point where the diagonals intersect, which demonstrates that this point of intersection is also the midpoint of $\overline{AC}$. Similarly, when you fold the parallelogram so that vertex *B* is on vertex *D*, the fold will pass through the point where the diagonals intersect, which demonstrates that this point of intersection is also the midpoint of $\overline{BD}$.

37.

STATEMENTS	REASONS
1. *ABCD* is a parallelogram.	**1.** Given
2. $\overline{AB} \parallel \overline{DC}$, $\overline{BC} \parallel \overline{AD}$	**2.** Definition of parallelogram
3. $\angle BDA \cong \angle DBC$, $\angle DBA \cong \angle BDC$	**3.** Alternate Interior Angles Theorem (Thm. 3.2)
4. $\overline{BD} \cong \overline{BD}$	**4.** Reflexive Property of Congruence (Thm. 2.1)
5. $\triangle ABD \cong \triangle CDB$	**5.** ASA Congruence Theorem (Thm. 5.10)
6. $\angle A \cong \angle C$	**6.** Corresponding parts of congruent triangles are congruent.
7. $m\angle BDA = m\angle DBC$, $m\angle DBA = m\angle BDC$	**7.** Definition of congruent angles
8. $m\angle B = m\angle DBC + m\angle DBA$, $m\angle D = m\angle BDA + m\angle BDC$	**8.** Angle Addition Postulate
9. $m\angle D = m\angle DBC + m\angle DBA$	**9.** Substitution Property of Equality
10. $m\angle D = m\angle B$	**10.** Transitive Property of Equality
11. $\angle D \cong \angle B$	**11.** Definition of congruent angles

39.



52 units

41. no; Two parallelograms with congruent corresponding sides may or may not have congruent corresponding angles.

43. 16° **45.** 3; (4, 0), (−2, 4), (8, 8)

47.

STATEMENTS	REASONS
1. $\overleftrightarrow{GH} \parallel \overleftrightarrow{JK} \parallel \overleftrightarrow{LM}$, $\overline{GJ} \cong \overline{JL}$	**1.** Given
2. Construct $\overline{PK}$ and $\overline{QM}$ such that $\overline{PK} \parallel \overleftrightarrow{GL} \parallel \overline{QM}$.	**2.** Construction
3. *GPKJ* and *JQML* are parallelograms.	**3.** Definition of parallelogram
4. $\angle GHK \cong \angle JKM$, $\angle PKQ \cong \angle QML$	**4.** Corresponding Angles Theorem (Thm. 3.1)
5. $\overline{GJ} \cong \overline{PK}$, $\overline{JL} \cong \overline{QM}$	**5.** Parallelogram Opposite Sides Theorem (Thm. 7.3)
6. $\overline{PK} \cong \overline{QM}$	**6.** Transitive Property of Congruence (Thm. 2.1)
7. $\angle HPK \cong \angle PKQ$, $\angle KQM \cong \angle QML$	**7.** Alternate Interior Angles Theorem (Thm. 3.2)
8. $\angle HPK \cong \angle QML$	**8.** Transitive Property of Congruence (Thm. 2.2)
9. $\angle HPK \cong \angle KQM$	**9.** Transitive Property of Congruence (Thm. 2.2)
10. $\triangle PHK \cong \triangle QKM$	**10.** AAS Congruence Theorem (Thm. 5.11)
11. $\overline{HK} \cong \overline{KM}$	**11.** Corresponding sides of congruent triangles are congruent.

7.2 Maintaining Mathematical Proficiency *(p. 374)*

49. yes; Alternate Exterior Angles Converse (Thm. 3.7)

7.3 Vocabulary and Core Concept Check *(p. 381)*

1. yes; If all four sides are congruent, then both pairs of opposite sides are congruent. So, the quadrilateral is a parallelogram by the Parallelogram Opposite Sides Converse (Thm. 7.7).

7.3 Monitoring Progress and Modeling with Mathematics *(pp. 381–384)*

3. Parallelogram Opposite Angles Converse (Thm. 7.8)

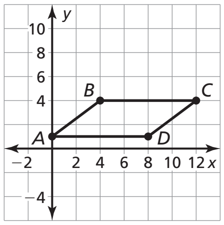
5. Parallelogram Diagonals Converse (Thm. 7.10)

7. Opposite Sides Parallel and Congruent Theorem (Thm. 7.9)

9. $x = 114$, $y = 66$ **11.** $x = 3$, $y = 4$ **13.** $x = 8$

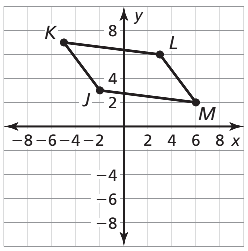
15. $x = 7$

17.



Because $BC = AD = 8$, $\overline{BC} \cong \overline{AD}$. Because both $\overline{BC}$ and $\overline{AD}$ are horizontal lines, their slope is 0, and they are parallel. $\overline{BC}$ and $\overline{AD}$ are opposite sides that are both congruent and parallel. So, *ABCD* is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

19.



Because $JK = LM = 5$ and $KL = JM = \sqrt{65}$, $\overline{JK} \cong \overline{LM}$ and $\overline{KL} \cong \overline{JM}$. Because both pairs of opposite sides are congruent, quadrilateral *JKLM* is a parallelogram by the Parallelogram Opposite Sides Converse (Thm. 7.7).

21. In order to be a parallelogram, the quadrilateral must have two pairs of opposite sides that are congruent, not consecutive sides; *DEFG* is not a parallelogram.

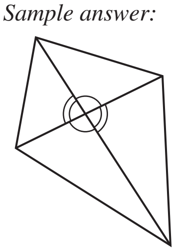
23. $x = 5$; The diagonals must bisect each other so you could solve for x using either $2x + 1 = x + 6$ or $4x - 2 = 3x + 3$. Also, the opposite sides must be congruent, so you could solve for x using either $3x + 1 = 4x - 4$ or $3x + 10 = 5x$.

25. A quadrilateral is a parallelogram if and only if both pairs of opposite sides are congruent.

27. A quadrilateral is a parallelogram if and only if the diagonals bisect each other.

29. Check students' work; Because the diagonals bisect each other, this quadrilateral is a parallelogram by the Parallelogram Diagonals Converse (Thm. 7.10).

31. *Sample answer:*



33. **a.** 27°; Because $\angle EAF$ is a right angle, the other two angles of $\triangle EAF$ must be complementary. So, $m\angle AFE = 90° - 63° = 27°$.

b. Because $\angle GDF$ is a right angle, the other two angles of $\triangle GDF$ must be complementary. So, $m\angle FGD = 90° - 27° = 63°$.

c. 27°; 27°

d. yes; $\angle HEF \cong \angle HGF$ because they both are adjacent to two congruent angles that together add up to 180°, and $\angle EHG \cong \angle GFE$ for the same reason. So, *EFGH* is a parallelogram by the Parallelogram Opposite Angles Converse (Thm. 7.8).

35. You can use the Alternate Interior Angles Converse (Thm. 3.6) to show that $\overline{AD} \parallel \overline{BC}$. Then, $\overline{AD}$ and $\overline{BC}$ are both congruent and parallel. So, *ABCD* is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm 7.9).

37. Use the Corresponding Angles Converse to show that $\overline{AD} \parallel \overline{BC}$ and the Alternate Interior Angles Converse to show that $\overline{AB} \parallel \overline{DC}$. So, *ABCD* is a parallelogram by definition.

39.

STATEMENTS	REASONS
1. $\angle A \cong \angle C$, $\angle B \cong \angle D$	**1.** Given
2. Let $m\angle A = m\angle C = x°$ and $m\angle B = m\angle D = y°$	**2.** Definition of congruent angles
3. $m\angle A + m\angle B + m\angle C + m\angle D = x° + y° + x° + y° = 360°$	**3.** Corollary to the Polygon Interior Angles Theorem (Cor. 7.1)
4. $2(x°) + 2(y°) = 360°$	**4.** Simplify
5. $2(x° + y°) = 360°$	**5.** Distributive Property
6. $x° + y° = 180°$	**6.** Division Property of Equality
7. $m\angle A + m\angle B = 180°$, $m\angle A + m\angle D = 180°$	**7.** Substitution Property of Equality
8. $\angle A$ and $\angle B$ are supplementary. $\angle A$ and $\angle D$ are supplementary.	**8.** Definition of supplementary angles
9. $\overline{BC} \parallel \overline{AD}$, $\overline{AB} \parallel \overline{DC}$	**9.** Consecutive Interior Angles Converse (Thm. 3.8)
10. *ABCD* is a parallelogram.	**10.** Definition of parallelogram

41.

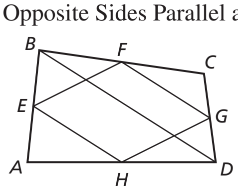
STATEMENTS	REASONS
1. Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other.	**1.** Given
2. $\overline{KP} \cong \overline{MP}$, $\overline{JP} \cong \overline{LP}$	**2.** Definition of segment bisector
3. $\angle KPL \cong \angle MPJ$	**3.** Vertical Angles Congruence Theorem (Thm. 2.6)
4. $\triangle KPL \cong \triangle MPJ$	**4.** SAS Congruence Theorem (Thm. 5.5)
5. $\angle MKL \cong \angle KMJ$, $\overline{KL} \cong \overline{MJ}$	**5.** Corresponding parts of congruent triangles are congruent.
6. $\overline{KL} \parallel \overline{MJ}$	**6.** Alternate Interior Angles Converse (Thm. 3.6)
7. *JKLM* is a parallelogram.	**7.** Opposite Sides Parallel and Congruent Theorem (Thm. 7.9)

43. no; The fourth angle will be 113° because of the Corollary to the Polygon Interior Angles Theorem (Cor. 7.1), but these could also be the angle measures of an isosceles trapezoid with base angles that are each 67°.

45. 8; By the Parallelogram Opposite Sides Theorem (Thm. 7.3), $\overline{AB} \cong \overline{CD}$. Also, $\angle ABE$ and $\angle CDF$ are congruent alternate interior angles of parallel segments $\overline{AB}$ and $\overline{CD}$. Then, you can use the Segment Addition Postulate (Post. 1.2), the Substitution Property of Equality, and the Reflexive Property of Congruence (Thm. 2.1) to show that $\overline{DF} \cong \overline{BE}$. So, $\triangle ABE \cong \triangle CDF$ by the SAS Congruence Theorem (Thm. 5.5), which means that $AE = CF = 8$ because corresponding parts of congruent triangles are congruent.

47. If every pair of consecutive angles of a quadrilateral is supplementary, then the quadrilateral is a parallelogram; In $ABCD$, you are given that $\angle A$ and $\angle B$ are supplementary, and $\angle B$ and $\angle C$ are supplementary. So, $m\angle A = m\angle C$. Also, $\angle B$ and $\angle C$ are supplementary, and $\angle C$ and $\angle D$ are supplementary. So, $m\angle B = m\angle D$. So, $ABCD$ is a parallelogram by the Parallelogram Opposite Angles Converse (Thm. 7.8).

49. Given quadrilateral $ABCD$ with midpoints E, F, G, and H that are joined to form a quadrilateral, you can construct diagonal $\overline{BD}$. Then $\overline{FG}$ is a midsegment of $\triangle BCD$, and $\overline{EH}$ is a midsegment of $\triangle DAB$. So, by the Triangle Midsegment Theorem (Thm. 6.8), $\overline{FG} \parallel \overline{BD}$, $FG = \frac{1}{2}BD$, $\overline{EH} \parallel \overline{BD}$, and $EH = \frac{1}{2}BD$. So, by the Transitive Property of Parallel Lines (Thm. 3.9), $\overline{EH} \parallel \overline{FG}$ and by the Transitive Property of Equality, $EH = FG$. Because one pair of opposite sides is both congruent and parallel, $EFGH$ is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).



7.3 Maintaining Mathematical Proficiency *(p. 384)*

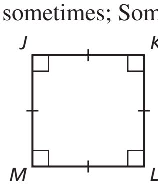
51. parallelogram **53.** square

7.4 Vocabulary and Core Concept Check *(p. 393)*

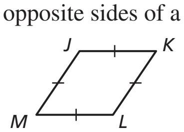
1. square

7.4 Monitoring Progress and Modeling with Mathematics *(pp. 393–396)*

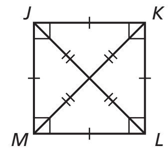
3. sometimes; Some rhombuses are squares.



5. always; By definition, a rhombus is a parallelogram, and opposite sides of a parallelogram are congruent.



7. sometimes; Some rhombuses are squares.



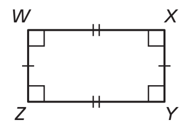
9. square; All of the sides are congruent, and all of the angles are congruent.

11. rectangle; Opposite sides are parallel and the angles are 90°.

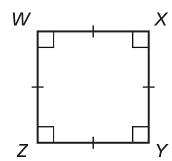
13. $m\angle 1 = m\angle 2 = m\angle 4 = 27°$, $m\angle 3 = 90°$; $m\angle 5 = m\angle 6 = 63°$

15. $m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 37°$; $m\angle 5 = 106°$

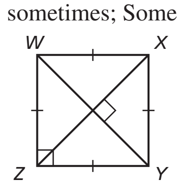
17. always; All angles of a rectangle are congruent.



19. sometimes; Some rectangles are squares.



21. sometimes; Some rectangles are squares.



23. no; All four angles are not congruent. **25.** 11 **27.** 4

29. rectangle, square **31.** rhombus, square

33. parallelogram, rectangle, rhombus, square

35. Diagonals do not necessarily bisect opposite angles of a rectangle;

$$m\angle QSR = 90° - m\angle QSP$$
$$x = 32$$

37. 53° **39.** 74° **41.** 6 **43.** 56° **45.** 56°

47. 10 **49.** 90° **51.** 45° **53.** 2

55. rectangle, rhombus, square; The diagonals are congruent and perpendicular.

57. rectangle; The sides are perpendicular and not congruent.

59. rhombus; The diagonals are perpendicular and not congruent.

61. rhombus; The sides are congruent; $x = 76$; $y = 4$

63. **a.** rhombus; rectangle; $HBDF$ has four congruent sides; $ACEG$ has four right angles.

b. $AE = GC$; $AJ = JE = CJ = JG$; The diagonals of a rectangle are congruent and bisect each other.

65. always; By the Square Corollary (Cor. 7.4), a square is a rhombus.

67. always; The diagonals of a rectangle are congruent by the Rectangle Diagonals Theorem (Thm. 7.13).

69. sometimes; Some rhombuses are squares.

71. Measure the diagonals to see if they are congruent.

73.

STATEMENTS	REASONS
1. *PQRS* is a parallelogram. $\overline{PR}$ bisects $\angle SPQ$ and $\angle QRS$. $\overline{SQ}$ bisects $\angle PSR$ and $\angle RQP$.	1. Given
2. $\angle SRT \cong \angle QRT$, $\angle RQT \cong \angle RST$	2. Definition of angle bisector
3. $\overline{TR} \cong \overline{TR}$	3. Reflexive Property of Congruence (Thm. 2.1)
4. $\triangle QRT \cong \triangle SRT$	4. AAS Congruence Theorem (Thm. 5.11)
5. $\overline{QR} \cong \overline{SR}$	5. Corresponding parts of congruent triangles are congruent.
6. $\overline{QR} \cong \overline{PS}$, $\overline{PQ} \cong \overline{SR}$	6. Parallelogram Opposite Sides Theorem (Thm. 7.3)
7. $\overline{PS} \cong \overline{QR} \cong \overline{SR} \cong \overline{PQ}$	7. Transitive Property of Congruence (Thm. 2.1)
8. *PQRS* is a rhombus.	8. Definition of rhombus

75. no; The diagonals of a square always create two right triangles.

77. square; A square has four congruent sides and four congruent angles.

79. no; yes; Corresponding angles of two rhombuses might not be congruent; Corresponding angles of two squares are congruent.

81. If a quadrilateral is a rhombus, then it has four congruent sides; If a quadrilateral has four congruent sides, then it is a rhombus; The conditional statement is true by the definition of rhombus. The converse is true because if a quadrilateral has four congruent sides, then both pairs of opposite sides are congruent. So, by the Parallelogram Opposite Sides Converse (Thm. 7.7), it is a parallelogram with four congruent sides, which is the definition of a rhombus.

83. If a quadrilateral is a square, then it is a rhombus and a rectangle; If a quadrilateral is a rhombus and a rectangle, then it is a square; The conditional statement is true because if a quadrilateral is a square, then by definition of a square, it has four congruent sides, which makes it a rhombus by the Rhombus Corollary (Cor. 7.2), and it has four right angles, which makes it a rectangle by the Rectangle Corollary (Cor. 7.3); The converse is true because if a quadrilateral is a rhombus and a rectangle, then by the Rhombus Corollary (Cor. 7.2), it has four congruent sides, and by the Rectangle Corollary (Cor. 7.3), it has four right angles. So, by the definition, it is a square.

85.

STATEMENTS	REASONS
1. $\triangle XYZ \cong \triangle XWZ$, $\angle XYW \cong \angle ZWY$	1. Given
2. $\angle YXZ \cong \angle WXZ$, $\angle YZX \cong \angle WZX$, $\overline{XY} \cong \overline{XW}$, $\overline{YZ} \cong \overline{WZ}$	2. Corresponding parts of congruent triangles are congruent.
3. $\overline{XZ}$ bisects $\angle WXY$ and $\angle WZY$.	3. Definition of angle bisector
4. $\angle XWY \cong \angle XYW$, $\angle WYZ \cong \angle ZWY$	4. Base Angles Theorem (Thm. 5.6)
5. $\angle XYW \cong \angle WYZ$, $\angle XWY \cong \angle ZWY$	5. Transitive Property of Congruence (Thm. 2.2)
6. $\overline{WY}$ bisects $\angle XWZ$ and $\angle XYZ$.	6. Definition of angle bisector
7. *WXYZ* is a rhombus.	7. Rhombus Opposite Angles Theorem (Thm. 7.12)

87.

STATEMENTS	REASONS
1. *PQRS* is a rectangle.	1. Given
2. *PQRS* is a parallelogram.	2. Def. of rectangle
3. $\overline{PS} \cong \overline{QR}$	3. Parallelogram Opposite Sides Thm. (Thm. 7.3)
4. $\angle PQR$ and $\angle QPS$ are right angles.	4. Def. of rectangle
5. $\angle PQR \cong \angle QPS$	5. Right Angles Congruence Thm. (Thm. 2.3)
6. $\overline{PQ} \cong \overline{PQ}$	6. Reflexive Property of Congruence (Thm. 2.1)
7. $\triangle PQR \cong \triangle QPS$	7. SAS Congruence Theorem (Thm. 5.5)
8. $\overline{PR} \cong \overline{SQ}$	8. Corresponding parts of congruent triangles are congruent.

7.4 Maintaining Mathematical Proficiency *(p. 396)*

89. $x = 10, y = 8$ **91.** $x = 9, y = 26$

7.5 Vocabulary and Core Concept Check *(p. 403)*

1. A trapezoid has exactly one pair of parallel sides, and a kite has two pairs of consecutive congruent sides.

7.5 Monitoring Progress and Modeling with Mathematics *(pp. 403–406)*

3. slope of $\overline{YZ}$ = slope of $\overline{XW}$ and slope of $\overline{XY} \neq$ slope of $\overline{WZ}$; $XY = WZ$, so *WXYZ* is isosceles.

5. slope of $\overline{MQ}$ = slope of $\overline{NP}$ and slope of $\overline{MN} \neq$ slope of $\overline{PQ}$; $MN \neq PQ$, so *MNPQ* is not isosceles.

7. $m\angle L = m\angle M = 62°, m\angle K = m\angle J = 118°$ **9.** 14

11. 4 **13.** $3\sqrt{13}$ **15.** 110° **17.** 80°

19. Because $MN = \frac{1}{2}(AB + DC)$, when you solve for DC, you should get $DC = 2(MN) - AB$; $DC = 2(8) - 14 = 2$.

21. rectangle; *JKLM* is a quadrilateral with 4 right angles.

Selected Answers

23. square; All four sides are congruent and the angles are 90°.

25. no; It could be a kite. **27.** 3 **29.** 26 in.

31. $\angle A \cong \angle D$, or $\angle B \cong \angle C$; $\overline{BC} \parallel \overline{AD}$, so base angles need to be congruent.

33. *Sample answer:* $\overline{BE} \cong \overline{DE}$; Then the diagonals bisect each other.

35.

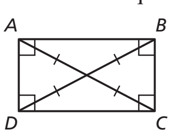
STATEMENTS	REASONS
1. $\overline{JL} \cong \overline{LN}$, $\overline{KM}$ is a midsegment of $\triangle JLN$.	**1.** Given
2. $\overline{KM} \parallel \overline{JN}$	**2.** Triangle Midsegment Theorem (Thm. 6.8)
3. $JKMN$ is a trapezoid.	**3.** Definition of trapezoid
4. $\angle LJN \cong \angle LNJ$	**4.** Base Angles Theorem (Thm. 5.6)
5. $JKMN$ is an isosceles trapezoid.	**5.** Isosceles Trapezoid Base Angles Converse (Thm. 7.15)

37. any point on $\overleftrightarrow{UV}$ such that $UV \neq SV$ and $SV \neq 0$

39. Given isosceles trapezoid $ABCD$ with $\overline{BC} \parallel \overline{AD}$, construct $\overline{CE}$ parallel to $\overline{BA}$. Then, $ABCE$ is a parallelogram by definition, so $\overline{AB} \cong \overline{EC}$. Because $\overline{AB} \cong \overline{CD}$ by the definition of an isosceles trapezoid, $\overline{CE} \cong \overline{CD}$ by the Transitive Property of Congruence (Thm. 2.1). So, $\angle CED \cong \angle D$ by the Base Angles Theorem (Thm. 5.6) and $\angle A \cong \angle CED$ by the Corresponding Angles Theorem (Thm. 3.1). So, $\angle A \cong \angle D$ by the Transitive Property of Congruence (Thm. 2.2). Next, by the Consecutive Interior Angles Theorem (Thm. 3.4), $\angle B$ and $\angle A$ are supplementary and so are $\angle BCD$ and $\angle D$. So, $\angle B \cong \angle BCD$ by the Congruent Supplements Theorem (Thm. 2.4).

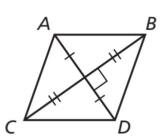
41. no; It could be a square or rectangle.

43. a.



rectangle; The diagonals are congruent, but not perpendicular.

b.



rhombus; The diagonals are perpendicular, but not congruent.

45. a. yes **b.** 75°, 75°, 105°, 105°

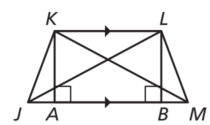
47. Given kite $EFGH$ with $\overline{EF} \cong \overline{FG}$ and $\overline{EH} \cong \overline{GH}$, construct diagonal $\overline{FH}$, which is congruent to itself by the Reflexive Property of Congruence (Thm. 2.1). So, $\triangle FGH \cong \triangle FEH$ by the SSS Congruence Theorem (Thm. 5.8), and $\angle E \cong \angle G$ because corresponding parts of congruent triangles are congruent. Next, assume temporarily that $\angle F \cong \angle H$. Then $EFGH$ is a parallelogram by the Parallelogram Opposite Angles Converse (Thm. 7.8), and opposite sides are congruent. However, this contradicts the definition of a kite, which says that opposite sides cannot be congruent. So, the assumption cannot be true and $\angle F$ is not congruent to $\angle H$.

49. By the Triangle Midsegment Theorem (Thm. 6.8), $\overline{BG} \parallel \overline{CD}$, $BG = \frac{1}{2}CD$, $\overline{GE} \parallel \overline{AF}$ and $GE = \frac{1}{2}AF$. By the Transitive Property of Parallel Lines (Thm. 3.9), $\overline{CD} \parallel \overline{BE} \parallel \overline{AF}$. Also, by the Segment Addition Postulate (Post. 1.2), $BE = BG + GE$. So, by the Substitution Property of Equality, $BE = \frac{1}{2}CD + \frac{1}{2}AF = \frac{1}{2}(CD + AF)$.

51. a.

STATEMENTS	REASONS
1. $JKLM$ is an isosceles trapezoid, $\overline{KL} \parallel \overline{JM}$, $\overline{JK} \cong \overline{LM}$	**1.** Given
2. $\angle JKL \cong \angle MLK$	**2.** Isosceles Trapezoid Base Angles Theorem (Thm. 7.14)
3. $\overline{KL} \cong \overline{KL}$	**3.** Reflexive Property of Congruence (Thm. 2.1)
4. $\triangle JKL \cong \triangle MLK$	**4.** SAS Congruence Theorem (Thm. 5.5)
5. $\overline{JL} \cong \overline{KM}$	**5.** Corresponding parts of congruent triangles are congruent.

b. If the diagonals of a trapezoid are congruent, then the trapezoid is isosceles. Let $JKLM$ be a trapezoid, $\overline{KL} \parallel \overline{JM}$ and $\overline{JL} \cong \overline{KM}$. Construct line segments through K and L perpendicular to $\overline{JM}$ as shown.



Because $\overline{KL} \parallel \overline{JM}$, $\angle AKL$ and $\angle KLB$ are right angles, so $KLBA$ is a rectangle and $\overline{AK} \cong \overline{BL}$. Then $\triangle JLB \cong \triangle MKA$ by the HL Congruence Theorem (Thm. 5.9). So, $\angle LJB \cong \angle KMA$. $\overline{JM} \cong \overline{JM}$ by the Reflexive Property of Congruence (Thm. 2.1). So, $\triangle KJM \cong \triangle LMJ$ by the SAS Congruence Theorem (Thm. 5.5). Then $\angle KJM \cong \angle LMJ$, and the trapezoid is isosceles by the Isosceles Trapezoid Base Angles Converse (Thm. 7.15).

7.5 Maintaining Mathematical Proficiency *(p. 406)*

53. *Sample answer:* translation 1 unit right followed by a dilation with a scale factor of 2

Chapter 7 Review *(pp. 408–410)*

1. 5040°; 168°; 12° **2.** 133 **3.** 82 **4.** 15

5. $a = 79$, $b = 101$ **6.** $a = 28$, $b = 87$

7. $c = 6$, $d = 10$ **8.** $(-2, -1)$ **9.** $M(2, -2)$

10. Parallelogram Opposite Sides Converse (Thm. 7.7)

11. Parallelogram Diagonals Converse (Thm. 7.10)

12. Parallelogram Opposite Angles Converse (Thm. 7.8)

13. $x = 1$, $y = 6$ **14.** 4

15. Because $WX = YZ = \sqrt{13}$, $\overline{WX} \cong \overline{YZ}$. Because the slopes of $\overline{WX}$ and $\overline{YZ}$ are both $\frac{2}{3}$, they are parallel. $\overline{WX}$ and $\overline{YZ}$ are opposite sides that are both congruent and parallel. So, $WXYZ$ is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Thm. 7.9).

16. rhombus; There are four congruent sides.

17. parallelogram; There are two pairs of parallel sides.

18. square; There are four congruent sides and the angles are 90°.

19. 10

20. rectangle, rhombus, square; The diagonals are congruent and perpendicular.

21. $m\angle Z = m\angle Y = 58°$, $m\angle W = m\angle X = 122°$ **22.** 26

23. $3\sqrt{5}$ **24.** $x = 15$; 105°

25. yes; Use the Isosceles Trapezoid Base Angles Converse (Thm. 7.15).

26. trapezoid;There is one pair of parallel sides.

27. rhombus; There are four congruent sides.

28. rectangle; There are four right angles.

Chapter 8

Chapter 8 Maintaining Mathematical Proficiency *(p. 415)*

1. yes **2.** yes **3.** no **4.** no **5.** yes

6. yes **7.** $k = \frac{3}{7}$ **8.** $k = \frac{8}{3}$ **9.** $k = 2$

10. yes; All of the ratios are equivalent by the Transitive Property of Equality.

8.1 Vocabulary and Core Concept Check *(p. 423)*

1. congruent; proportional

8.1 Monitoring Progress and Modeling with Mathematics *(pp. 423–426)*

3. $\frac{4}{3}$; $\angle A \cong \angle L$, $\angle B \cong \angle M$, $\angle C \cong \angle N$; $\frac{LM}{AB} = \frac{MN}{BC} = \frac{NL}{CA}$

5. $x = 30$ **7.** $x = 11$ **9.** altitude; 24 **11.** 2 : 3

13. 72 cm **15.** 20 yd **17.** 288 ft, 259.2 ft

19. 108 ft^2 **21.** 4 in.^{2}

23. Because the first ratio has a side length of B over a side length of A, the second ratio should have the perimeter of B over the perimeter of A;

$$\frac{5}{10} = \frac{x}{28}$$

$$x = 14$$

25. no; Corresponding angles are not congruent. **27.** A, D

29. $\frac{5}{2}$ **31.** 34, 85 **33.** 60.5, 378.125 **35.** B, D

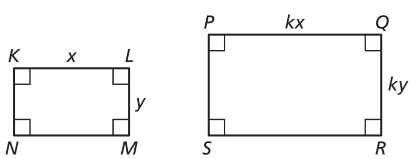
37. $x = 35.25$, $y = 20.25$ **39.** 30 m **41.** 7.5 ft

43. sometimes **45.** sometimes **47.** sometimes

49. yes; All four angles of each rectangle will always be congruent right angles.

51. about 1116 mi

53.



Let $KLMN$ and $PQRS$ be similar rectangles as shown. The ratio of corresponding side lengths is $\frac{KL}{PQ} = \frac{x}{kx} = \frac{1}{k}$. The area of $KLMN$ is xy and the area of $PQRS$ is $(kx)(ky) = k^2xy$. So, the ratio of the areas is $\frac{xy}{k^2xy} = \frac{1}{k^2} = \left(\frac{1}{k}\right)^2$. Because the ratio of corresponding side lengths is $\frac{1}{k}$, any pair of corresponding side lengths can be substituted for $\frac{1}{k}$. So,

$$\frac{\text{Area of } KLMN}{\text{Area of } PQRS} = \left(\frac{KL}{PQ}\right)^2 = \left(\frac{LM}{QR}\right)^2 = \left(\frac{MN}{RS}\right)^2 = \left(\frac{NK}{SP}\right)^2.$$

55. $x = \frac{1 + \sqrt{5}}{2}$; $x = \frac{1 + \sqrt{5}}{2}$ satisfies the proportion $\frac{1}{x} = \frac{x - 1}{1}$.

8.1 Maintaining Mathematical Proficiency *(p. 426)*

57. $x = 63$ **59.** $x = 64$

8.2 Vocabulary and Core Concept Check *(p. 431)*

1. similar

8.2 Monitoring Progress and Modeling with Mathematics *(pp. 431–432)*

3. yes; $\angle H \cong \angle J$ and $\angle F \cong \angle K$, so $\triangle FGH \sim \triangle KLJ$.

5. no; $m\angle N = 50°$

7. $\angle N \cong \angle Z$ and $\angle MYN \cong \angle XYZ$, so $\triangle MYN \sim \triangle XYZ$.

9. $\angle Y \cong \angle Y$ and $\angle YZX \cong \angle W$, so $\triangle XYZ \sim \triangle UYW$.

11. $\triangle CAG \sim \triangle CEF$ **13.** $\triangle ACB \sim \triangle ECD$

15. $m\angle ECD = m\angle ACB$ **17.** $BC = 4\sqrt{2}$

19. The AA Similarity Theorem (Thm. 8.3) does not apply to quadrilaterals. There is not enough information to determine whether or not quadrilaterals $ABCD$ and $EFGH$ are similar.

21. 78 m; Corresponding angles are congruent, so the triangles are similar.

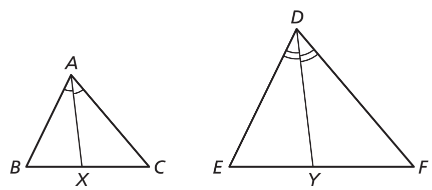
23. yes; Corresponding angles are congruent.

25. no; 94° + 87° > 180°

27. *Sample answer:* Because the triangles are similar, the ratios of the vertical sides to the horizontal sides are equal.

29. The angle measures are 60°.

31.



Let $\triangle ABC \sim \triangle DEF$ with a scale factor of k, and $\overline{AX}$ and $\overline{DY}$ be angle bisectors as shown. Then $\angle C \cong \angle F$, $m\angle CAB = m\angle FDE$, $2m\angle CAX = m\angle CAB$ and $2m\angle FDY = m\angle FDE$. By the Substitution Property of Equality, $2m\angle CAX = 2m\angle FDY$, so $m\angle CAX = m\angle FDY$. Then $\triangle ACX \sim \triangle DFY$ by the AA Similarity Theorem (Thm. 8.3), and because corresponding side lengths are proportional,

$$\frac{AX}{DY} = \frac{AC}{DF} = k.$$

33. about 17.1 ft; $\triangle AED \sim \triangle CEB$, so $\frac{DE}{BE} = \frac{4}{3}$. $\triangle DEF \sim \triangle DBC$, so $\frac{EF}{30} = \frac{DE}{DB} = \frac{4}{7}$ and $EF = \frac{120}{7}$.

8.2 Maintaining Mathematical Proficiency *(p. 432)*

35. yes; Use the SSS Congruence Theorem (Thm. 5.8).

8.3 Vocabulary and Core Concept Check *(p. 441)*

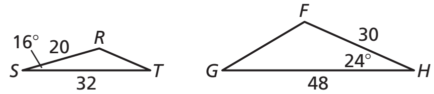
1. $\frac{QR}{XY} = \frac{RS}{YZ} = \frac{QS}{XZ}$

8.3 Monitoring Progress and Modeling with Mathematics *(pp. 441–444)*

3. $\triangle RST$ **5.** $x = 4$ **7.** $\frac{12}{18} = \frac{10}{15} = \frac{8}{12} = \frac{2}{3}$

9. similar; $\triangle DEF \sim \triangle WXY$; $\frac{4}{3}$

11.



no

Selected Answers

13. $\frac{HG}{HF} = \frac{HJ}{HK} = \frac{GJ}{FK}$, so $\triangle GHJ \sim \triangle FHK$.

15. $\angle X \cong \angle D$ and $\frac{XY}{DJ} = \frac{XZ}{DG}$, so $\triangle XYZ \sim \triangle DJG$.

17. 24, 26

19. Because $\overline{AB}$ corresponds to $\overline{RQ}$ and $\overline{BC}$ corresponds to $\overline{QP}$, the proportionality statement should be $\triangle ABC \sim \triangle RQP$.

21. 61° **23.** 30° **25.** 91°

27. no; The included angles are not congruent.

29. D; $\angle M \cong \angle M$ so, $\triangle MNP \sim \triangle MRQ$ by the SAS Similarity Theorem (Thm. 8.5).

31. a. $\frac{CD}{CE} = \frac{BC}{AC}$ **b.** $\angle CBD \cong \angle CAE$

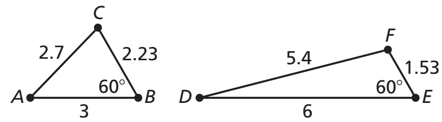
33.

STATEMENTS	REASONS
1. $\angle A \cong \angle D$, $\frac{AB}{DE} = \frac{AC}{DF}$	**1.** Given
2. Draw $\overline{PQ}$ so that P is on $\overline{AB}$, Q is on $\overline{AC}$, $\overline{PQ} \parallel \overline{BC}$, and $AP = DE$.	**2.** Parallel Postulate (Post. 3.1)
3. $\angle APQ \cong \angle ABC$	**3.** Corresponding Angles Theorem (Thm. 3.1)
4. $\angle A \cong \angle A$	**4.** Reflexive Property of Congruence (Thm. 2.2)
5. $\triangle APQ \sim \triangle ABC$	**5.** AA Similarity Theorem (Thm. 8.3)
6. $\frac{AB}{AP} = \frac{AC}{AQ} = \frac{BC}{PQ}$	**6.** Corresponding sides of similar figures are proportional.
7. $\frac{AB}{DE} = \frac{AC}{AQ}$	**7.** Substitution Property of Equality
8. $AQ \cdot \frac{AB}{DE} = AC$, $DF \cdot \frac{AB}{DE} = AC$	**8.** Multiplication Property of Equality
9. $AQ = AC \cdot \frac{DE}{AB}$, $DF = AC \cdot \frac{DE}{AB}$	**9.** Multiplication Property of Equality
10. $AQ = DF$	**10.** Transitive Property of Equality
11. $\overline{AQ} \cong \overline{DF}$, $\overline{AP} \cong \overline{DE}$	**11.** Definition of congruent segments
12. $\triangle APQ \cong \triangle DEF$	**12.** SAS Congruence Theorem (Thm. 5.5)
13. $\overline{PQ} \cong \overline{EF}$	**13.** Corresponding parts of congruent triangles are congruent.
14. $PQ = EF$	**14.** Definition of congruent segments
15. $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$	**15.** Substitution Property of Equality
16. $\triangle ABC \sim \triangle DEF$	**16.** SSS Similarity Theorem (Thm. 8.4)

35. no; no; The sum of the angle measures would not be 180°.

37. If two angles are congruent, then the triangles are similar by the AA Similarity Theorem (Thm. 8.3).

39. *Sample answer:*



41. the Substitution Property of Equaltiy; $\frac{BC}{EF} = \frac{AC}{DF}$; $\angle ACB \cong \angle DFE$; SAS Similarity Theorem (Thm. 8.5); Corresponding Angles Converse (Thm. 3.5)

8.3 Maintaining Mathematical Proficiency *(p. 444)*

43. $P(0, 3)$ **45.** $P(5, 6)$

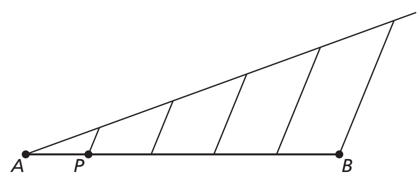
8.4 Vocabulary and Core Concept Check *(p. 450)*

1. parallel, Converse of the Triangle Proportionality Theorem (Thm. 8.7)

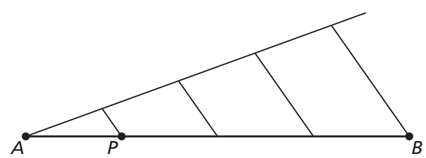
8.4 Monitoring Progress and Modeling with Mathematics *(pp. 450–452)*

3. 9 **5.** yes **7.** no

9.



11.



13. CE **15.** BD **17.** 6 **19.** 12 **21.** 27

23. The proportion should show that AD corresponds with DC and BA corresponds with BC;

$$\frac{AD}{DC} = \frac{BA}{BC}$$

$$\frac{x}{14} = \frac{10}{16}$$

$$x = 8.75$$

25. $x = 3$

27.

STATEMENTS	REASONS
1. $\overline{QS} \parallel \overline{TU}$	1. Given
2. $\angle RQS \cong \angle RTU$, $\angle RSQ \cong \angle RUT$	2. Corresponding Angles Theorem (Thm. 3.1)
3. $\triangle RQS \sim \triangle RTU$	3. AA Similarity Theorem (Thm. 8.3)
4. $\dfrac{QR}{TR} = \dfrac{SR}{UR}$	4. Corresponding side lengths of similar figures are proportional.
5. $QR = QT + TR$, $SR = SU + UR$	5. Segment Addition Postulate (Post. 1.2)
6. $\dfrac{QT + TR}{TR} = \dfrac{SU + UR}{UR}$	6. Substitution Property of Equality
7. $\dfrac{QT}{TR} + \dfrac{TR}{TR} = \dfrac{SU}{UR} + \dfrac{UR}{UR}$	7. Rewrite the proportion.
8. $\dfrac{QT}{TR} + 1 = \dfrac{SU}{UR} + 1$	8. Simplify.
9. $\dfrac{QT}{TR} = \dfrac{SU}{UR}$	9. Subtraction Property of Equality

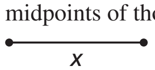
29. **a.** about 50.9 yd, about 58.4 yd, about 64.7 yd **b.** Lot C
c. about \$287,000, about \$318,000; $\dfrac{50.9}{250{,}000} \approx \dfrac{58.4}{287{,}000}$ and $\dfrac{50.9}{250{,}000} \approx \dfrac{64.7}{318{,}000}$

31. Because $\overline{DJ}, \overline{EK}, \overline{FL}$, and $\overline{GB}$ are cut by a transversal $\overrightarrow{AC}$, and $\angle ADJ \cong \angle DEK \cong \angle EFL \cong \angle FGB$ by construction, $\overline{DJ} \parallel \overline{EK} \parallel \overline{FL} \parallel \overline{GB}$ by the Corresponding Angles Converse (Thm. 3.5).

33. isosceles; By the Triangle Angle Bisector Theorem (Thm. 8.9), the ratio of the lengths of the segments of $\overline{LN}$ equals the ratio of the other two side lengths. Because $\overline{LN}$ is bisected, the ratio is 1, and $ML = MN$.

35. Because $\overline{WX} \parallel \overline{ZA}$, $\angle XAZ \cong \angle YXW$ by the Corresponding Angles Theorem (Thm. 3.1) and $\angle WXZ \cong \angle XZA$ by the Alternate Interior Angles Theorem (Thm. 3.2). So, by the Transitive Property of Congruence (Thm. 2.2), $\angle XAZ \cong \angle XZA$. Then $\overline{XA} \cong \overline{XZ}$ by the Converse of the Base Angles Theorem (Thm. 5.7), and by the Triangle Proportionality Theorem (Thm. 8.6), $\dfrac{YW}{WZ} = \dfrac{XY}{XA}$. Because $XA = XZ$, $\dfrac{YW}{WZ} = \dfrac{XY}{XZ}$.

37. The Triangle Midsegment Theorem (Thm. 6.8) is a specific case of the Triangle Proportionality Theorem (Thm. 8.6) when the segment parallel to one side of a triangle that connects the other two sides also happens to pass through the midpoints of those two sides.

39.


8.4 Maintaining Mathematical Proficiency *(p. 452)*

41. a, b **43.** $x = \pm 11$ **45.** $x = \pm 7$

Chapter 8 Review *(pp 454–456)*

1. $\frac{3}{4}$; $\angle A \cong \angle E, \angle B \cong \angle F, \angle C \cong \angle G, \angle D \cong \angle H$; $\dfrac{EF}{AB} = \dfrac{FG}{BC} = \dfrac{GH}{CD} = \dfrac{EH}{AD}$

2. $\frac{2}{5}$; $\angle X \cong \angle R, \angle Y \cong \angle P, \angle Z \cong \angle Q$; $\dfrac{RP}{XY} = \dfrac{PQ}{YZ} = \dfrac{RQ}{XZ}$

3. 14.4 in. **4.** $P = 32$ m; $A = 80$ m^2

5. $\angle Q \cong \angle T$ and $\angle RSQ \cong \angle UST$, so $\triangle RSQ \sim \triangle UST$.

6. $\angle C \cong \angle F$ and $\angle B \cong \angle E$, so $\triangle ABC \sim \triangle DEF$.

7. 324 ft

8. $\angle C \cong \angle C$ and $\dfrac{CD}{CE} = \dfrac{CB}{CA}$, so $\triangle CBD \sim \triangle CAE$.

9. $\dfrac{QU}{QT} = \dfrac{QR}{QS} = \dfrac{UR}{TS}$, so $\triangle QUR \sim \triangle QTS$. **10.** $x = 4$

11. no **12.** yes **13.** 11.2 **14.** 10.5 **15.** 7.2

Chapter 9

Chapter 9 Maintaining Mathematical Proficiency *(p. 461)*

1. $5\sqrt{3}$ **2.** $3\sqrt{30}$ **3.** $3\sqrt{15}$ **4.** $\dfrac{2\sqrt{7}}{7}$ **5.** $\dfrac{5\sqrt{2}}{2}$

6. $2\sqrt{6}$ **7.** $x = 9$ **8.** $x = 7.5$ **9.** $x = 32$

10. $x = 9.2$ **11.** $x = 2$ **12.** $x = 17$

13. no; no; Because square roots have to do with factors, the rule allows you to simplify with products, not sums and differences.

9.1 Vocabulary and Core Concept Check *(p. 468)*

1. A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

9.1 Monitoring Progress and Modeling with Mathematics *(pp. 468–470)*

3. $x = \sqrt{170} \approx 13.0$; no **5.** $x = 41$; yes

7. $x = 15$; yes **9.** $x = 14$; yes

11. Exponents cannot be distributed as shown in the third line; $c^2 = a^2 + b^2$; $x^2 = 7^2 + 24^2$; $x^2 = 49 + 576$; $x^2 = 625$; $x = 25$

13. about 14.1 ft **15.** yes **17.** no **19.** no

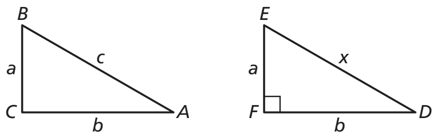
21. yes; acute **23.** yes; right **25.** yes; acute

27. yes; obtuse **29.** about 127.3 ft **31.** 120 m^2

33. 48 cm^2

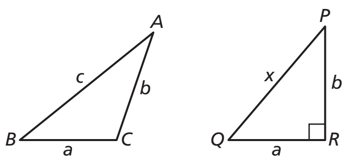
35. The horizontal distance between any two points is given by $(x_2 - x_1)$, and the vertical distance is given by $(y_2 - y_1)$. The horizontal and vertical segments that represent these distances form a right angle, with the segment between the two points being the hypotenuse. So, you can use the Pythagorean Theorem (Thm. 9.1) to say $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, and when you solve for d, you get the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

37. 2 packages

39. 

Let $\triangle ABC$ be any triangle so that the square of the length, c, of the longest side of the triangle is equal to the sum of the squares of the lengths, a and b, of the other two sides: $c^2 = a^2 + b^2$. Let $\triangle DEF$ be any right triangle with leg lengths of a and b. Let x represent the length of its hypotenuse. Because $\triangle DEF$ is a right triangle, by the Pythagorean Theorem (Thm. 9.1), $a^2 + b^2 = x^2$. So, by the Transitive Property, $c^2 = x^2$. By taking the positive square root of each side, you get $c = x$. So, $\triangle ABC \cong \triangle DEF$ by the SSS Congruence Theorem (Thm. 5.8).

41. no; They can be part of a Pythagorean triple if 75 is the hypotenuse: $21^2 + 72^2 = 75^2$

43. 

STATEMENTS	REASONS
1. In $\triangle ABC$, $c^2 > a^2 + b^2$, where c is the length of the longest side. $\triangle PQR$ has side lengths a, b, and x, where x is the length of the hypotenuse and $\angle R$ is a right angle.	**1.** Given
2. $a^2 + b^2 = x^2$	**2.** Pythagorean Theorem (Thm. 9.1)
3. $c^2 > x^2$	**3.** Substitution Property
4. $c > x$	**4.** Take the positive square root of each side.
5. $m\angle R = 90°$	**5.** Definition of a right angle
6. $m\angle C > m\angle R$	**6.** Converse of the Hinge Theorem (Thm. 6.13)
7. $m\angle C > 90°$	**7.** Substitution Property
8. $\angle C$ is an obtuse angle.	**8.** Definition of obtuse angle
9. $\triangle ABC$ is an obtuse triangle.	**9.** Definition of obtuse triangle

9.1 Maintaining Mathematical Proficiency *(p. 470)*

45. $\frac{14\sqrt{3}}{3}$ **47.** $4\sqrt{3}$

9.2 Vocabulary and Core Concept Check *(p. 475)*

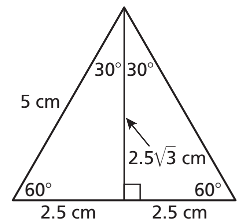
1. 45°-45°-90°, 30°-60°-90°

9.2 Monitoring Progress and Modeling with Mathematics *(pp. 475–476)*

3. $x = 7\sqrt{2}$ **5.** $x = 3$ **7.** $x = 9\sqrt{3}$, $y = 18$

9. $x = 12\sqrt{3}$, $y = 12$

11. The hypotenuse of a 30°-60°-90° triangle is equal to the shorter leg times 2; hypotenuse = shorter leg • 2 = 7 • 2 = 14; So, the length of the hypotenuse is 14 units.

13. 

about 4.3 cm

15. 32 ft^2 **17.** 142 ft; about 200.82 ft; about 245.95 ft

19. Because $\triangle DEF$ is a 45°-45°-90° triangle, by the Converse of the Base Angles Theorem (Thm. 5.7), $\overline{DF} \cong \overline{FE}$. So, let $x = DF = FE$. By the Pythagorean Theorem (Thm. 9.1), $x^2 + x^2 = c^2$, where c is the length of the hypotenuse. So, $2x^2 = c^2$ by the Distributive Property. Take the positive square root of each side to get $x\sqrt{2} = c$. So, the hypotenuse is $\sqrt{2}$ times as long as each leg.

21. Given $\triangle JKL$, which is a 30°-60°-90° triangle, whose shorter leg, $\overline{KL}$, has length x, construct $\triangle JML$, which is congruent and adjacent to $\triangle JKL$. Because corresponding parts of congruent triangles are congruent, $LM = KL = x$, $m\angle M = m\angle K = 60°$, $m\angle MJL = m\angle KJL = 30°$, and $JM = JK$. Also, by the Angle Addition Postulate (Post. 1.4), $m\angle KJM = m\angle KJL + m\angle MJL$, and by substituting, $m\angle KJM = 30° + 30° = 60°$. So, $\triangle JKM$ has three 60° angles, which means that it is equiangular by definition, and by the Corollary to the Converse of the Base Angles Theorem (Cor. 5.3), it is also equilateral. By the Segment Addition Postulate (Post. 1.2), $KM = KL + LM$, and by substituting, $KM = x + x = 2x$. So, by the definition of an equilateral triangle, $JM = JK = KM = 2x$. By the Pythagorean Theorem (Thm. 9.1), $(JL)^2 + (KL)^2 = (JK)^2$. By substituting, we get $(JL)^2 + x^2 = (2x)^2$, which is equivalent to $(JL)^2 + x^2 = 4x^2$, when simplified. When the Subtraction Property of Equality is applied, we get $(JL)^2 = 4x^2 - x^2$, which is equivalent to $(JL)^2 = 3x^2$. By taking the positive square root of each side, $JL = x\sqrt{3}$. So, the hypotenuse of the 30°-60°-90° triangle, $\triangle JKL$, is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

23. *Sample answer:* Because all isosceles right triangles are 45°-45°-90° triangles, they are similar by the AA Similarity Theorem (Thm. 8.3). Because both legs of an isosceles right triangle are congruent, the legs will always be proportional. So, 45°-45°-90° triangles are all similar by the SAS Similarity Theorem (Thm. 8.5) also.

25. $T(1.5, 1.5\sqrt{3} - 1)$

9.2 Maintaining Mathematical Proficiency *(p. 476)*

27. $x = 2$

9.3 Vocabulary and Core Concept Check *(p. 482)*

1. each other

9.3 Monitoring Progress and Modeling with Mathematics *(pp. 482–484)*

3. $\triangle HFE \sim \triangle GHE \sim \triangle GFH$ **5.** $x = \frac{168}{25} = 6.72$

7. $x = \frac{180}{13} \approx 13.8$ **9.** About 11.2 ft **11.** 16

13. $2\sqrt{70} \approx 16.7$ **15.** 20 **17.** $6\sqrt{17} \approx 24.7$

19. $x = 8$ **21.** $y = 27$

23. $x = 3\sqrt{5} \approx 6.7$ **25.** $z = \frac{729}{16} \approx 45.6$

27. The length of leg z should be the geometric mean of the length of the hypotenuse, $(w + v)$, and the segment of the hypotenuse that is adjacent to z, which is v, not w; $z^2 = v \cdot (w + v)$

29. about 14.9 ft **31.** $a = 3$ **33.** $x = 9, y = 15, z = 20$

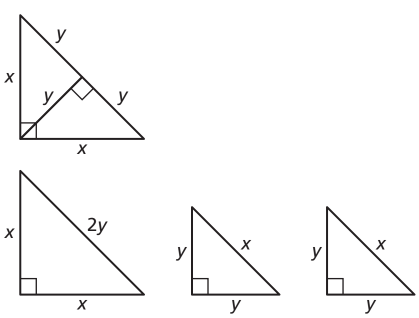
35. A, D **37.** $AC = 25, BD = 12$

39. given; Geometric Mean (Leg) Theorem (Thm. 9.8); a^2; Substitution Property of Equality; Distributive Property; c; Substitution Property of Equality

41.

STATEMENTS	REASONS
1. Draw $\triangle ABC$, $\angle BCA$ is a right angle.	1. Given
2. Draw a perpendicular segment (altitude) from C to $\overline{AB}$, and label the new point on $\overline{AB}$ as D.	2. Perpendicular Postulate (Post. 3.2)
3. $\triangle ADC \sim \triangle CDB$	3. Right Triangle Similarity Theorem (Thm. 9.6)
4. $\frac{BD}{CD} = \frac{CD}{AD}$	4. Corresponding sides of similar figures are proportional.
5. $CD^2 = AD \cdot BD$	5. Cross Products Property

43.



The two smaller triangles are congruent; Their corresponding sides lengths are represented by the same variables. So, they are congruent by the SSS Congruence Theorem (Thm. 5.8).

45.

STATEMENTS	REASONS
1. $\triangle ABC$ is a right triangle. Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.	1. Given
2. $\angle BCA$ is a right angle.	2. Definition of right triangle
3. $\angle ADC$ and $\angle BDC$ are right angles.	3. Definition of perpendicular lines
4. $\angle BCA \cong \angle ADC \cong \angle BDC$	4. Right Angles Congruence Theorem (Thm. 2.3)
5. $\angle A$ and $\angle ACD$ are complementary. $\angle B$ and $\angle BCD$ are complementary.	5. Corollary to the Triangle Sum Theorem (Cor. 5.1)
6. $\angle ACD$ and $\angle BCD$ are complementary.	6. Definition of complementary angles
7. $\angle A \cong \angle BCD$, $\angle B \cong \angle ACD$	7. Congruent Complements Theorem (Thm. 2.5)
8. $\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ACD$	8. AA Similarity Theorem (Thm. 8.3)

9.3 Maintaining Mathematical Proficiency *(p. 484)*

47. $x = 116$ **49.** $x = \frac{23}{6} \approx 3.8$

9.4 Vocabulary and Core Concept Check *(p. 491)*

1. the opposite leg, the adjacent leg

9.4 Monitoring Progress and Modeling with Mathematics *(pp. 491–492)*

3. $\tan R = \frac{45}{28} \approx 1.6071$, $\tan S = \frac{28}{45} \approx 0.6222$

5. $\tan G = \frac{2}{1} = 2.0000$, $\tan H = \frac{1}{2} = 0.5000$

7. $x \approx 13.8$ **9.** $x \approx 13.7$

11. The tangent ratio should be the length of the leg opposite $\angle D$ to the length of the leg adjacent to $\angle D$, not the length of the hypotenuse; $\tan D = \frac{35}{12}$

13. 1 **15.** about 555 ft **17.** $\frac{5}{12} \approx 0.4167$

19. it increases; The opposite side gets longer.

21. no; The Sun's rays form a right triangle with the length of the awning and the height of the door. The tangent of the angle of elevation equals the height of the door divided by the length of the awning, so the length of the awning equals the quotient of the height of the door, 8 feet, and the tangent of the angle of elevation, 70°: $x = \frac{8}{\tan 70^\circ} \approx 2.9$ ft

23. You cannot find the tangent of a right angle, because each right angle has two adjacent legs, and the opposite side is the hypotenuse. So, you do not have an opposite leg and an adjacent leg. If a triangle has an obtuse angle, then it cannot be a right triangle, and the tangent ratio only works for right triangles.

25. **a.** about 33.4 ft
b. 3 students at each end; The triangle formed by the 60° angle has an opposite leg that is about 7.5 feet longer than the opposite leg of the triangle formed by the 50° angle. Because each student needs 2 feet of space, 3 more students can fit on each end with about 1.5 feet of space left over.

9.4 Maintaining Mathematical Proficiency *(p. 492)*

27. $x = 2\sqrt{3} \approx 3.5$ **29.** $x = 5\sqrt{2} \approx 7.1$

9.5 Vocabulary and Core Concept Check *(p. 498)*

1. the opposite leg, the hypotenuse

9.5 Monitoring Progress and Modeling with Mathematics *(pp. 498– 500)*

3. $\sin D = \frac{4}{5} = 0.8000$, $\sin E = \frac{3}{5} = 0.6000$,
$\cos D = \frac{3}{5} = 0.6000$, $\cos E = \frac{4}{5} = 0.8000$

5. $\sin D = \frac{28}{53} \approx 0.5283$, $\sin E = \frac{45}{53} \approx 0.8491$,
$\cos D = \frac{45}{53} \approx 0.8491$, $\cos E = \frac{28}{53} \approx 0.5283$

7. $\sin D = \dfrac{\sqrt{3}}{2} \approx 0.8660$, $\sin E = \dfrac{1}{2} = 0.5000$,
$\cos D = \dfrac{1}{2} = 0.5000$, $\cos E = \dfrac{\sqrt{3}}{2} \approx 0.8660$

9. cos 53° **11.** cos 61° **13.** sin 31° **15.** sin 17°

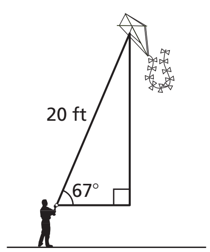
17. $x \approx 9.5$, $y \approx 15.3$ **19.** $v \approx 4.7$, $w \approx 1.6$

21. $a \approx 14.9$, $b \approx 11.1$ **23.** $\sin X = \cos X = \sin Z = \cos Z$

25. The sine of $\angle A$ should be equal to the ratio of the length of the leg opposite the angle, to the length of the hypotenuse; $\sin A = \frac{12}{13}$

27. about 15 ft

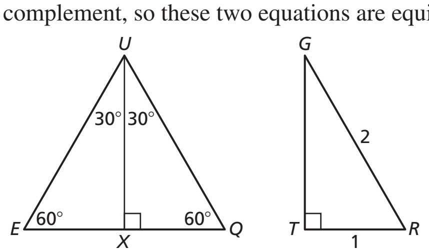
29. a.



b. about 23.4 ft; The higher you hold the spool, the farther the kite is from the ground.

31. both; The sine of an acute angle is equal to the cosine of its complement, so these two equations are equivalent.

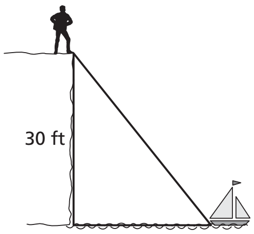
33.



Because $\triangle EQU$ is an equilateral triangle, all three angles have a measure of 60°. When an altitude, $\overline{UX}$, is drawn from U to $\overline{EQ}$ as shown, two congruent 30°-60°-90° triangles are formed, where $m\angle E = 60°$. So, $\sin E = \sin 60° = \dfrac{\sqrt{3}}{2}$. Also, in $\triangle RGT$, because the hypotenuse is twice as long as one of the legs, it is also a 30°-60°-90° triangle. Because $\angle G$ is across from the shorter leg, it must have a measure of 30°, which means that $\cos G = \cos 30° = \dfrac{\sqrt{3}}{2}$. So, $\sin E = \cos G$.

35. If you knew how to take the inverse of the trigonometric ratios, you could first find the respective ratio of sides and then take the inverse of the trigonometric ratio to find the measure of the angle.

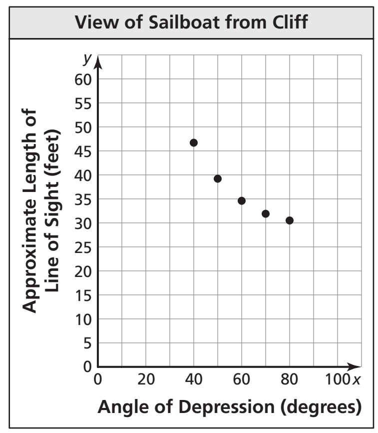
37. a.



b.

Angle of depression	40°	50°	60°	70°	80°
Approximate length of line of sight (feet)	46.7	39.2	34.6	31.9	30.5

c.



d. 60 ft

39. a. $$\frac{\sin A}{\cos A} = \frac{\dfrac{\text{length of side opposite } A}{\text{length of hypotenuse}}}{\dfrac{\text{length of side adjacent to } A}{\text{length of hypotenuse}}} \bullet \frac{\text{length of hypotenuse}}{\text{length of hypotenuse}}$$
$$= \frac{\text{length of side opposite } A}{\text{length of side adjacent to } A}$$
$$= \tan A$$

b. $(\sin A)^2 + (\cos A)^2$
$$= \left(\frac{\text{length of side opposite } A}{\text{length of hypotenuse}}\right)^2 + \left(\frac{\text{length of side adjacent to } A}{\text{length of hypotenuse}}\right)^2$$
$$= \frac{(\text{length of side opposite } A)^2 + (\text{length of side adjacent to } A)^2}{(\text{length of hypotenuse})^2}.$$
By the Pythagorean Theorem (Thm. 9.1),
$(\text{length of side opposite } A)^2 + (\text{length of side adjacent to } A)^2 = (\text{length of hypotenuse})^2$.
So, $(\sin A)^2 + (\cos A)^2 = \dfrac{(\text{length of hypotenuse})^2}{(\text{length of hypotenuse})^2} = 1$.

9.5 Maintaining Mathematical Proficiency *(p. 500)*

41. $x = 8$; yes **43.** $x = 45$; yes

9.6 Vocabulary and Core Concept Check *(p. 505)*

1. sides, angles

9.6 Monitoring Progress and Modeling with Mathematics *(pp. 505–506)*

3. $\angle C$ **5.** $\angle A$ **7.** about 48.6° **9.** about 70.7°

11. about 15.6° **13.** $AB = 15$, $m\angle A \approx 53.1°$, $m\angle B \approx 36.9°$

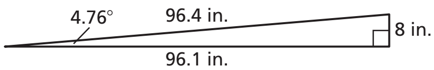
15. $YZ \approx 8.5$, $m\angle X \approx 70.5°$, $m\angle Z \approx 19.5°$

17. $KL \approx 5.1$, $ML \approx 6.1$, $m\angle K = 50°$

19. The sine ratio should be the length of the opposite side to the length of the hypotenuse, not the adjacent side; $\sin^{-1} \frac{8}{17} = m\angle T$

21. about 59.7°

23.



25. about 36.9°; $PQ = 3$ centimeters and $PR = 4$ centimeters, so $m\angle R = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9°$.

27. $KM \approx 7.8$ ft, $JK \approx 11.9$ ft, $m\angle JKM = 49°$; $ML \approx 19.5$ ft, $m\angle MKL \approx 68.2°$, $m\angle L \approx 21.8°$

29. a. *Sample answer:* $\tan^{-1} \frac{3}{1}$; about 71.6°

b. *Sample answer:* $\tan^{-1} \frac{4}{3}$; about 53.1°

31. Because the sine is the ratio of the length of a leg to the length of the hypotenuse, and the hypotenuse is always longer than either of the legs, the sine cannot have a value greater than 1.

9.6 Maintaining Mathematical Proficiency *(p. 506)*

33. $x = 8$ **35.** $x = 2.46$

9.7 Vocabulary and Core Concept Check *(p. 513)*

1. Both the Law of Sines (Thm. 9.9) and the Law of Cosines (Thm. 9.10) can be used to solve any triangle.

9.7 Monitoring Progress and Modeling with Mathematics *(pp. 513–516)*

3. about 0.7986 **5.** about −0.7547 **7.** about −0.2679

9. about 81.8 square units **11.** about 147.3 square units

13. $m\angle A = 48°$, $b \approx 25.5$, $c \approx 18.7$

15. $m\angle B = 66°$, $a \approx 14.3$, $b \approx 24.0$

17. $m\angle A \approx 80.9°$, $m\angle C \approx 43.1°$, $a \approx 20.2$

19. $a \approx 5.2$, $m\angle B \approx 50.5°$, $m\angle C \approx 94.5°$

21. $m\angle A \approx 81.1°$, $m\angle B \approx 65.3°$, $m\angle C \approx 33.6°$

23. $b \approx 35.8$, $m\angle A \approx 46.2°$, $m\angle C \approx 70.8°$

25. According to the Law of Sines (Thm. 9.9), the ratio of the sine of an angle's measure to the length of its opposite side should be equal to the ratio of the sine of another angle measure to the length of its opposite side; $\dfrac{\sin C}{5} = \dfrac{\sin 55°}{6}$, $\sin C = \dfrac{5 \sin 55°}{6}$, $m\angle C \approx 43.0°$

27. Law of Sines (Thm. 9.9); given two angle measures and the length of a side; $m\angle C = 64°$, $a \approx 19.2$, $c \approx 18.1$

29. Law of Cosines (Thm. 9.10); given the lengths of two sides and the measure of the included angle; $c \approx 19.3$, $m\angle A \approx 34.3°$, $m\angle B \approx 80.7°$

31. Law of Sines (Thm. 9.9); given the lengths of two sides and the measure of a nonincluded angle; $m\angle A \approx 111.2°$, $m\angle B \approx 28.8°$, $a \approx 52.2$

33. about 10.7 ft **35.** about 5.1 mi

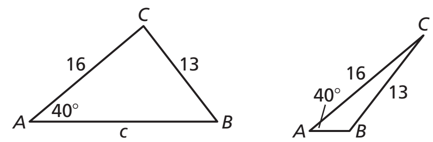
37. cousin; You are given the lengths of two sides and the measure of their included angle.

39. yes; The area of any triangle is given by one-half the product of the lengths of two sides times the sine of their included angle. For $\triangle QRS$, $A = \frac{1}{2}qr \sin S = \frac{1}{2}(25)(17)\sin 79° \approx 208.6$ square units.

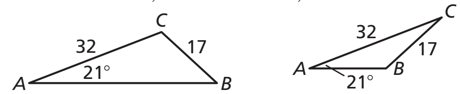
41. a. about 163.4 yd **b.** about 3.5°

43. $x = 99$, $y \approx 20.1$ **45.** $c^2 = a^2 + b^2$

47. a. $m\angle B \approx 52.3°$, $m\angle C \approx 87.7°$, $c \approx 20.2$; $m\angle B \approx 127.7°$, $m\angle C \approx 12.3°$, $c \approx 4.3$

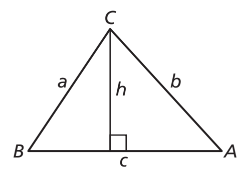


b. $m\angle B \approx 42.4°$, $m\angle C \approx 116.6°$, $c \approx 42.4$; $m\angle B \approx 137.6°$, $m\angle C \approx 21.4°$, $c \approx 17.3$

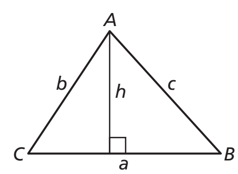


49. about 523.8 mi

51. a.



The formula for the area of $\triangle ABC$ with altitude h drawn from C to $\overline{AB}$ as shown is Area $= \dfrac{1}{2}ch$. Because $\sin A = \dfrac{h}{b}$, $h = b \sin A$. By substituting, you get Area $= \dfrac{1}{2}c(b \sin A) = \dfrac{1}{2}bc \sin A$.



The formula for the area of $\triangle ABC$ with altitude h drawn from A to $\overline{BC}$ as shown is Area $= \dfrac{1}{2}ah$. Because $\sin B = \dfrac{h}{c}$, $h = c \sin B$. By substituting, you get Area $= \dfrac{1}{2}a(c \sin B) = \dfrac{1}{2}ac \sin B$. See Exercise 50 for Area $= \dfrac{1}{2}ab \sin C$.

b. They are all expressions for the area of the same triangle, so they are all equal to each other by the Transitive Property.

c. By the Multiplication Property of Equality, multiply all three expressions by 2 to get $bc \sin A = ac \sin B = ab \sin C$. By the Division Property of Equality, divide all three expressions by abc to get $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Selected Answers

9.7 Maintaining Mathematical Proficiency *(p. 516)*

53. $r = 4$ ft, $d = 8$ ft **55.** $r = 1$ ft, $d = 2$ ft

Chapter 9 Review *(pp. 518–522)*

1. $x = 2\sqrt{34} \approx 11.7$; no **2.** $x = 12$; yes

3. $x = 2\sqrt{30} \approx 11.0$; no **4.** yes; acute **5.** yes; right

6. yes; obtuse **7.** $x = 6\sqrt{2}$ **8.** $x = 7$

9. $x = 16\sqrt{3}$ **10.** $\triangle GFH \sim \triangle FEH \sim \triangle GEF$; $x = 13.5$

11. $\triangle KLM \sim \triangle JKM \sim \triangle JLK$; $x = 2\sqrt{6} \approx 4.9$

12. $\triangle QRS \sim \triangle PQS \sim \triangle PRQ$; $x = 3\sqrt{3} \approx 5.2$

13. $\triangle TUV \sim \triangle STV \sim \triangle SUT$; $x = 25$ **14.** 15

15. $24\sqrt{3} \approx 41.6$ **16.** $6\sqrt{14} \approx 22.4$

17. $\tan J = \frac{11}{60} \approx 0.1833$, $\tan L = \frac{60}{11} \approx 5.4545$

18. $\tan N = \frac{12}{35} \approx 0.3429$, $\tan P = \frac{35}{12} \approx 2.9167$

19. $\tan A = \dfrac{7\sqrt{2}}{8} \approx 1.2374$, $\tan B = \dfrac{4\sqrt{2}}{7} \approx 0.8081$

20. $x \approx 44.0$ **21.** $x \approx 9.3$ **22.** $x \approx 12.8$

23. about 15 ft

24. $\sin X = \frac{3}{5} = 0.600$, $\sin Z = \frac{4}{5} = 0.8000$, $\cos X = \frac{4}{5} = 0.8000$, $\cos Z = \frac{3}{5} = 0.6000$

25. $\sin X = \dfrac{7\sqrt{149}}{149} \approx 0.5735$, $\sin Z = \dfrac{10\sqrt{149}}{149} \approx 0.8192$, $\cos X = \dfrac{10\sqrt{149}}{149} \approx 0.8192$, $\cos Z = \dfrac{7\sqrt{149}}{149} \approx 0.5735$

26. $\sin X = \frac{55}{73} \approx 0.7534$, $\sin Z = \frac{48}{73} \approx 0.6575$, $\cos X = \frac{48}{73} \approx 0.6575$, $\cos Z = \frac{55}{73} \approx 0.7534$

27. $s \approx 31.3$, $t \approx 13.3$ **28.** $r \approx 4.0$, $s \approx 2.9$

29. $v \approx 9.4$, $w \approx 3.4$ **30.** $\cos 18°$ **31.** $\sin 61°$

32. $m\angle Q \approx 71.3°$ **33.** $m\angle Q \approx 65.5°$ **34.** $m\angle Q \approx 2.3°$

35. $m\angle A \approx 48.2°$, $m\angle B \approx 41.8°$, $BC \approx 11.2$

36. $m\angle L = 53°$, $ML \approx 4.5$, $NL \approx 7.5$

37. $m\angle X \approx 46.1°$, $m\angle Z \approx 43.9°$, $XY \approx 17.3$

38. about 41.0 square units **39.** about 42.2 square units

40. about 208.6 square units

41. $m\angle B \approx 24.3°$, $m\angle C \approx 43.7°$, $c \approx 6.7$

42. $m\angle C = 88°$, $a \approx 25.8$, $b \approx 49.5$

43. $m\angle A \approx 99.9°$, $m\angle B \approx 32.1°$, $a \approx 37.1$

44. $b \approx 5.4$, $m\angle A \approx 141.4°$, $m\angle C \approx 13.6°$

45. $m\angle A = 35°$, $a \approx 12.3$, $c \approx 14.6$

46. $m\angle A \approx 42.6°$, $m\angle B \approx 11.7°$, $m\angle C \approx 125.7°$

Chapter 10

Chapter 10 Maintaining Mathematical Proficiency *(p. 527)*

1. $x^2 + 11x + 28$ **2.** $a^2 - 4a - 5$ **3.** $3q^2 - 31q + 36$

4. $10v^2 - 33v - 7$ **5.** $4h^2 + 11h + 6$

6. $18b^2 - 54b + 40$ **7.** $x \approx -1.45$; $x \approx 3.45$

8. $r \approx -9.24$; $r \approx -0.76$ **9.** $w = -1$, $w = 9$

10. $p \approx -10.39$; $p \approx 0.39$ **11.** $k \approx -1.32$; $k \approx 5.32$

12. $z = 1$

13. *Sample answer:* $(2n + 1)(2n + 3)$; $2n + 1$ is positive and odd when n is a nonnegative integer. The next positive, odd integer is $2n + 3$.

10.1 Vocabulary and Core Concept Check *(p. 534)*

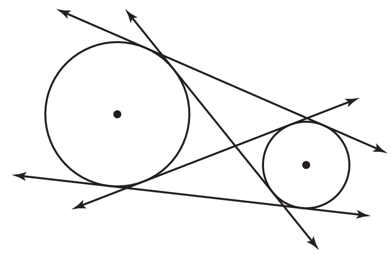
1. They both intersect the circle in two points; Chords are segments and secants are lines.

3. concentric circles

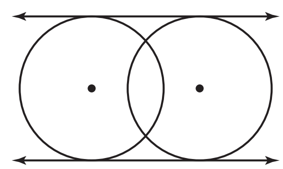
10.1 Monitoring Progress and Modeling with Mathematics *(pp. 534–536)*

5. $\odot C$ **7.** $\overline{BH}, \overline{AD}$ **9.** $\overleftrightarrow{KG}$

11. 4



13. 2

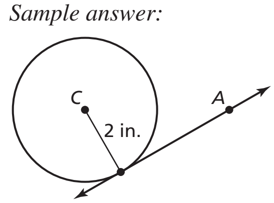


15. external **17.** internal

19. yes; $\triangle ABC$ is a right triangle.

21. no; $\triangle ABD$ is not a right triangle. **23.** 10 **25.** 10.5

27. *Sample answer:*



29. 5 **31.** ± 3

33. $\angle Z$ is a right angle, not $\angle YXZ$; $\overline{XY}$ is not tangent to $\odot Z$.

35. 2; 1; 0; *Sample answer:* There are two possible points of tangency from a point outside the circle, one from a point on the circle, and none from a point inside the circle.

37. 25.6 units **39.** yes; $\overline{PE}$ and $\overline{PM}$ are radii, so $\overline{PE} \cong \overline{PM}$.

41. *Sample answer:* Every point is the same distance from the center, so the farthest two points can be from each other is opposite sides of the center.

43. $\angle ARC \cong \angle BSC$ and $\angle ACR \cong \angle BCS$, so $\triangle ARC \sim \triangle BSC$ by the AA Similarity Theorem (Thm. 8.3). Because corresponding sides of similar figures are proportional, $\dfrac{AC}{BC} = \dfrac{RC}{SC}$.

45. $x = 13$, $y = 5$; $2x - 5 = x + 8$ and $2x + 4y - 6 = 2x + 14$.

47. **a.** Assume m is not perpendicular to $\overline{QP}$. The perpendicular segment from Q to m intersects m at some other point R. Then $QR < QP$, so R must be inside $\odot Q$, and m must be a secant line. This is a contradiction, so m must be perpendicular to $\overline{QP}$.

b. Assume m is not tangent to $\odot Q$. Then m must intersect $\odot Q$ at a second point R. $\overline{QP}$ and $\overline{QR}$ are both radii of $\odot Q$, so $\overline{QP} \cong \overline{QR}$. Because $m \perp \overline{QP}$, $QP < QR$. This is a contradiction, so m must be tangent to $\odot Q$.

10.1 Maintaining Mathematical Proficiency *(p. 536)*

49. 43°

10.2 Vocabulary and Core Concept Check *(p. 542)*

1. congruent arcs

10.2 Monitoring Progress and Modeling with Mathematics *(pp. 542–544)*

3. $\overset{\frown}{AB}$, 135°; $\overset{\frown}{ADB}$, 225° **5.** $\overset{\frown}{JL}$, 120°; $\overset{\frown}{JKL}$, 240°

7. minor arc; 70° **9.** minor arc; 45°

11. semicircle; 180° **13.** major arc; 290°

15. a. 132° **b.** 147° **c.** 200° **d.** 160°

17. a. 103° **b.** 257° **c.** 196° **d.** 305° **e.** 79° **f.** 281°

19. congruent; They are in the same circle and $m\overset{\frown}{AB} = m\overset{\frown}{CD}$.

21. congruent; The circles are congruent and $m\overset{\frown}{VW} = m\overset{\frown}{XY}$.

23. 70; 110°

25. your friend; The arcs must be in the same circle or congruent circles.

27. $\overset{\frown}{AD}$ is the minor arc; $\overset{\frown}{ABD}$ **29.** 340°; 160° **31.** 18°

33. Translate $\odot A$ left a units so that point A maps to point O. The image of $\odot A$ is $\odot A'$ with center O, so $\odot A'$ and $\odot O$ are concentric circles. Dilate $\odot A'$ using center of dilation O and scale factor $\frac{r}{s}$, which maps the points s units from point O to the points $\frac{r}{s}(s) = r$ units from point O. So, this dilation maps $\odot A'$ to $\odot O$. Because a similarity transformation maps $\odot A$ to $\odot O$, $\odot O \sim \odot A$.

35. a. Translate $\odot B$ so that point B maps to point A. The image of $\odot B$ is $\odot B'$ with center A. Because $\overline{AC} \cong \overline{BD}$, this translation maps $\odot B'$ to $\odot A$. A rigid motion maps $\odot B$ to $\odot A$, so $\odot A \cong \odot B$.

b. Because $\odot A \cong \odot B$, the distance from the center of the circle to a point on the circle is the same for each circle. So, $\overline{AC} \cong \overline{BD}$.

37. a. $m\overset{\frown}{BC} = m\angle BAC$, $m\overset{\frown}{DE} = m\angle DAE$ and $m\angle BAC = m\angle DAE$, so $m\overset{\frown}{BC} = m\overset{\frown}{DE}$. Because $\overset{\frown}{BC}$ and $\overset{\frown}{DE}$ are in the same circle, $\overset{\frown}{BC} \cong \overset{\frown}{DE}$.

b. $m\overset{\frown}{BC} = m\angle BAC$ and $m\overset{\frown}{DE} = m\angle DAE$. Because $\overset{\frown}{BC} \cong \overset{\frown}{DE}$, $\angle BAC \cong \angle DAE$.

10.2 Maintaining Mathematical Proficiency *(p. 544)*

39. 15; yes **41.** about 13.04; no

10.3 Vocabulary and Core Concept Check *(p. 549)*

1. Split the chord into two segments of equal length.

10.3 Monitoring Progress and Modeling with Mathematics *(pp. 549–550)*

3. 75° **5.** 170° **7.** 8 **9.** 5

11. $\overline{AC}$ and $\overline{DB}$ are not perpendicular; $\overset{\frown}{BC}$ is not congruent to $\overset{\frown}{CD}$.

13. yes; The triangles are congruent, so $\overline{AB}$ is a perpendicular bisector of $\overline{CD}$.

15. 17

17. about 13.9 in.; The perpendicular bisectors intersect at the center, so the right triangle with legs of 6 inches and 3.5 inches have a hypotenuse equal to the length of the radius.

19. a. Because $PA = PB = PC = PD$, $\triangle PDC \cong \triangle PAB$ by the SSS Congruence Theorem (Thm. 5.8). So, $\angle DPC \cong \angle APB$ and $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

b. $PA = PB = PC = PD$, and because $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, $\angle DPC \cong \angle APB$. By the SAS Congruence Theorem (Thm. 5.5), $\triangle PDC \cong \triangle PAB$, so $\overline{AB} \cong \overline{CD}$.

21. about 16.26°; *Sample answer:* $AB = 2\sqrt{2}$ and $PA = PB = 10$, so $m\angle APB \approx 16.26°$ by the Law of Cosines (Thm. 9.10).

23. $\overline{TP} \cong \overline{PR}$, $\overline{LP} \cong \overline{LP}$, and $\overline{LT} \cong \overline{LR}$, so $\triangle LPR \cong \triangle LPT$ by the SSS Congruence Theorem (Thm. 5.8). Then $\angle LPT \cong \angle LPR$, so $m\angle LPT = m\angle LPR = 90°$. By definition, $\overline{LP}$ is a perpendicular bisector of $\overline{RT}$, so L lies on $\overline{QS}$. Because $\overline{QS}$ contains the center, $\overline{QS}$ is a diameter of $\odot L$.

25. If $\overline{AB} \cong \overline{CD}$, then $\overline{GC} \cong \overline{FA}$. Because $\overline{EC} \cong \overline{EA}$, $\triangle ECG \cong \triangle EAF$ by the HL Congruence Theorem (Thm. 5.9), so $\overline{EF} \cong \overline{EG}$ and $EF = EG$.
If $EF = EG$, then because $\overline{EC} \cong \overline{ED} \cong \overline{EA} \cong \overline{EB}$, $\triangle AEF \cong \triangle BEF \cong \triangle DEG \cong \triangle CEG$ by the HL Congruence Theorem (Thm. 5.9). Then $\overline{AF} \cong \overline{BF} \cong \overline{DG} \cong \overline{CG}$, so $\overline{AB} \cong \overline{CD}$.

10.3 Maintaining Mathematical Proficiency *(p. 550)*

27. 259°

10.4 Vocabulary and Core Concept Check *(p. 558)*

1. inscribed polygon

10.4 Monitoring Progress and Modeling with Mathematics *(pp. 558–560)*

3. 42° **5.** 10° **7.** 120°

9. $\angle ACB \cong \angle ADB$, $\angle DAC \cong \angle DBC$ **11.** 51°

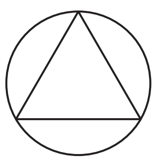
13. $x = 100$, $y = 85$ **15.** $a = 20$, $b = 22$

17. The inscribed angle was not doubled; $m\angle BAC = 2(53°) = 106°$

19. $x = 25$, $y = 5$; 130°, 75°, 50°, 105°

21. $x = 30$, $y = 20$; 60°, 60°, 60°

23.

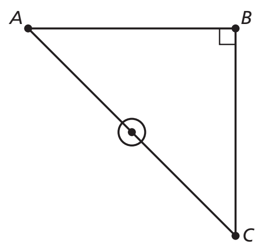


25. yes; Opposite angles are always supplementary.

27. no; Opposite angles are not always supplementary.

29. no; Opposite angles are not always supplementary.

31.



220,000 km

33. double the radius

35. Each diagonal splits the rectangle into two right triangles.

37. **a.** $\overline{QB} \cong \overline{QA}$, so $\triangle ABC$ is isosceles. By the Base Angles Theorem (Thm. 5.6), $\angle QBA \cong \angle QAB$, so $m\angle BAQ = x°$. By the Exterior Angles Theorem (Thm. 5.2), $m\angle AQC = 2x°$. Then $m\overset{\frown}{AC} = 2x°$, so $m\angle ABC = x° = \frac{1}{2}(2x)° = \frac{1}{2}m\overset{\frown}{AC}$.

b. Given: $\angle ABC$ is inscribed in $\odot Q$. $\overline{DB}$ is a diameter; Prove: $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$; By Case 1, proved in part (a), $m\angle ABD = \frac{1}{2}m\overset{\frown}{AD}$ and $m\angle CBD = \frac{1}{2}m\overset{\frown}{CD}$. By the Arc Addition Postulate (Post. 10.1), $m\overset{\frown}{AD} + m\overset{\frown}{CD} = m\overset{\frown}{AC}$. By the Angle Addition Postulate (Post. 1.4), $m\angle ABD + m\angle CBD = m\angle ABC$.

Then $m\angle ABC = \frac{1}{2}m\overset{\frown}{AD} + \frac{1}{2}m\overset{\frown}{CD}$
$= \frac{1}{2}\left(m\overset{\frown}{AD} + m\overset{\frown}{CD}\right)$
$= \frac{1}{2}m\overset{\frown}{AC}$.

c. Given: $\angle ABC$ is inscribed in $\odot Q$. $\overline{DB}$ is a diameter; Prove: $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$; By Case 1, proved in part (a), $m\angle DBA = \frac{1}{2}m\overset{\frown}{AD}$ and $m\angle DBC = \frac{1}{2}m\overset{\frown}{CD}$. By the Arc Addition Postulate (Post. 10.1), $m\overset{\frown}{AC} + m\overset{\frown}{CD} = m\overset{\frown}{AD}$, so $m\overset{\frown}{AC} = m\overset{\frown}{AD} - m\overset{\frown}{CD}$. By the Angle Addition Postulate (Post. 1.4), $m\angle DBC + m\angle ABC = m\angle DBA$, so $m\angle ABC = m\angle DBA - m\angle DBC$. Then

$m\angle ABC = \frac{1}{2}m\overset{\frown}{AD} - \frac{1}{2}m\overset{\frown}{CD}$
$= \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{CD}\right)$
$= \frac{1}{2}m\overset{\frown}{AC}$.

39. To prove the conditional, find the measure of the intercepted arc of the right angle and the definition of a semicircle to show the hypotenuse of the right triangle must be the diameter of the circle. To prove the converse, use the definition of a semicircle to find the measure of the angle opposite the diameter.

41. 2.4 units

10.4 Maintaining Mathematical Proficiency *(p. 560)*

43. $x = \frac{145}{3}$ **45.** $x = 120$

10.5 Vocabulary and Core Concept Check *(p. 566)*

1. outside

10.5 Monitoring Progress and Modeling with Mathematics *(pp. 566–568)*

3. 130° **5.** 130° **7.** 115 **9.** 56

11. 40 **13.** 34

15. $\angle SUT$ is not a central angle; $m\angle SUT = \frac{1}{2}\left(m\overset{\frown}{QR} + m\overset{\frown}{ST}\right) = 41.5°$

17. 60°; When a tangent and a secant intersect outside the circle, the measure of the angle is half of the difference of the intercepted arcs.

19. 30°; Because the sum of the angles of a triangle always equals 180°, solve the equation $60 + 90 + x = 180$.

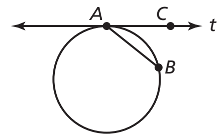
21. 30°; This angle is complementary to $\angle 2$, which is 60°.

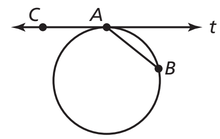
23. about 2.8° **25.** $360 - 10x$; 160°

27. $m\angle LPJ < 90$; The difference of $m\overset{\frown}{JL}$ and $m\overset{\frown}{LK}$ must be less than 180°, so $m\angle LPJ < 90$.

29. By the Angles Inside a Circle Theorem (Thm. 10.15), $m\angle JPN = \frac{1}{2}\left(m\overset{\frown}{JN} + m\overset{\frown}{KM}\right)$. By the Angles Outside the Circle Theorem (Thm. 10.16), $m\angle JLN = \frac{1}{2}\left(m\overset{\frown}{JN} - m\overset{\frown}{KM}\right)$. Because the angle measures are positive, $\frac{1}{2}\left(m\overset{\frown}{JN} + m\overset{\frown}{KM}\right) > \frac{1}{2}m\overset{\frown}{JN} > \frac{1}{2}\left(m\overset{\frown}{JN} - m\overset{\frown}{KM}\right)$, so, $m\angle JPN > m\angle JLN$.

31. **a.**



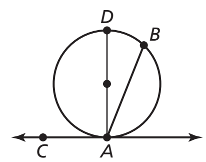


b. $m\overset{\frown}{AB} = 2m\angle BAC$, $m\overset{\frown}{AB} = 360° - 2m\angle BAC$

c. 90°; $2m\angle BAC = 360° - 2m\angle BAC$ when $m\angle BAC = 90°$.

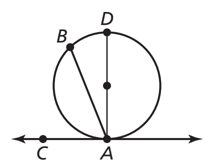
33. **a.** By the Tangent Line to Circle Theorem (Thm. 10.1), $m\angle BAC$ is 90°, which is half the measure of the semicircular arc.

b.



By the Tangent Line to Circle Theorem (Thm. 10.1), $m\angle CAD = 90°$. $m\angle DAB = \frac{1}{2}m\overset{\frown}{DB}$ and by part (a), $m\angle CAD = \frac{1}{2}m\overset{\frown}{AD}$. By the Angle Addition Postulate (Post. 1.4), $m\angle BAC = m\angle BAD + m\angle CAD$. So, $m\angle BAC = \frac{1}{2}m\overset{\frown}{DB} + \frac{1}{2}m\overset{\frown}{AD} = \frac{1}{2}\left(m\overset{\frown}{DB} + m\overset{\frown}{AD}\right)$. By the Arc Addition Postulate (Post. 10.1), $m\overset{\frown}{DB} + m\overset{\frown}{AD} = m\overset{\frown}{ADB}$, so $m\angle BAC = \frac{1}{2}\left(m\overset{\frown}{ADB}\right)$.

c.

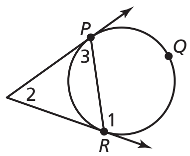


By the Tangent Line to Circle Theorem (Thm. 10.1), $m\angle CAD = 90°$. $m\angle DAB = \frac{1}{2}m\overset{\frown}{DB}$ and by part (a), $m\angle DAC = \frac{1}{2}m\overset{\frown}{ABD}$. By the Angle Addition Postulate (Post. 1.4), $m\angle BAC = m\angle DAC - m\angle DAB$. So, $m\angle BAC = \frac{1}{2}m\overset{\frown}{ABD} - \frac{1}{2}m\overset{\frown}{DB} = \frac{1}{2}\left(m\overset{\frown}{ABD} - m\overset{\frown}{DB}\right)$. By the Arc Addition Postulate (Post. 10.1), $m\overset{\frown}{ABD} - m\overset{\frown}{DB} = m\overset{\frown}{AB}$, so $m\angle BAC = \frac{1}{2}\left(m\overset{\frown}{AB}\right)$.

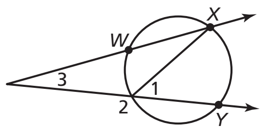
35.

STATEMENTS	REASONS
1. Chords $\overline{AC}$ and $\overline{BD}$ intersect.	**1.** Given
2. $m\angle ACB = \frac{1}{2}m\overset{\frown}{AB}$ and $m\angle DBC = \frac{1}{2}m\overset{\frown}{DC}$	**2.** Measure of an Inscribed Angle Theorem (Thm. 10.10)
3. $m\angle 1 = m\angle DBC + m\angle ACB$	**3.** Exterior Angle Theorem (Thm. 5.2)
4. $m\angle 1 = \frac{1}{2}m\overset{\frown}{DC} + \frac{1}{2}m\overset{\frown}{AB}$	**4.** Substitution Property of Equality
5. $m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{DC} + m\overset{\frown}{AB}\right)$	**5.** Distributive Property

37. By the Exterior Angle Theorem (Thm. 5.2), $m\angle 2 = m\angle 1 + m\angle ABC$, so $m\angle 1 = m\angle 2 - m\angle ABC$. By the Tangent and Intersected Chord Theorem (Thm. 10.14), $m\angle 2 = \frac{1}{2}m\widehat{BC}$ and by the Measure of an Inscribed Angle Theorem (Thm. 10.10), $m\angle ABC = \frac{1}{2}m\widehat{AC}$. By the Substitution Property, $m\angle 1 = \frac{1}{2}m\widehat{BC} - \frac{1}{2}m\widehat{AC} = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$;



By the Exterior Angle Theorem (Thm. 5.2), $m\angle 1 = m\angle 2 + m\angle 3$, so $m\angle 2 = m\angle 1 - m\angle 3$. By the Tangent and Intersected Chord Theorem (Thm. 10.14), $m\angle 1 = \frac{1}{2}m\widehat{PQR}$ and $m\angle 3 = \frac{1}{2}m\widehat{PR}$. By the Substitution Property, $m\angle 2 = \frac{1}{2}m\widehat{PQR} - \frac{1}{2}m\widehat{PR} = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$;



By the Exterior Angle Theorem (Thm. 5.2), $m\angle 1 = m\angle 3 + m\angle WXZ$, so $m\angle 3 = m\angle 1 - m\angle WXZ$. By the Measure of an Inscribed Angle Theorem (Thm. 10.10), $m\angle 1 = \frac{1}{2}m\widehat{XY}$ and $m\angle WXZ = \frac{1}{2}m\widehat{WZ}$. By the Substitution Property, $m\angle 3 = \frac{1}{2}m\widehat{XY} - \frac{1}{2}m\widehat{WZ} = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$.

39. 20°; *Sample answer:* $m\widehat{WY} = 160°$ and $m\widehat{WX} = m\widehat{ZY}$, so

$$\begin{aligned} m\angle P &= \tfrac{1}{2}(m\widehat{WZ} - m\widehat{XY}) \\ &= \tfrac{1}{2}\big((200° - m\widehat{ZY}) - (160° - m\widehat{WX})\big) \\ &= \tfrac{1}{2}(40°). \end{aligned}$$

10.5 Maintaining Mathematical Proficiency *(p. 568)*

41. $x = -4, x = 3$ **43.** $x = -3, x = -1$

10.6 Vocabulary and Core Concept Check *(p. 573)*

1. external segment

10.6 Monitoring Progress and Modeling with Mathematics *(pp. 573–574)*

3. 5 **5.** 4 **7.** 4 **9.** 5 **11.** 12 **13.** 4

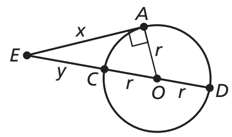
15. The chords were used instead of the secant segments; $CF \cdot DF = BF \cdot AF$; $CD = 2$

17. about 124.5 ft

19.

STATEMENTS	REASONS
1. $\overline{AB}$ and $\overline{CD}$ are chords intersecting in the interior of the circle.	**1.** Given
2. $\angle AEC \cong \angle DEB$	**2.** Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\angle ACD \cong \angle ABD$	**3.** Inscribed Angles of a Circle Theorem (Thm. 10.11)
4. $\triangle AEC \sim \triangle DEB$	**4.** AA Similarity Theorem (Thm. 8.3)
5. $\frac{EA}{ED} = \frac{EC}{EB}$	**5.** Corresponding side lengths of similar triangles are proportional.
6. $EB \cdot EA = EC \cdot ED$	**6.** Cross Products Property

21.



By the Tangent Line to Circle Theorem (Thm. 10.1), $\angle EAO$ is a right angle, which makes $\triangle AEO$ a right triangle. By the Pythagorean Theorem (Thm. 9.1), $(r + y)^2 = r^2 + x^2$. So, $r^2 + 2yr + y^2 = r^2 + x^2$. By the Subtraction Property of Equality, $2yr + y^2 = x^2$. Then $y(2r + y) = x^2$, so $EC \cdot ED = EA^2$.

23. $BC = \dfrac{AD^2 + (AD)(DE) - AB^2}{AB}$ **25.** $2\sqrt{10}$

10.6 Maintaining Mathematical Proficiency *(p. 574)*

27. $x = -9, x = 5$ **29.** $x = -7, x = 1$

10.7 Vocabulary and Core Concept Check *(p. 579)*

1. $(x - h)^2 + (y - k)^2 = r^2$

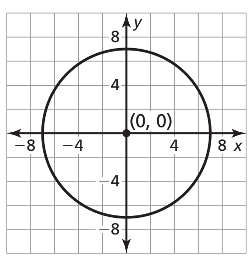
10.7 Monitoring Progress and Modeling with Mathematics *(pp. 579–580)*

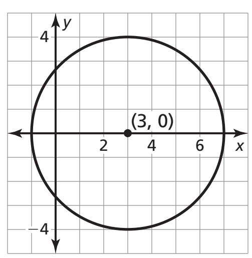
3. $x^2 + y^2 = 4$ **5.** $x^2 + y^2 = 49$

7. $(x + 3)^2 + (y - 4)^2 = 1$ **9.** $x^2 + y^2 = 36$

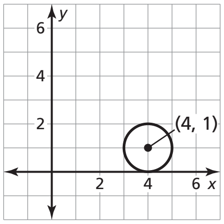
11. $x^2 + y^2 = 58$

13. center: (0, 0), radius: 7 **15.** center: (3, 0), radius: 4





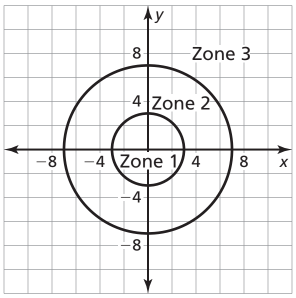
17. center: (4, 1), radius: 1



19. The radius of the circle is 8. $\sqrt{(2-0)^2+(3-0)^2}=\sqrt{13}$, so (2, 3) does not lie on the circle.

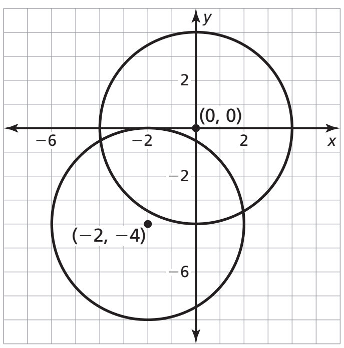
21. The radius of the circle is $\sqrt{10}$. $\sqrt{(\sqrt{6}-0)^2+(2-0)^2}=\sqrt{10}$, so $(\sqrt{6}, 2)$ does lie on the circle.

23. **a.**



b. zone 2, zone 3, zone 1, zone 1, zone 2

25.



The equation of the image is $(x+2)^2+(y+4)^2=16$; The equation of the image of a circle after a translation m units to the left and n units down is $(x+m)^2+(y+n)^2=r^2$.

27. $(x-4)^2+(y-9)^2=16$; $m\angle Z=90°$, so $\overline{XY}$ is a diameter.

29. tangent; The system has one solution.

31. secant; The system has two solutions, and (5, −1) is not on the line.

33. yes; The diameter perpendicularly bisects the chord from (−1, 0) to (1, 0), so the center is on the y-axis at (0, k) and the radius is k^2+1.

10.7 Maintaining Mathematical Proficiency *(p. 580)*

35. minor arc; 53° **37.** major arc; 270°

39. semicircle; 180°

Chapter 10 Review *(pp. 582–586)*

1. radius **2.** chord **3.** tangent **4.** diameter

5. secant **6.** radius **7.** internal **8.** external

9. 2 **10.** 2 **11.** 12 **12.** tangent; $20^2+48^2=52^2$

13. 100° **14.** 60° **15.** 160° **16.** 80°

17. not congruent; The circles are not congruent.

18. congruent; The circles are congruent and $m\widehat{AB}=m\widehat{EF}$.

19. 61° **20.** 65° **21.** 91° **22.** 26 **23.** 80

24. $q=100$, $r=20$ **25.** 5 **26.** $y=30$, $z=10$

27. $m=44$, $n=39$ **28.** 28 **29.** 70 **30.** 106

31. 16 **32.** 240° **33.** 5 **34.** 3 **35.** 10

36. about 10.7 ft **37.** $(x-4)^2+(y+1)^2=9$

38. $(x-8)^2+(y-6)^2=36$ **39.** $x^2+y^2=16$

40. $x^2+y^2=81$ **41.** $(x+5)^2+(y-2)^2=1.69$

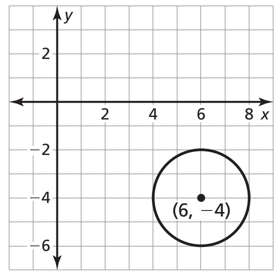
42. $(x-6)^2+(y-21)^2=16$

43. $(x+3)^2+(y-2)^2=256$

44. $(x-10)^2+(y-7)^2=12.25$ **45.** $x^2+y^2=27.04$

46. $(x+7)^2+(y-6)^2=25$

47. center: (6, −4), radius: 2



48. The radius of the circle is 5. $d=\sqrt{(0-4)^2+(0+3)^2}=5$, so (4, −3) is on the circle.

Chapter 11

Chapter 11 Maintaining Mathematical Proficiency *(p. 591)*

1. 158 ft^2 **2.** 144 m^2 **3.** 184 cm^2 **4.** 9 in.

5. 2 cm **6.** 12 ft **7.** $S=6x^2$; cube

11.1 Vocabulary and Core Concept Check *(p. 598)*

1. πd

11.1 Monitoring Progress and Modeling with Mathematics *(pp. 598–600)*

3. about 37.70 in. **5.** 14 units **7.** about 3.14 ft

9. about 35.53 m

11. The diameter was used as the radius; $C=\pi d=9\pi$ in.

13. 182 ft **15.** about 44.85 units **17.** about 20.57 units

19. $\frac{7\pi}{18}$ rad **21.** 165° **23.** about 27.19 min

25. 8π units **27.** about 7.85 units

29. yes; *Sample answer:* The arc length also depends on the radius.

31. B **33.** $2\frac{1}{3}$

35. arc length of $\widehat{AB}=r\theta$; about 9.42 in.

37. yes; *Sample answer:* The circumference of the red circle can be found using $2=\frac{30°}{360°}C$. The circumference of the blue circle is double the circumference of the red circle.

39. 28 units

41. *Sample answer:*

STATEMENTS	REASONS
1. $\overline{FG} \cong \overline{GH}$, $\angle JFK \cong \angle KFL$	**1.** Given
2. $FG = GH$	**2.** Definition of congruent segments
3. $FH = FG + GH$	**3.** Segment Addition Postulate (Post. 1.2)
4. $FH = 2FG$	**4.** Substitution Property of Equality
5. $m\angle JFK = m\angle KFL$	**5.** Definition of congruent angles
6. $m\angle JFL = m\angle JFK + m\angle KFL$	**6.** Angle Addition Postulate (Post. 1.4)
7. $m\angle JFL = 2m\angle JFK$	**7.** Substitution Property of Equality
8. $\angle NFG \cong \angle JFL$	**8.** Vertical Angles Congruence Theorem (Thm. 2.6)
9. $m\angle NFG = m\angle JFL$	**9.** Definition of congruent angles
10. $m\angle NFG = 2m\angle JFK$	**10.** Substitution Property of Equality
11. arc length of $\overparen{JK} = \frac{m\angle JFK}{360^\circ} \cdot 2\pi FH$, arc length of $\overparen{NG} = \frac{m\angle NFG}{360^\circ} \cdot 2\pi FG$	**11.** Formula for arc length
12. arc length of $\overparen{JK} = \frac{m\angle JFK}{360^\circ} \cdot 2\pi(2FG)$, arc length of $\overparen{NG} = \frac{2m\angle JFK}{360^\circ} \cdot 2\pi FG$	**12.** Substitution Property of Equality
13. arc length of $\overparen{NG}$ = arc length of $\overparen{JK}$	**13.** Transitive Property of Equality

11.1 Maintaining Mathematical Proficiency *(p. 600)*

43. 15 square units

11.2 Vocabulary and Core Concept Check *(p. 606)*

1. sector

11.2 Monitoring Progress and Modeling with Mathematics *(pp. 606–608)*

3. about 0.50 cm^2 **5.** about 78.54 in.^{2}

7. about 5.32 ft **9.** about 4.01 in.

11. about 464 people per mi^2 **13.** about 319,990 people

15. about 52.36 in.^{2}; about 261.80 in.^{2}

17. about 937.31 m^2; about 1525.70 m^2

19. The diameter was substituted in the formula for area as the radius; $A = \pi(6)^2 \approx 113.10$ ft^2

21. about 66.04 cm^2 **23.** about 1696.46 m^2

25. about 43.98 ft^2 **27.** about 26.77 in.^{2}

29. about 192.48 ft^2

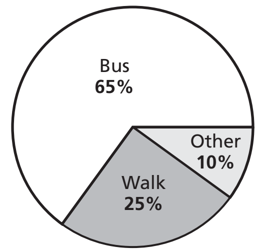
31. **a.** about 285 ft^2 **b.** about 182 ft^2

33. *Sample answer:* change side lengths to radii and perimeter to circumference; Different terms need to be used because a circle is not a polygon.

35. **a.** *Sample answer:* The total is 100%.

b. bus 234°; walk 90°; other 36°

How Students Get To School



c. bus: about 8.17 in.^{2}; walk: about 3.14 in.^{2}; other: about 1.26 in.^{2}

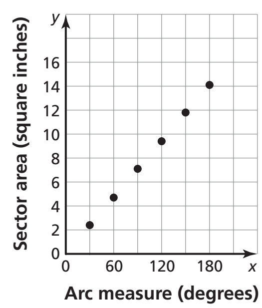
37. **a.** You should buy two 14-inch pizzas; *Sample answer:* The area is 98π square inches and the cost is \$25.98.

b. You should buy two 10-inch pizzas and one 14-inch pizza; *Sample answer:* Buying three 10-inch pizzas is the only cheaper option, and it would not be enough pizza.

c. You should buy four 10-inch pizzas; *Sample answer:* The total circumference is 20π inches.

39. **a.** 2.4 in.^{2}; 4.7 in.^{2}; 7.1 in.^{2}; 9.4 in.^{2}; 11.8 in.^{2}; 14.1 in.^{2}

b.



c. yes; *Sample answer:* The rate of change is constant.

d. yes; no; *Sample answer:* The rate of change will still be constant.

41. *Sample answer:* Let $2a$ and $2b$ represent the lengths of the legs of the triangle. The areas of the semicircles are $\frac{1}{2}\pi a^2$, $\frac{1}{2}\pi b^2$, and $\frac{1}{2}\pi(a^2 + b^2)$. $\frac{1}{2}\pi a^2 + \frac{1}{2}\pi b^2 = \frac{1}{2}\pi(a^2 + b^2)$, and subtracting the areas of the unshaded regions from both sides leaves the area of the crescents on the left and the area of the triangle on the right.

11.2 Maintaining Mathematical Proficiency *(p. 608)*

43. 49 ft^2 **45.** 15 ft^2

11.3 Vocabulary and Core Concept Check *(p. 614)*

1. Divide 360° by the number of sides.

11.3 Monitoring Progress and Modeling with Mathematics *(pp. 614–616)*

3. 361 square units **5.** 70 square units **7.** *P*

9. 5 units **11.** 36° **13.** 15° **15.** 45°

17. 67.5° **19.** about 62.35 square units

21. about 20.87 square units **23.** about 342.24 square units

25. The side lengths were used instead of the diagonals; $A = \frac{1}{2}(8)(4) = 16$

27. about 79.60 square units **29.** about 117.92 square units

31. about 166 in.^{2}

33. true; *Sample answer:* As the number of sides increases, the polygon fills more of the circle.

35. false; *Sample answer:* The radius can be less than or greater than the side length.

37. about 59.44 square units **39.** $x^2 = 324$; 18 in.; 36 in.

41. yes; about 24.73 in.^{2}; *Sample answer:* Each side length is 2 inches, and the central angle is 40°.

43. *Sample answer:* Let $QT = x$ and $TS = y$. The area of $PQRS$ is $\frac{1}{2}d_2x + \frac{1}{2}d_2y = \frac{1}{2}d_2(x + y) = \frac{1}{2}d_2d_1$.

45. $A = \frac{1}{2}d^2$; $A = \frac{1}{2}d^2 = \frac{1}{2}(s^2 + s^2) = \frac{1}{2}(2s^2) = s^2$

47. about 6.47 cm **49.** about 52

51. about 43 square units; *Sample answer:* $A = \frac{1}{2}aP$; There are fewer calculations.

11.3 Maintaining Mathematical Proficiency *(p. 616)*

53. line symmetry; 1 **55.** rotational symmetry; 180°

11.4 Vocabulary and Core Concept Check *(p. 621)*

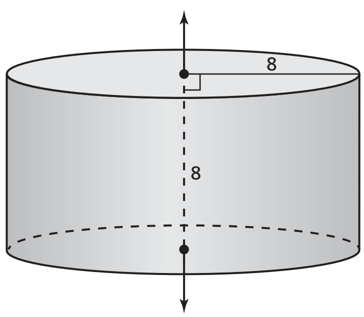
1. polyhedron

11.4 Monitoring Progress and Modeling with Mathematics *(pp. 621–622)*

3. B **5.** A **7.** yes; pentagonal pyramid **9.** no

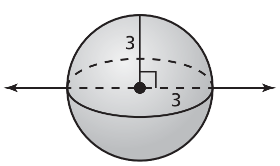
11. circle **13.** triangle

15.



cylinder with height 8 and base radius 8

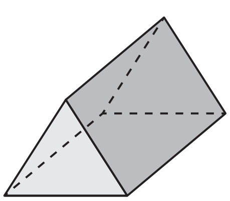
17.



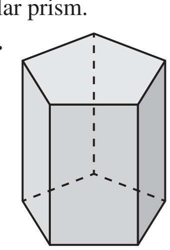
sphere with radius 3

19. There are two parallel, congruent bases, so it is a prism, not a pyramid; The solid is a triangular prism.

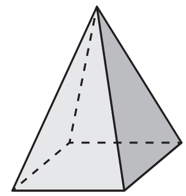
21.



23.



25.



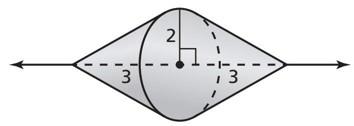
27. your cousin; The sides come together at a point.

29. no

31. yes; *Sample answer:* The plane is parallel to a face.

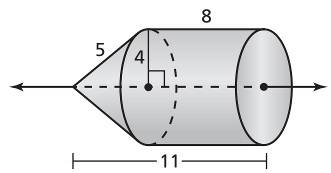
33. yes; *Sample answer:* The plane passes through six faces.

35. a.



two cones with heights 3 and base radii 2

b.



cone with height 3 and base radius 4 and cylinder with height 8 and base radius 4

11.4 Maintaining Mathematical Proficiency *(p. 622)*

37. yes; SSS Congruence Theorem (Thm. 5.8)

39. yes; ASA Congruence Theorem (Thm. 5.10)

11.5 Vocabulary and Core Concept Check *(p. 631)*

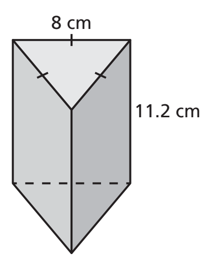
1. cubic units

11.5 Monitoring Progress and Modeling with Mathematics *(pp. 631–634)*

3. 6.3 cm^3 **5.** 175 in.^{3} **7.** about 288.40 ft^3

9. about 628.32 ft^3

11.



310.38 cm^3

13. copper

15. The base circumference was used instead of the base area; $V = \pi r^2 h = 48\pi$ ft^3

17. 10 ft **19.** 4 cm **21.** about 11.04 ft

23. 14 in.^{2}; *Sample answer:* length: 7 in., width: 2 in.

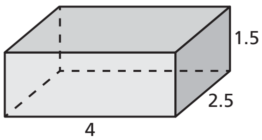
25. 99 cm^3 **27.** 2 cm **29.** 150 ft^3

31. about 1900.66 in.^{3} **33.** about 2,350,000,000 gal

35. 2 **37. a.** 75 in.^{3} **b.** 20

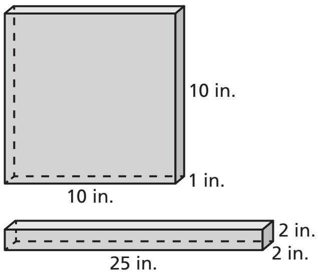
39. *Sample answer:* The stacks have the same height and the rectangles have the same lengths, so the stacks have the same area.

41.



15 cubic units

43. *Sample answer:*



45. the solid produced by rotating around the vertical line; *Sample answer:* The solid produced by rotating around the horizontal line has a volume of 45π cubic inches and the solid produced by rotating around the vertical line has a volume of 75π cubic inches.

47. about 7.33 in.^{3} **49.** Increase the height by 25%.

51. yes; *Sample answer:* Density is proportional to mass when the volume is constant.

53. 36 ft, 15 ft

11.5 Maintaining Mathematical Proficiency *(p. 634)*

55. 16 m^2 **57.** 680.4 in.^{2}

11.6 Vocabulary and Core Concept Check *(p. 639)*

1. *Sample answer:* A triangular prism has two parallel bases that are triangles. A triangular pyramid has one base that is a triangle, and the other faces all intersect at a single point.

11.6 Monitoring Progress and Modeling with Mathematics *(pp. 639–640)*

3. 448 m^3 **5.** 6 m **7.** 16 in.

9. One side length was used in the formula as the base area; $V = \frac{1}{3}(6^2)(5) = 60 \text{ ft}^3$

11. 5 ft **13.** 12 yd **15.** 4 ft^3 **17.** 72 in.^{3}

19. about 213.33 cm^3

21. **a.** The volume doubles. **b.** The volume is 4 times greater.

c. yes; *Sample answer:*

Square Pyramid: $V = \frac{1}{3}s^2h$

Double height: $V = \frac{1}{3}s^2(2h) = 2\left(\frac{1}{3}s^2h\right)$

Double side length of base: $V = \frac{1}{3}(2s)^2h = 4\left(\frac{1}{3}s^2h\right)$

23. about 9.22 ft^3 **25.** about 78 in.^{3}

11.6 Maintaining Mathematical Proficiency *(p. 640)*

27. 12.6 **29.** 16.0

11.7 Vocabulary and Core Concept Check *(p. 645)*

1. *Sample answer:* pyramids have a polygonal base, cones have a circular base; They both have sides that meet at a single vertex.

11.7 Monitoring Progress and Modeling with Mathematics *(pp. 645–646)*

3. about 603.19 in.^{2} **5.** about 678.58 in.^{2}

7. about 1361.36 mm^3 **9.** about 526.27 in.^{3}

11. $\ell \approx 5.00$ cm; $h \approx 4.00$ cm **13.** 256π ft^3

15. about 226.19 cm^3

17. $2h$; $r\sqrt{2}$; *Sample answer:* The original volume is $V = \frac{1}{3}\pi r^2 h$ and the new volume is $V = \frac{2}{3}\pi r^2 h$.

19. about 3716.85 ft^3

21. yes; *Sample answer:* The automatic pet feeder holds about 12 cups of food.

23. It is half; about 60°

25. yes; *Sample answer:* The base areas are the same and the total heights are the same.

11.7 Maintaining Mathematical Proficiency *(p. 646)*

27. about 153.94 ft^2 **29.** 32 m

11.8 Vocabulary and Core Concept Check *(p. 652)*

1. The plane must contain the center of the sphere.

11.8 Monitoring Progress and Modeling with Mathematics *(pp. 652–654)*

3. about 201.06 ft^2 **5.** about 1052.09 m^2 **7.** 1 ft

9. 30 m **11.** about 235.62 m^2 **13.** about 2144.66 m^3

15. about 5575.28 yd^3 **17.** about 4188.79 cm^3

19. about 33.51 ft^3

21. The radius was squared instead of cubed; $V = \frac{4}{3}\pi(6)^3 \approx 904.78 \text{ ft}^3$

23. about 445.06 in.^{3} **25.** about 7749.26 cm^3

27. $S \approx 226.98$ in.^{2}; $V \approx 321.56$ in.^{3}

29. $S \approx 45.84$ in.^{2}; $V \approx 29.18$ in.^{3}

31. $S \approx 215.18$ in.^{2}; $V \approx 296.80$ in.^{3}

33. no; The surface area is quadrupled.

35. about 20,944 ft^3

37. **a.** 144π in.^{2}, 288π in.^{3}; 324π in.^{2}, 972π in.^{3}; 576π in.^{2}, 2304π in.^{3}

b. It is multiplied by 4; It is multiplied by 9; It is multiplied by 16.

c. It is multiplied by 8; It is multiplied by 27; It is multiplied by 64.

39. **a.** Earth: about 197.1 million mi^2; moon: about 14.7 million mi^2

b. The surface area of the Earth is about 13.4 times greater than the surface area of the moon.

c. about 137.9 million mi^2

41. about 50.27 in.^{2}; *Sample answer:* The side length of the cube is the diameter of the sphere.

43. $V = \frac{1}{3}rS$

45. *Sample answer:* radius 1 in. and height $\frac{4}{3}$ in.; radius $\frac{1}{3}$ in. and height 12 in.; radius 2 in. and height $\frac{1}{3}$ in.

47. $S \approx 113.10$ in.^{2}, $V \approx 75.40$ in.^{3}

11.8 Maintaining Mathematical Proficiency *(p. 654)*

49. $A = 35°$, $a \approx 12.3$, $c \approx 14.6$

51. $a \approx 31.0$, $B \approx 28.1°$, $C \approx 48.9°$

Chapter 11 Review *(pp. 656–660)*

1. about 30.00 ft **2.** about 56.57 cm

3. about 26.09 in. **4.** 218 ft **5.** about 169.65 in.^{2}

6. about 17.72 in.^{2} **7.** 173.166 ft^2

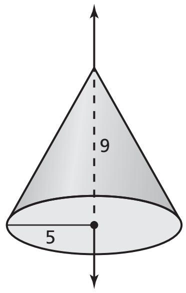
8. 130 square units **9.** 96 square units

10. 105 square units **11.** about 201.20 square units

12. about 167.11 square units

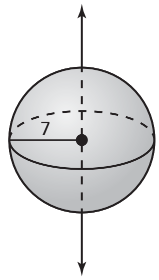
13. about 37.30 square units 14. about 119.29 in.^{2}

15.



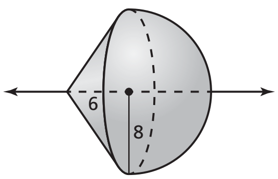
cone with height 9 and base radius 5

16.



sphere with radius 7

17.



cone with height 6 and base radius 8 and hemisphere with radius 8

18. rectangle 19. square 20. triangle

21. 11.34 m^3 22. about 100.53 mm^3

23. about 27.53 yd^3 24. 189 ft^3 25. 400 yd^3

26. 300 m^3 27. about 3.46 in. 28. 12 in.

29. $S \approx 678.58$ cm^2; $V \approx 1017.88$ cm^3

30. $S \approx 2513.27$ cm^2; $V \approx 8042.48$ cm^3

31. $S \approx 439.82$ m^2; $V \approx 562.10$ m^3 32. 15 cm

33. $S \approx 615.75$ in.^{2}; $V \approx 1436.76$ in.^{3}

34. $S \approx 907.92$ ft^2; $V \approx 2572.44$ ft^3

35. $S \approx 2827.43$ ft^2; $V \approx 14{,}137.17$ ft^3

36. $S \approx 74.8$ million km^2; $V \approx 60.8$ billion km^3

37. about 272.55 m^3

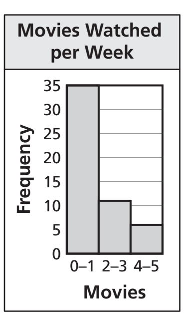
Chapter 12

Chapter 12 Maintaining Mathematical Proficiency *(p. 665)*

1. $\frac{6}{30} = \frac{p}{100}$, 20% 2. $\frac{a}{25} = \frac{68}{100}$, 17

3. $\frac{34.4}{86} = \frac{p}{100}$, 40%

4.



5. no; The sofa will cost 80% of the retail price and the arm chair will cost 81% of the retail price.

12.1 Vocabulary and Core Concept Check *(p. 672)*

1. probability

12.1 Monitoring Progress and Modeling with Mathematics *(pp. 672–674)*

3. 48; 1HHH, 1HHT, 1HTH, 1THH, 1HTT, 1THT, 1TTH, 1TTT, 2HHH, 2HHT, 2HTH, 2THH, 2HTT, 2THT, 2TTH, 2TTT, 3HHH, 3HHT, 3HTH, 3THH, 3HTT, 3THT, 3TTH, 3TTT, 4HHH, 4HHT, 4HTH, 4THH, 4HTT, 4THT, 4TTH, 4TTT, 5HHH, 5HHT, 5HTH, 5THH, 5HTT, 5THT, 5TTH, 5TTT, 6HHH, 6HHT, 6HTH, 6THH, 6HTT, 6THT, 6TTH, 6TTT

5. 12; R1, R2, R3, R4, W1, W2, W3, W4, B1, B2, B3, B4

7. $\frac{5}{16}$, or about 31.25%

9. **a.** $\frac{11}{12}$, or about 92% **b.** $\frac{13}{18}$, or about 72%

11. There are 4 outcomes, not 3; The probability is $\frac{1}{4}$.

13. about 0.56, or about 56% 15. 4

17. **a.** $\frac{9}{10}$, or 90% **b.** $\frac{2}{3}$, or about 67%
   c. The probability in part (b) is based on trials, not possible outcomes.

19. about 0.08, or about 8% 21. C, A, D, B

23. **a.** 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
   b. 2: $\frac{1}{36}$, 3: $\frac{1}{18}$, 4: $\frac{1}{12}$, 5: $\frac{1}{9}$, 6: $\frac{5}{36}$, 7: $\frac{1}{6}$, 8: $\frac{5}{36}$, 9: $\frac{1}{9}$, 10: $\frac{1}{12}$, 11: $\frac{1}{18}$, 12: $\frac{1}{36}$
   c. *Sample answer:* The probabilities are similar.

25. $\frac{\pi}{6}$, or about 52%

27. $\frac{3}{400}$, or 0.75%; about 113; $(0.0075)15{,}000 = 112.5$

12.1 Maintaining Mathematical Proficiency *(p. 674)*

29. $2x$ 31. $\frac{4x^6}{3}$ 33. $81p^4q^4$

12.2 Vocabulary and Core Concept Check *(p. 680)*

1. When two events are dependent, the occurrence of one event affects the other. When two events are independent, the occurrence of one event does not affect the other. *Sample answer:* choosing two marbles from a bag without replacement; rolling two dice

12.2 Monitoring Progress and Modeling with Mathematics *(pp. 680–682)*

3. dependent; The occurrence of event A affects the occurrence of event B.

5. dependent; The occurrence of event A affects the occurrence of event B.

7. yes 9. yes 11. about 2.8% 13. about 34.7%

15. The probabilities were added instead of multiplied; $P(A \text{ and } B) = (0.6)(0.2) = 0.12$

17. 0.325

19. **a.** about 1.2% **b.** about 1.0%
   You are about 1.2 times more likely to select 3 face cards when you replace each card before you select the next card.

21. **a.** about 17.1% **b.** about 81.4% 23. about 53.5%

25. **a.** *Sample answer:* Put 20 pieces of paper with each of the 20 students' names in a hat and pick one; 5%
   b. *Sample answer:* Put 45 pieces of paper in a hat with each student's name appearing once for each hour the student worked. Pick one piece; about 8.9%

27. yes; The chance that it will be rescheduled is $(0.7)(0.75) = 0.525$, which is a greater than a 50% chance.

29. **a.** wins: 0%; loses: 1.99%; ties: 98.01%
b. wins: 20.25%; loses: 30.25%; ties: 49.5%
c. yes; Go for 2 points after the first touchdown, and then go for 1 point if they were successful the first time or 2 points if they were unsuccessful the first time; winning: 44.55%; losing: 30.25%

12.2 Maintaining Mathematical Proficiency *(p. 682)*

31. $x = 0.2$ **33.** $x = 0.15$

12.3 Vocabulary and Core Concept Check *(p. 688)*

1. two-way table

12.3 Monitoring Progress and Modeling with Mathematics *(p. 688–690)*

3. 34; 40; 4; 6; 12

5.

		Gender		
		Male	**Female**	**Total**
Response	**Yes**	132	151	283
	No	39	29	68
	Total	171	180	351

351 people were surveyed, 171 males were surveyed, 180 females were surveyed, 283 people said yes, 68 people said no.

7.

		Dominant Hand		
		Left	**Right**	**Total**
Gender	**Female**	0.048	0.450	0.498
	Male	0.104	0.398	0.502
	Total	0.152	0.848	1

9.

		Gender		
		Male	**Female**	**Total**
Response	**Yes**	0.376	0.430	0.806
	No	0.111	0.083	0.194
	Total	0.487	0.513	1

11.

		Breakfast	
		Ate	**Did Not Eat**
Feeling	**Tired**	0.091	0.333
	Not Tired	0.909	0.667

13. **a.** about 0.789 **b.** 0.168
c. The events are independent.

15. The value for $P(\text{yes})$ was used in the denominator instead of the value for $P(\text{Tokyo})$;

$\frac{0.049}{0.39} \approx 0.126$

17. Route B; It has the best probability of getting to school on time.

19. *Sample answer:*

		Transportation to School			
		Rides Bus	**Walks**	**Car**	**Total**
Gender	**Male**	6	9	4	19
	Female	5	2	4	11
	Total	11	11	8	30

		Transportation to School			
		Rides Bus	**Walks**	**Car**	**Total**
Gender	**Male**	0.2	0.3	0.133	0.633
	Female	0.167	0.067	0.133	0.367
	Total	0.367	0.367	0.266	1

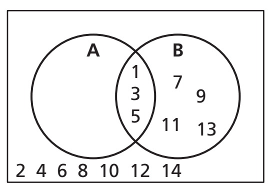
21. Routine B is the best option, but your friend's reasoning of why is incorrect; Routine B is the best choice because there is a 66.7% chance of reaching the goal, which is higher than the chances of Routine A (62.5%) and Routine C (63.6%).

23. **a.** about 0.438 **b.** about 0.387

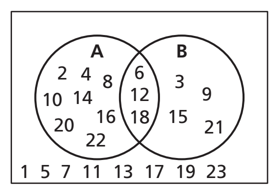
25. **a.** More of the current consumers prefer the leader, so they should improve the new snack before marketing it.
b. More of the new consumers prefer the new snack than the leading snack, so there is no need to improve the snack.

12.3 Maintaining Mathematical Proficiency *(p. 690)*

27.



29.



12.4 Vocabulary and Core Concept Check *(p. 697)*

1. yes; $\overline{A}$ is everything not in A; *Sample answer:* event A: you win the game, event $\overline{A}$: you do not win the game

12.4 Monitoring Progress and Modeling with Mathematics *(p. 697–698)*

3. 0.4 **5.** $\frac{7}{12}$, or about 0.58 **7.** $\frac{9}{20}$, or 0.45

9. $\frac{7}{10}$, or 0.7

11. forgot to subtract $P(\text{heart and face card})$; $P(\text{heart}) + P(\text{face card}) - P(\text{heart and face card}) = \frac{11}{26}$

13. $\frac{2}{3}$ **15.** 10% **17.** 0.4742, or 47.42% **19.** $\frac{13}{18}$

21. $\frac{3}{20}$

23. no; Until all cards, numbers, and colors are known, the conclusion cannot be made.

12.4 Maintaining Mathematical Proficiency *(p. 698)*

25. $4x^2 + 36x + 81$ **27.** $9a^2 - 42ab + 49b^2$

12.5 Vocabulary and Core Concept Check *(p. 704)*

1. permutation

12.5 Monitoring Progress and Modeling with Mathematics *(p. 704–706)*

3. a. 2 **b.** 2 **5. a.** 24 **b.** 12

7. a. 720 **b.** 30 **9.** 20 **11.** 9 **13.** 20,160

15. 870 **17.** 990 **19.** $\frac{1}{56}$ **21.** 4 **23.** 20

25. 5 **27.** 1 **29.** 220 **31.** 6435 **33.** 635,376

35. The factorial in the denominator was left out;

$$_{11}P_7 = \frac{11!}{(11-7)!} = 1{,}663{,}200$$

37. combinations; The order is not important; 45

39. permutations; The order is important; 132,600

41. $_{50}C_9 = {}_{50}C_{41}$; For each combination of 9 objects, there is a corresponding combination of the 41 remaining objects.

43.

	$r = 0$	$r = 1$	$r = 2$	$r = 3$
$_3P_r$	1	3	6	6
$_3C_r$	1	3	3	1

$_nP_r \geq {}_nC_r$; Because $_nP_r = \dfrac{n!}{(n-r)!}$ and $_nC_r = \dfrac{n!}{(n-r)! \cdot r!}$, $_nP_r > {}_nC_r$ when $r > 1$ and $_nP_r = {}_nC_r$ when $r = 0$ or $r = 1$.

45. $\dfrac{1}{44{,}850}$ **47.** $\dfrac{1}{15{,}890{,}700}$

49. a. $_nC_{n-2} - n$ **b.** $\dfrac{n(n-3)}{2}$

51. 30

53. a. $\frac{1}{2}$ **b.** $\frac{1}{2}$; The probabilities are the same.

55. a. $\frac{1}{90}$ **b.** $\frac{9}{10}$

57. $\frac{1}{406}$; There are $_{30}C_5$ possible groups. The number of groups that will have you and your two best friends is $_{27}C_2$.

12.5 Maintaining Mathematical Proficiency *(p. 706)*

59. $\frac{1}{5}$

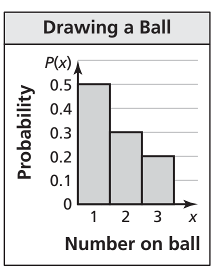
12.6 Vocabulary and Core Concept Check *(p. 711)*

1. a variable whose value is determined by the outcomes of a probability experiment

12.6 Monitoring Progress and Modeling with Mathematics *(pp. 711–712)*

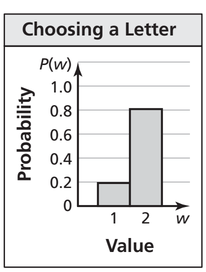
3.

x (value)	1	2	3
Outcomes	5	3	2
P(*x*)	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{5}$



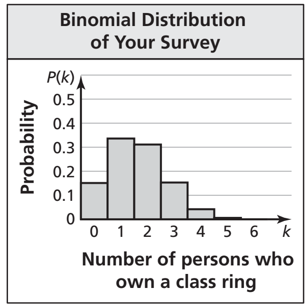
5.

w (value)	1	2
Outcomes	5	21
P(*w*)	$\frac{5}{26}$	$\frac{21}{26}$



7. a. 2 **b.** $\frac{5}{8}$ **9.** about 0.00002 **11.** about 0.00018

13. a.



b. The most likely outcome is that 1 of the 6 students owns a ring.

c. about 0.798

15. The exponents are switched;
$P(k = 3) = {}_5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{5-3} \approx 0.032$

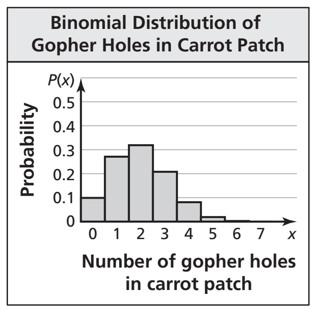
17. a. $P(0) \approx 0.099$, $P(1) \approx 0.271$, $P(2) \approx 0.319$, $P(3) \approx 0.208$, $P(4) \approx 0.081$, $P(5) \approx 0.019$, $P(6) \approx 0.0025$, $P(7) \approx 0.00014$

b.

x	0	1	2	3	4
P(*x*)	0.099	0.271	0.319	0.208	0.081

x	5	6	7
P(*x*)	0.019	0.0025	0.00014

c.

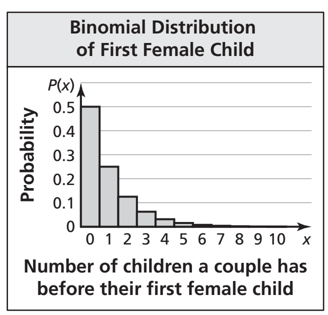


19. no; The data is skewed right, so the probability of failure is greater.

21. a. The statement is not valid, because having a male and having a female are independent events.

b. 0.03125

c.



skewed right

12.6 Maintaining Mathematical Proficiency *(p. 712)*

23. FFF, FFM FMF, FMM, MMM, MMF, MFM, MFF

Chapter 12 Review *(pp. 714–716)*

1. $\frac{2}{9}$; $\frac{7}{9}$ **2.** 20 points

3. **a.** 0.15625 **b.** about 0.1667

You are about 1.07 times more likely to pick a red then a green if you do not replace the first marble.

4. **a.** about 0.0586 **b.** 0.0625

You are about 1.07 times more likely to pick a blue then a red if you do not replace the first marble.

5. **a.** 0.25 **b.** about 0.2333

You are about 1.07 times more likely to pick a green and then another green if you replace the first marble.

6. about 0.529

7.

		Gender		
		Men	**Women**	**Total**
Response	**Yes**	200	230	430
	No	20	40	60
	Total	220	270	490

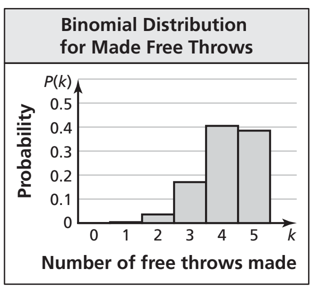
About 44.9% of responders were men, about 55.1% of responders were women, about 87.8% of responders thought it was impactful, about 12.2% of responders thought it was not impactful.

8. 0.68 **9.** 0.02 **10.** 5040 **11.** 1,037,836,800

12. 15 **13.** 70 **14.** 40,320

15. $\frac{1}{84}$ **16.** about 0.12

17.



The most likely outcome is that 4 of the 5 free throw shots will be made.

Additional Topic

Vocabulary and Core Concept Check *(p. 726)*

1. focus; directrix

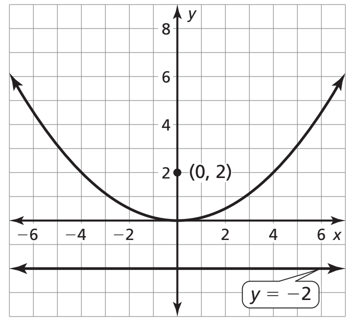
Monitoring Progress and Modeling with Mathematics *(pp. 726–728)*

3. $y = \frac{1}{4}x^2$ **5.** $y = -\frac{1}{8}x^2$ **7.** $y = \frac{1}{24}x^2$

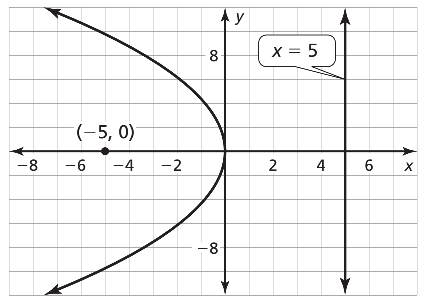
9. $y = -\frac{1}{40}x^2$

11. A, B and D; Each has a value for p that is negative. Substituting in a negative value for p in $y = \dfrac{1}{4p}x^2$ results in a parabola that has been reflected across the x-axis.

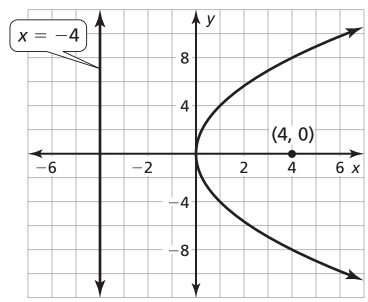
13. The focus is (0, 2). The directrix is $y = -2$. The axis of symmetry is the y-axis.



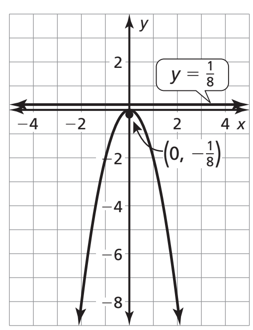
15. The focus is (−5, 0). The directrix is $x = 5$. The axis of symmetry is the x-axis.



17. The focus is (4, 0). The directrix is $x = -4$. The axis of symmetry is the x-axis.

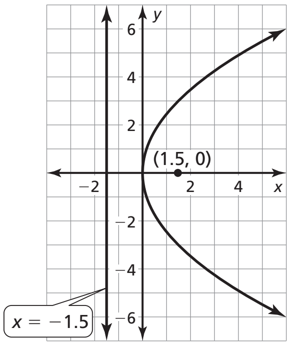


19. The focus is $\left(0, -\frac{1}{8}\right)$. The directrix is $y = \frac{1}{8}$. The axis of symmetry is the y-axis.



Selected Answers

21. Instead of a vertical axis of symmetry, the graph should have a horizontal axis of symmetry.



23. 9.5 in.; The receiver should be placed at the focus. The distance from the vertex to the focus is $p = \frac{38}{4} = 9.5$ in.

25. $y = \frac{1}{32}x^2$ **27.** $x = -\frac{1}{10}y^2$ **29.** $x = \frac{1}{12}y^2$

31. $x = \frac{1}{40}y^2$ **33.** $y = -\frac{3}{20}x^2$ **35.** $y = \frac{7}{24}x^2$

37. $x = -\frac{1}{16}y^2 - 4$ **39.** $y = \frac{1}{6}x^2 + 1$

41. The vertex is (3, 2). The focus is (3, 4). The directrix is $y = 0$. The axis of symmetry is $x = 3$. The graph is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 3 units right and 2 units up.

43. The vertex is (1, 3). The focus is (5, 3). The directrix is $x = -3$. The axis of symmetry is $y = 3$. The graph is a horizontal shrink by a factor of $\frac{1}{4}$ followed by a translation 1 unit right and 3 units up.

45. The vertex is $(2, -4)$. The focus is $\left(\frac{23}{12}, -4\right)$. The directrix is $x = \frac{25}{12}$. The axis of symmetry is $y = -4$. The graph is a horizontal stretch by a factor of 12 followed by a reflection in the y-axis and a translation 2 units right and 4 units down.

47. $x = \dfrac{1}{5.2}y^2$; about 3.08 in.

49. As $|p|$ increases, the graph gets wider; As $|p|$ increases, the constant in the function gets smaller which results in a vertical shrink, making the graph wider.

51. $y = \dfrac{1}{4}x^2$ **53.** $x = \dfrac{1}{4p}y^2$