12.4 Probability of Disjoint and Overlapping Events

Essential Question How can you find probabilities of disjoint and overlapping events?

Two events are **disjoint**, or **mutually exclusive**, when they have no outcomes in common. Two events are **overlapping** when they have one or more outcomes in common.

EXPLORATION 1 **Disjoint Events and Overlapping Events**

Work with a partner. A six-sided die is rolled. Draw a Venn diagram that relates the two events. Then decide whether the events are disjoint or overlapping.

a. Event A: The result is an even number.
Event B: The result is a prime number.

b. Event A: The result is 2 or 4.
Event B: The result is an odd number.

MODELING WITH MATHEMATICS

To be proficient in math, you need to map the relationships between important quantities in a practical situation using such tools as diagrams.

EXPLORATION 2 **Finding the Probability that Two Events Occur**

Work with a partner. A six-sided die is rolled. For each pair of events, find (a) $P(A)$, (b) $P(B)$, (c) $P(A \text{ and } B)$, and (d) $P(A \text{ or } B)$.

a. Event A: The result is an even number.
Event B: The result is a prime number.

b. Event A: The result is 2 or 4.
Event B: The result is an odd number.

EXPLORATION 3 **Discovering Probability Formulas**

Work with a partner.

a. In general, if event A and event B are disjoint, then what is the probability that event A or event B will occur? Use a Venn diagram to justify your conclusion.

b. In general, if event A and event B are overlapping, then what is the probability that event A or event B will occur? Use a Venn diagram to justify your conclusion.

c. Conduct an experiment using a six-sided die. Roll the die 50 times and record the results. Then use the results to find the probabilities described in Exploration 2. How closely do your experimental probabilities compare to the theoretical probabilities you found in Exploration 2?

Communicate Your Answer

4. How can you find probabilities of disjoint and overlapping events?

5. Give examples of disjoint events and overlapping events that do not involve dice.

12.4 Lesson

Core Vocabulary

compound event, *p. 694*
overlapping events, *p. 694*
disjoint or mutually exclusive events, *p. 694*

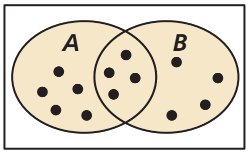
Previous
Venn diagram

What You Will Learn

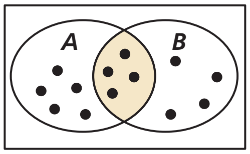
- Find probabilities of compound events.
- Use more than one probability rule to solve real-life problems.

Compound Events

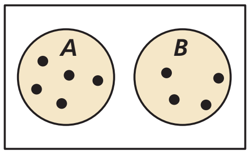
When you consider all the outcomes for either of two events A and B, you form the *union* of A and B, as shown in the first diagram. When you consider only the outcomes shared by both A and B, you form the *intersection* of A and B, as shown in the second diagram. The union or intersection of two events is called a **compound event**.



Union of A and B



Intersection of A and B



Intersection of A and B is empty.

To find $P(A \text{ or } B)$ you must consider what outcomes, if any, are in the intersection of A and B. Two events are **overlapping** when they have one or more outcomes in common, as shown in the first two diagrams. Two events are **disjoint**, or **mutually exclusive**, when they have no outcomes in common, as shown in the third diagram.

STUDY TIP

If two events A and B are overlapping, then the outcomes in the intersection of A and B are counted *twice* when $P(A)$ and $P(B)$ are added. So, $P(A \text{ and } B)$ must be subtracted from the sum.

Core Concept

Probability of Compound Events

If A and B are any two events, then the probability of A or B is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

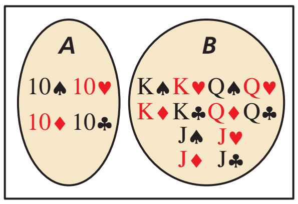
If A and B are disjoint events, then the probability of A or B is

$$P(A \text{ or } B) = P(A) + P(B).$$

EXAMPLE 1 Finding the Probability of Disjoint Events

A card is randomly selected from a standard deck of 52 playing cards. What is the probability that it is a 10 *or* a face card?

SOLUTION



Let event A be selecting a 10 and event B be selecting a face card. From the diagram, A has 4 outcomes and B has 12 outcomes. Because A and B are disjoint, the probability is

$$
\begin{aligned}
P(A \text{ or } B) &= P(A) + P(B) && \text{Write disjoint probability formula.} \\
&= \frac{4}{52} + \frac{12}{52} && \text{Substitute known probabilities.} \\
&= \frac{16}{52} && \text{Add.} \\
&= \frac{4}{13} && \text{Simplify.} \\
&\approx 0.308. && \text{Use a calculator.}
\end{aligned}
$$

COMMON ERROR

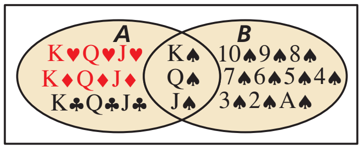
When two events A and B overlap, as in Example 2, $P(A \text{ or } B)$ does not equal $P(A) + P(B)$.

EXAMPLE 2 Finding the Probability of Overlapping Events

A card is randomly selected from a standard deck of 52 playing cards. What is the probability that it is a face card *or* a spade?

SOLUTION

Let event A be selecting a face card and event B be selecting a spade. From the diagram, A has 12 outcomes and B has 13 outcomes. Of these, 3 outcomes are common to A and B. So, the probability of selecting a face card or a spade is



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$ Write general formula.

$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52}$$ Substitute known probabilities.

$$= \frac{22}{52}$$ Add.

$$= \frac{11}{26}$$ Simplify.

$$\approx 0.423.$$ Use a calculator.

EXAMPLE 3 Using a Formula to Find $P(A \text{ and } B)$

Out of 200 students in a senior class, 113 students are either varsity athletes or on the honor roll. There are 74 seniors who are varsity athletes and 51 seniors who are on the honor roll. What is the probability that a randomly selected senior is both a varsity athlete *and* on the honor roll?

SOLUTION

Let event A be selecting a senior who is a varsity athlete and event B be selecting a senior on the honor roll. From the given information, you know that $P(A) = \frac{74}{200}$, $P(B) = \frac{51}{200}$, and $P(A \text{ or } B) = \frac{113}{200}$. The probability that a randomly selected senior is both a varsity athlete *and* on the honor roll is $P(A \text{ and } B)$.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$ Write general formula.

$$\frac{113}{200} = \frac{74}{200} + \frac{51}{200} - P(A \text{ and } B)$$ Substitute known probabilities.

$$P(A \text{ and } B) = \frac{74}{200} + \frac{51}{200} - \frac{113}{200}$$ Solve for $P(A \text{ and } B)$.

$$P(A \text{ and } B) = \frac{12}{200}$$ Simplify.

$$P(A \text{ and } B) = \frac{3}{50}, \text{ or } 0.06$$ Simplify.

Monitoring Progress

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A card is randomly selected from a standard deck of 52 playing cards. Find the probability of the event.

1. selecting an ace *or* an 8

2. selecting a 10 *or* a diamond

3. **WHAT IF?** In Example 3, suppose 32 seniors are in the band and 64 seniors are in the band or on the honor roll. What is the probability that a randomly selected senior is both in the band and on the honor roll?

Using More Than One Probability Rule

In the first four sections of this chapter, you have learned several probability rules. The solution to some real-life problems may require the use of two or more of these probability rules, as shown in the next example.

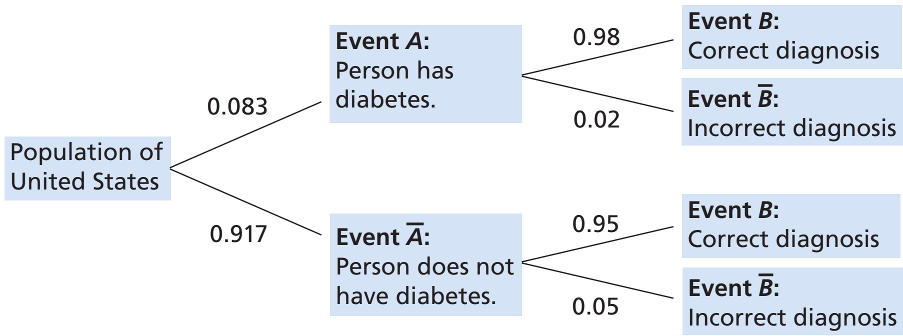
EXAMPLE 4 **Solving a Real-Life Problem**

The American Diabetes Association estimates that 8.3% of people in the United States have diabetes. Suppose that a medical lab has developed a simple diagnostic test for diabetes that is 98% accurate for people who have the disease and 95% accurate for people who do not have it. The medical lab gives the test to a randomly selected person. What is the probability that the diagnosis is correct?

SOLUTION

Let event A be "person has diabetes" and event B be "correct diagnosis." Notice that the probability of B depends on the occurrence of A, so the events are dependent. When A occurs, $P(B) = 0.98$. When A does not occur, $P(B) = 0.95$.

A probability tree diagram, where the probabilities are given along the branches, can help you see the different ways to obtain a correct diagnosis. Use the complements of events A and B to complete the diagram, where $\overline{A}$ is "person does not have diabetes" and $\overline{B}$ is "incorrect diagnosis." Notice that the probabilities for all branches from the same point must sum to 1.



To find the probability that the diagnosis is correct, follow the branches leading to event B.

$$
\begin{aligned}
P(B) &= P(A \text{ and } B) + P(\overline{A} \text{ and } B) && \text{Use tree diagram.}\\
&= P(A) \cdot P(B|A) + P(\overline{A}) \cdot P(B|\overline{A}) && \text{Probability of dependent events}\\
&= (0.083)(0.98) + (0.917)(0.95) && \text{Substitute.}\\
&\approx 0.952 && \text{Use a calculator.}
\end{aligned}
$$

▶ The probability that the diagnosis is correct is about 0.952, or 95.2%.

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4. In Example 4, what is the probability that the diagnosis is *incorrect*?

5. A high school basketball team leads at halftime in 60% of the games in a season. The team wins 80% of the time when they have the halftime lead, but only 10% of the time when they do not. What is the probability that the team wins a particular game during the season?

12.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Are the events A and $\overline{A}$ disjoint? Explain. Then give an example of a real-life event and its complement.

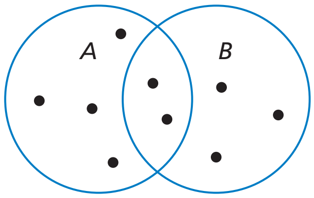
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

How many outcomes are in the intersection of A and B?

How many outcomes are shared by both A and B?

How many outcomes are in the union of A and B?

How many outcomes in B are also in A?



Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, events A and B are disjoint. Find $P(A \text{ or } B)$.

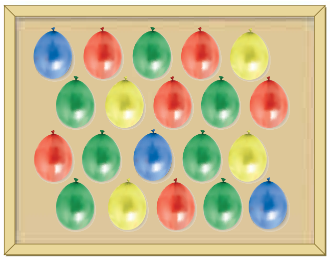
3. $P(A) = 0.3$, $P(B) = 0.1$

4. $P(A) = 0.55$, $P(B) = 0.2$

5. $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$

6. $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{5}$

7. **PROBLEM SOLVING** Your dart is equally likely to hit any point inside the board shown. You throw a dart and pop a balloon. What is the probability that the balloon is red or blue? *(See Example 1.)*



8. **PROBLEM SOLVING** You and your friend are among several candidates running for class president. You estimate that there is a 45% chance you will win and a 25% chance your friend will win. What is the probability that you or your friend win the election?

9. **PROBLEM SOLVING** You are performing an experiment to determine how well plants grow under different light sources. Of the 30 plants in the experiment, 12 receive visible light, 15 receive ultraviolet light, and 6 receive both visible and ultraviolet light. What is the probability that a plant in the experiment receives visible or ultraviolet light? *(See Example 2.)*

10. **PROBLEM SOLVING** Of 162 students honored at an academic awards banquet, 48 won awards for mathematics and 78 won awards for English. There are 14 students who won awards for both mathematics and English. A newspaper chooses a student at random for an interview. What is the probability that the student interviewed won an award for English or mathematics?

ERROR ANALYSIS In Exercises 11 and 12, describe and correct the error in finding the probability of randomly drawing the given card from a standard deck of 52 playing cards.

11.
$$P(\text{heart or face card}) = P(\text{heart}) + P(\text{face card}) = \frac{13}{52} + \frac{12}{52} = \frac{25}{52}$$

12.
$$P(\text{club or } 9) = P(\text{club}) + P(9) + P(\text{club and } 9) = \frac{13}{52} + \frac{4}{52} + \frac{1}{52} = \frac{9}{26}$$

In Exercises 13 and 14, you roll a six-sided die. Find $P(A \text{ or } B)$.

13. Event A: Roll a 6.
Event B: Roll a prime number.

14. Event A: Roll an odd number.
Event B: Roll a number less than 5.

15. **DRAWING CONCLUSIONS** A group of 40 trees in a forest are not growing properly. A botanist determines that 34 of the trees have a disease or are being damaged by insects, with 18 trees having a disease and 20 being damaged by insects. What is the probability that a randomly selected tree has both a disease and is being damaged by insects? *(See Example 3.)*

16. **DRAWING CONCLUSIONS** A company paid overtime wages or hired temporary help during 9 months of the year. Overtime wages were paid during 7 months, and temporary help was hired during 4 months. At the end of the year, an auditor examines the accounting records and randomly selects one month to check the payroll. What is the probability that the auditor will select a month in which the company paid overtime wages and hired temporary help?

17. **DRAWING CONCLUSIONS** A company is focus testing a new type of fruit drink. The focus group is 47% male. Of the responses, 40% of the males and 54% of the females said they would buy the fruit drink. What is the probability that a randomly selected person would buy the fruit drink? *(See Example 4.)*

18. **DRAWING CONCLUSIONS** The Redbirds trail the Bluebirds by one goal with 1 minute left in the hockey game. The Redbirds' coach must decide whether to remove the goalie and add a frontline player. The probabilities of each team scoring are shown in the table.

	Goalie	No goalie
Redbirds score	0.1	0.3
Bluebirds score	0.1	0.6

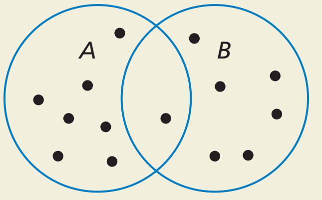
a. Find the probability that the Redbirds score and the Bluebirds do not score when the coach leaves the goalie in.

b. Find the probability that the Redbirds score and the Bluebirds do not score when the coach takes the goalie out.

c. Based on parts (a) and (b), what should the coach do?

19. **PROBLEM SOLVING** You can win concert tickets from a radio station if you are the first person to call when the song of the day is played, or if you are the first person to correctly answer the trivia question. The song of the day is announced at a random time between 7:00 and 7:30 A.M. The trivia question is asked at a random time between 7:15 and 7:45 A.M. You begin listening to the radio station at 7:20. Find the probability that you miss the announcement of the song of the day or the trivia question.

20. **HOW DO YOU SEE IT?** Are events *A* and *B* disjoint events? Explain your reasoning.



21. **PROBLEM SOLVING** You take a bus from your neighborhood to your school. The express bus arrives at your neighborhood at a random time between 7:30 and 7:36 A.M. The local bus arrives at your neighborhood at a random time between 7:30 and 7:40 A.M. You arrive at the bus stop at 7:33 A.M. Find the probability that you missed both the express bus and the local bus.

22. **THOUGHT PROVOKING** Write a general rule for finding *P*(*A* or *B* or *C*) for (a) disjoint and (b) overlapping events *A*, *B*, and *C*.

23. **MAKING AN ARGUMENT** A bag contains 40 cards numbered 1 through 40 that are either red or blue. A card is drawn at random and placed back in the bag. This is done four times. Two red cards are drawn, numbered 31 and 19, and two blue cards are drawn, numbered 22 and 7. Your friend concludes that red cards and even numbers must be mutually exclusive. Is your friend correct? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

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24. $(n - 12)^2$ **25.** $(2x + 9)^2$ **26.** $(-5z + 6)^2$ **27.** $(3a - 7b)^2$