11.8 Surface Areas and Volumes of Spheres

Essential Question How can you find the surface area and the volume of a sphere?

EXPLORATION 1 Finding the Surface Area of a Sphere

Work with a partner. Remove the covering from a baseball or softball.

You will end up with two "figure 8" pieces of material, as shown above. From the amount of material it takes to cover the ball, what would you estimate the surface area S of the ball to be? Express your answer in terms of the radius r of the ball.

$S =$ ______ Surface area of a sphere

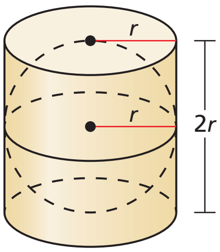
USING TOOLS STRATEGICALLY

To be proficient in math, you need to identify relevant external mathematical resources, such as content located on a website.

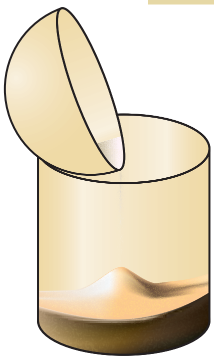
Use the Internet or some other resource to confirm that the formula you wrote for the surface area of a sphere is correct.

EXPLORATION 2 Finding the Volume of a Sphere

Work with a partner. A cylinder is circumscribed about a sphere, as shown. Write a formula for the volume V of the cylinder in terms of the radius r.



$V =$ ______ Volume of cylinder



When half of the sphere (a *hemisphere*) is filled with sand and poured into the cylinder, it takes three hemispheres to fill the cylinder. Use this information to write a formula for the volume V of a sphere in terms of the radius r.

$V =$ ______ Volume of a sphere

Communicate Your Answer

3. How can you find the surface area and the volume of a sphere?

4. Use the results of Explorations 1 and 2 to find the surface area and the volume of a sphere with a radius of (a) 3 inches and (b) 2 centimeters.

11.8 Lesson

Core Vocabulary

chord of a sphere, *p. 648*
great circle, *p. 648*

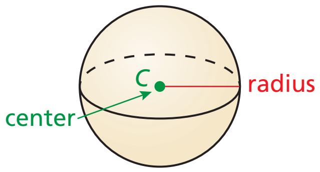
Previous
sphere
center of a sphere
radius of a sphere
diameter of a sphere
hemisphere

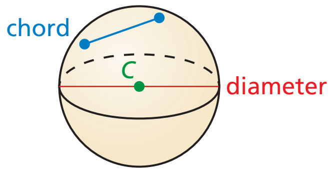
What You Will Learn

- Find surface areas of spheres.
- Find volumes of spheres.

Finding Surface Areas of Spheres

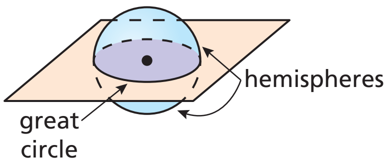
A *sphere* is the set of all points in space equidistant from a given point. This point is called the *center* of the sphere. A *radius* of a sphere is a segment from the center to a point on the sphere. A **chord of a sphere** is a segment whose endpoints are on the sphere. A *diameter* of a sphere is a chord that contains the center.





As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

If a plane intersects a sphere, then the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. The circumference of a great circle is the circumference of the sphere. Every great circle of a sphere separates the sphere into two congruent halves called *hemispheres*.



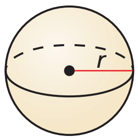
Core Concept

Surface Area of a Sphere

The surface area S of a sphere is

$$S = 4\pi r^2$$

where r is the radius of the sphere.



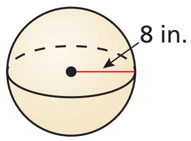
$S = 4\pi r^2$

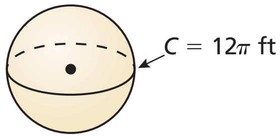
To understand the formula for the surface area of a sphere, think of a baseball. The surface area of a baseball is sewn from two congruent shapes, each of which resembles two joined circles.

So, the entire covering of the baseball consists of four circles, each with radius r. The area A of a circle with radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the formula for the surface area of a sphere.

EXAMPLE 1 Finding the Surface Areas of Spheres

Find the surface area of each sphere.

a. 

b. 

SOLUTION

a. $S = 4\pi r^2$ — Formula for surface area of a sphere

$= 4\pi(8)^2$ — Substitute 8 for r.

$= 256\pi$ — Simplify.

≈ 804.25 — Use a calculator.

▶ The surface area is 256π, or about 804.25 square inches.

b. The circumference of the sphere is 12π, so the radius of the sphere is $\frac{12\pi}{2\pi} = 6$ feet.

$S = 4\pi r^2$ — Formula for surface area of a sphere

$= 4\pi(6)^2$ — Substitute 6 for r.

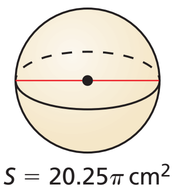
$= 144\pi$ — Simplify.

≈ 452.39 — Use a calculator.

▶ The surface area is 144π, or about 452.39 square feet.

EXAMPLE 2 Finding the Diameter of a Sphere

Find the diameter of the sphere.



SOLUTION

$S = 4\pi r^2$ — Formula for surface area of a sphere

$20.25\pi = 4\pi r^2$ — Substitute 20.25π for S.

$5.0625 = r^2$ — Divide each side by 4π.

$2.25 = r$ — Find the positive square root.

▶ The diameter is $2r = 2 \cdot 2.25 = 4.5$ centimeters.

COMMON ERROR

Be sure to multiply the value of r by 2 to find the diameter.

Monitoring Progress

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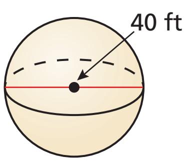
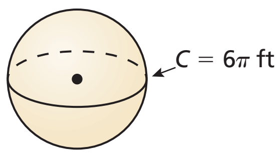
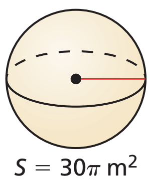
Find the surface area of the sphere.

1. 

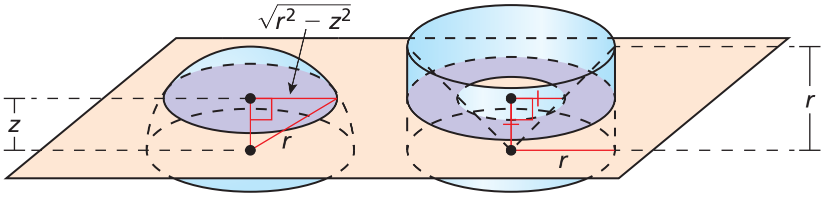
2. 

3. Find the radius of the sphere.



Finding Volumes of Spheres

The figure shows a hemisphere and a cylinder with a cone removed. A plane parallel to their bases intersects the solids z units above their bases.



Using the AA Similarity Theorem (Theorem 8.3), you can show that the radius of the cross section of the cone at height z is z. The area of the cross section formed by the plane is $\pi(r^2 - z^2)$ for both solids. Because the solids have the same height and the same cross-sectional area at every level, they have the same volume by Cavalieri's Principle.

$$\begin{aligned} V_{\text{hemisphere}} &= V_{\text{cylinder}} - V_{\text{cone}} \\ &= \pi r^2(r) - \tfrac{1}{3}\pi r^2(r) \\ &= \tfrac{2}{3}\pi r^3 \end{aligned}$$

So, the volume of a sphere of radius r is

$$2 \cdot V_{\text{hemisphere}} = 2 \cdot \tfrac{2}{3}\pi r^3 = \tfrac{4}{3}\pi r^3.$$

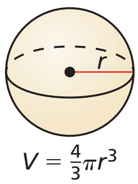
Core Concept

Volume of a Sphere

The volume V of a sphere is

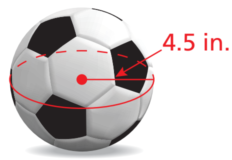
$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.



EXAMPLE 3 Finding the Volume of a Sphere

Find the volume of the soccer ball.



SOLUTION

$V = \frac{4}{3}\pi r^3$	Formula for volume of a sphere
$= \frac{4}{3}\pi(4.5)^3$	Substitute 4.5 for r.
$= 121.5\pi$	Simplify.
≈ 381.70	Use a calculator.

▶ The volume of the soccer ball is 121.5π, or about 381.70 cubic inches.

EXAMPLE 4 Finding the Volume of a Sphere

The surface area of a sphere is 324π square centimeters. Find the volume of the sphere.

SOLUTION

Step 1 Use the surface area to find the radius.

$S = 4\pi r^2$	Formula for surface area of a sphere
$324\pi = 4\pi r^2$	Substitute 324π for S.
$81 = r^2$	Divide each side by 4π.
$9 = r$	Find the positive square root.

The radius is 9 centimeters.

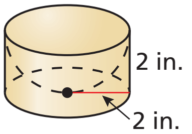
Step 2 Use the radius to find the volume.

$V = \frac{4}{3}\pi r^3$	Formula for volume of a sphere
$= \frac{4}{3}\pi(9)^3$	Substitute 9 for r.
$= 972\pi$	Simplify.
≈ 3053.63	Use a calculator.

▶ The volume is 972π, or about 3053.63 cubic centimeters.

EXAMPLE 5 Finding the Volume of a Composite Solid

Find the volume of the composite solid.



SOLUTION

Volume of solid = Volume of cylinder − Volume of hemisphere

$= \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	Write formulas.
$= \pi(2)^2(2) - \frac{2}{3}\pi(2)^3$	Substitute.
$= 8\pi - \frac{16}{3}\pi$	Multiply.
$= \frac{24}{3}\pi - \frac{16}{3}\pi$	Rewrite fractions using least common denominator.
$= \frac{8}{3}\pi$	Subtract.
≈ 8.38	Use a calculator.

▶ The volume is $\frac{8}{3}\pi$, or about 8.38 cubic inches.

Monitoring Progress

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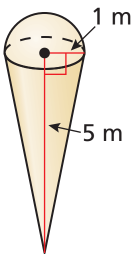
4. The radius of a sphere is 5 yards. Find the volume of the sphere.

5. The diameter of a sphere is 36 inches. Find the volume of the sphere.

6. The surface area of a sphere is 576π square centimeters. Find the volume of the sphere.

7. Find the volume of the composite solid at the left.



11.8 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

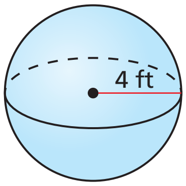
Vocabulary and Core Concept Check

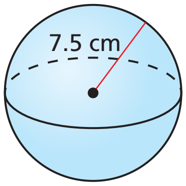
1. **VOCABULARY** When a plane intersects a sphere, what must be true for the intersection to be a great circle?

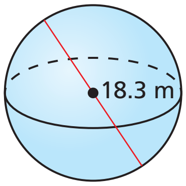
2. **WRITING** Explain the difference between a sphere and a hemisphere.

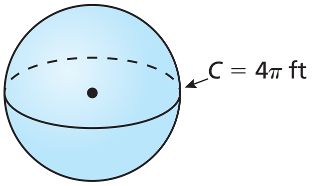
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the surface area of the sphere. *(See Example 1.)*

3. 

4. 

5. 

6. 

In Exercises 7–10, find the indicated measure. *(See Example 2.)*

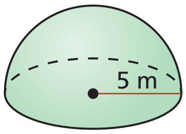
7. Find the radius of a sphere with a surface area of 4π square feet.

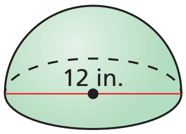
8. Find the radius of a sphere with a surface area of 1024π square inches.

9. Find the diameter of a sphere with a surface area of 900π square meters.

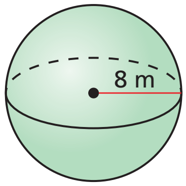
10. Find the diameter of a sphere with a surface area of 196π square centimeters.

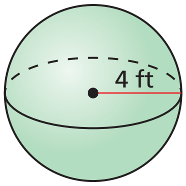
In Exercises 11 and 12, find the surface area of the hemisphere.

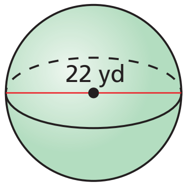
11. 

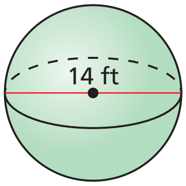
12. 

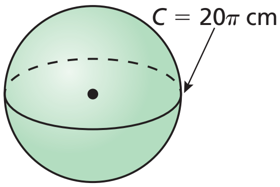
In Exercises 13–18, find the volume of the sphere. *(See Example 3.)*

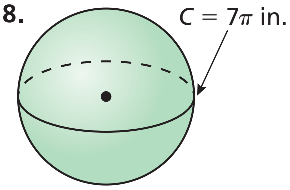
13. 

14. 

15. 

16. 

17. 

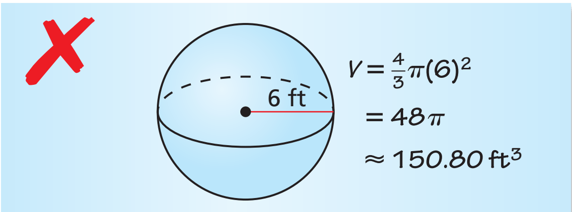
18. 

In Exercises 19 and 20, find the volume of the sphere with the given surface area. *(See Example 4.)*

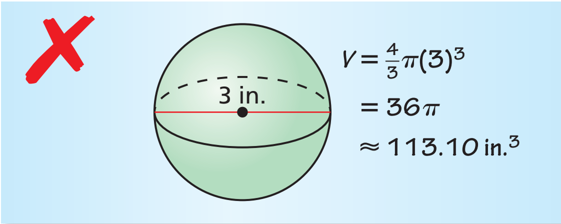
19. Surface area $= 16\pi$ ft^2

20. Surface area $= 484\pi$ cm^2

21. **ERROR ANALYSIS** Describe and correct the error in finding the volume of the sphere.

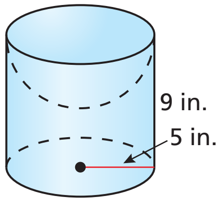


22. ERROR ANALYSIS Describe and correct the error in finding the volume of the sphere.



In Exercises 23–26, find the volume of the composite solid. *(See Example 5.)*

23.



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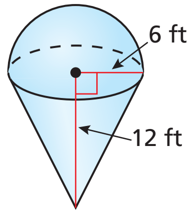

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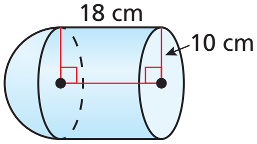

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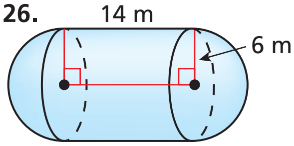


In Exercises 27–32, find the surface area and volume of the ball.

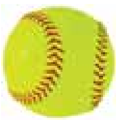
27. bowling ball

$d = 8.5$ in.

28. basketball

$C = 29.5$ in.

29. softball



$C = 12$ in.

30. golf ball

$d = 1.7$ in.

31. volleyball

$C = 26$ in.

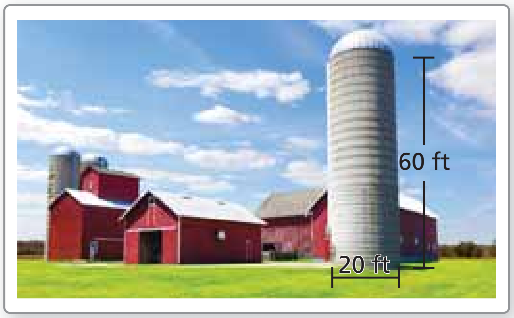
32. baseball

$C = 9$ in.

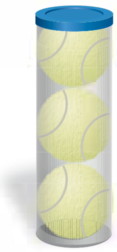
33. MAKING AN ARGUMENT You friend claims that if the radius of a sphere is doubled, then the surface area of the sphere will also be doubled. Is your friend correct? Explain your reasoning.

34. REASONING A semicircle with a diameter of 18 inches is rotated about its diameter. Find the surface area and the volume of the solid formed.

35. MODELING WITH MATHEMATICS A silo has the dimensions shown. The top of the silo is a hemispherical shape. Find the volume of the silo.



36. MODELING WITH MATHEMATICS Three tennis balls are stored in a cylindrical container with a height of 8 inches and a radius of 1.43 inches. The circumference of a tennis ball is 8 inches.



a. Find the volume of a tennis ball.

b. Find the amount of space within the cylinder not taken up by the tennis balls.

37. ANALYZING RELATIONSHIPS Use the table shown for a sphere.

Radius	Surface area	Volume
3 in.	36π in.^{2}	36π in.^{3}
6 in.		
9 in.		
12 in.		

a. Copy and complete the table. Leave your answers in terms of π.

b. What happens to the surface area of the sphere when the radius is doubled? tripled? quadrupled?

c. What happens to the volume of the sphere when the radius is doubled? tripled? quadrupled?

38. MATHEMATICAL CONNECTIONS A sphere has a diameter of $4(x + 3)$ centimeters and a surface area of 784π square centimeters. Find the value of x.

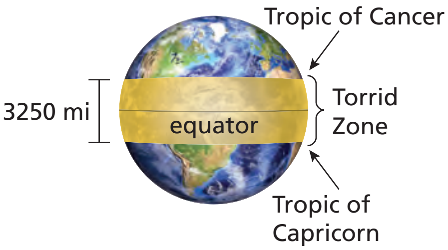
39. **MODELING WITH MATHEMATICS** The radius of Earth is about 3960 miles. The radius of the moon is about 1080 miles.

a. Find the surface area of Earth and the moon.

b. Compare the surface areas of Earth and the moon.

c. About 70% of the surface of Earth is water. How many square miles of water are on Earth's surface?

40. **MODELING WITH MATHEMATICS** The Torrid Zone on Earth is the area between the Tropic of Cancer and the Tropic of Capricorn. The distance between these two tropics is about 3250 miles. You can estimate the distance as the height of a cylindrical belt around the Earth at the equator.

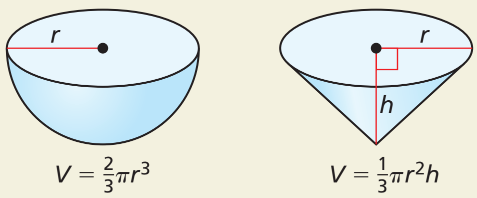


a. Estimate the surface area of the Torrid Zone. (The radius of Earth is about 3960 miles.)

b. A meteorite is equally likely to hit anywhere on Earth. Estimate the probability that a meteorite will land in the Torrid Zone.

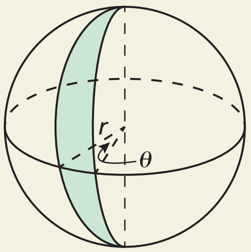
41. **ABSTRACT REASONING** A sphere is inscribed in a cube with a volume of 64 cubic inches. What is the surface area of the sphere? Explain your reasoning.

42. **HOW DO YOU SEE IT?** The formula for the volume of a hemisphere and a cone are shown. If each solid has the same radius and $r = h$, which solid will have a greater volume? Explain your reasoning.



43. **CRITICAL THINKING** Let V be the volume of a sphere, S be the surface area of the sphere, and r be the radius of the sphere. Write an equation for V in terms of r and S. $\left(\textit{Hint}\text{: Start with the ratio } \frac{V}{S}.\right)$

44. **THOUGHT PROVOKING** A *spherical lune* is the region between two great circles of a sphere. Find the formula for the area of a lune.



45. **CRITICAL THINKING** The volume of a right cylinder is the same as the volume of a sphere. The radius of the sphere is 1 inch. Give three possibilities for the dimensions of the cylinder.

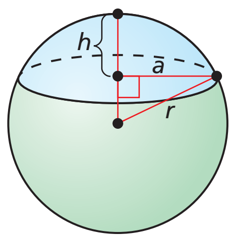
46. **PROBLEM SOLVING** A *spherical cap* is a portion of a sphere cut off by a plane. The formula for the volume of a spherical cap is $V = \frac{\pi h}{6}(3a^2 + h^2)$, where a is the radius of the base of the cap and h is the height of the cap. Use the diagram and given information to find the volume of each spherical cap.

a. $r = 5$ ft, $a = 4$ ft

b. $r = 34$ cm, $a = 30$ cm

c. $r = 13$ m, $h = 8$ m

d. $r = 75$ in., $h = 54$ in.



47. **CRITICAL THINKING** A sphere with a radius of 2 inches is inscribed in a right cone with a height of 6 inches. Find the surface area and the volume of the cone.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the triangle. Round decimal answers to the nearest tenth. *(Section 9.7)*

48. $A = 26°, C = 35°, b = 13$

49. $B = 102°, C = 43°, b = 21$

50. $a = 23, b = 24, c = 20$

51. $A = 103°, b = 15, c = 24$