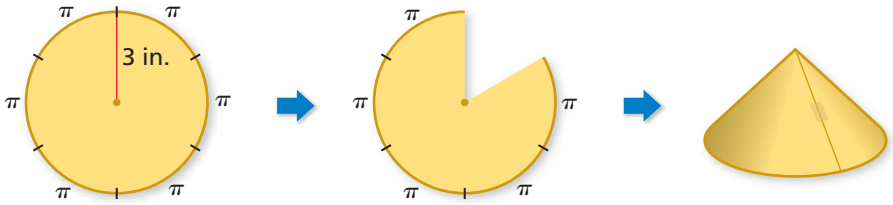
11.7 Surface Areas and Volumes of Cones

Essential Question How can you find the surface area and the volume of a cone?

EXPLORATION 1 Finding the Surface Area of a Cone

Work with a partner. Construct a circle with a radius of 3 inches. Mark the circumference of the circle into six equal parts, and label the length of each part. Then cut out one sector of the circle and make a cone.



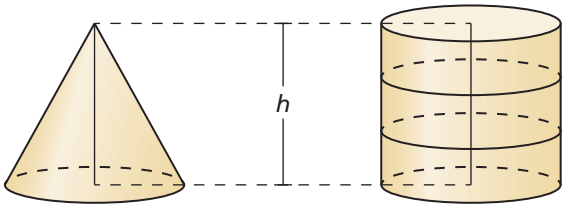
a. Explain why the base of the cone is a circle. What are the circumference and radius of the base?

b. What is the area of the original circle? What is the area with one sector missing?

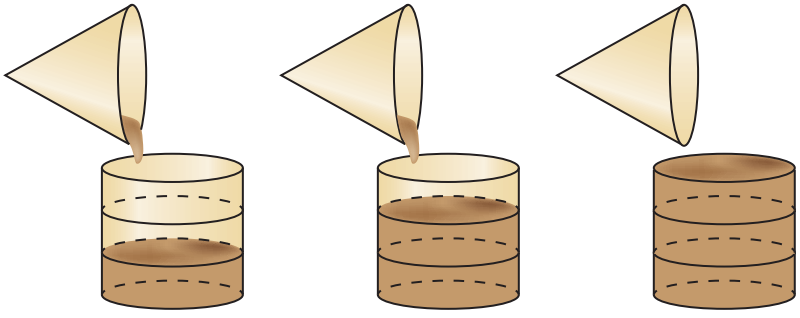
c. Describe the surface area of the cone, including the base. Use your description to find the surface area.

EXPLORATION 2 Finding the Volume of a Cone

Work with a partner. The cone and the cylinder have the same height and the same circular base.



When the cone is filled with sand and poured into the cylinder, it takes three cones to fill the cylinder.



CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results in constructing arguments.

Use this information to write a formula for the volume V of a cone.

Communicate Your Answer

3. How can you find the surface area and the volume of a cone?

4. In Exploration 1, cut another sector from the circle and make a cone. Find the radius of the base and the surface area of the cone. Repeat this three times, recording your results in a table. Describe the pattern.

11.7 Lesson

Core Vocabulary

lateral surface of a cone, *p. 642*

Previous

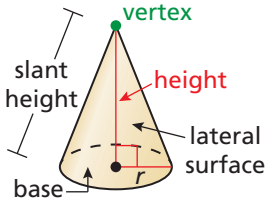
cone
net
composite solid

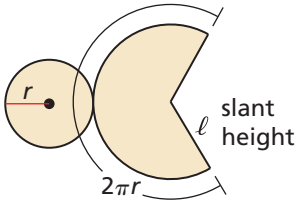
What You Will Learn

- Find surface areas of right cones.
- Find volumes of cones.
- Use volumes of cones.

Finding Surface Areas of Right Cones

Recall that a *circular cone*, or *cone*, has a circular *base* and a *vertex* that is not in the same plane as the base. The *altitude*, or *height*, is the perpendicular distance between the vertex and the base. In a *right cone*, the height meets the base at its center and the *slant height* is the distance between the vertex and a point on the base edge.





The **lateral surface of a cone** consists of all segments that connect the vertex with points on the base edge. When you cut along the slant height and lay the right cone flat, you get the net shown at the left. In the net, the circular base has an area of πr^2 and the lateral surface is a sector of a circle. You can find the area of this sector by using a proportion, as shown below.

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}}$$ Set up proportion.

$$\frac{\text{Area of sector}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell}$$ Substitute.

$$\text{Area of sector} = \pi \ell^2 \cdot \frac{2\pi r}{2\pi \ell}$$ Multiply each side by $\pi \ell^2$.

$$\text{Area of sector} = \pi r \ell$$ Simplify.

The surface area of a right cone is the sum of the base area and the lateral area, $\pi r \ell$.

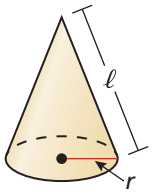
Core Concept

Surface Area of a Right Cone

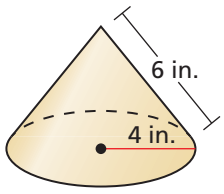
The surface area S of a right cone is

$$S = \pi r^2 + \pi r \ell$$

where r is the radius of the base and ℓ is the slant height.



EXAMPLE 1 Finding Surface Areas of Right Cones



Find the surface area of the right cone.

SOLUTION

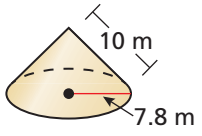
$$S = \pi r^2 + \pi r \ell = \pi \cdot 4^2 + \pi(4)(6) = 40\pi \approx 125.66$$

The surface area is 40π, or about 125.66 square inches.

Monitoring Progress

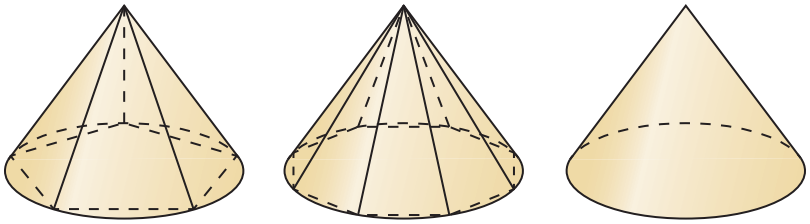
Help in English and Spanish at *BigIdeasMath.com*

1. Find the surface area of the right cone.



Finding Volumes of Cones

Consider a cone with a regular polygon inscribed in the base. The pyramid with the same vertex as the cone has volume $V = \frac{1}{3}Bh$. As you increase the number of sides of the polygon, it approaches the base of the cone and the pyramid approaches the cone. The volume approaches $\frac{1}{3}\pi r^2 h$ as the base area B approaches πr^2.



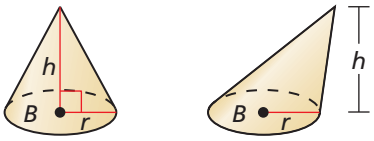
Core Concept

Volume of a Cone

The volume V of a cone is

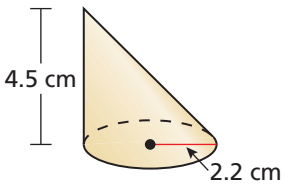
$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$

where B is the area of the base, h is the height, and r is the radius of the base.



EXAMPLE 2 Finding the Volume of a Cone

Find the volume of the cone.



SOLUTION

$V = \frac{1}{3}\pi r^2 h$ — Formula for volume of a cone

$= \frac{1}{3}\pi \cdot (2.2)^2 \cdot 4.5$ — Substitute.

$= 7.26\pi$ — Simplify.

≈ 22.81 — Use a calculator.

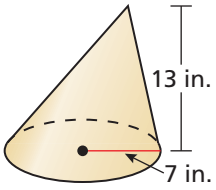
▶ The volume is 7.26π, or about 22.81 cubic centimeters.

Monitoring Progress

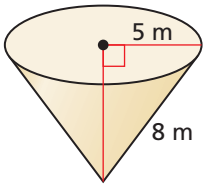
Help in English and Spanish at *BigIdeasMath.com*

Find the volume of the cone.

2.



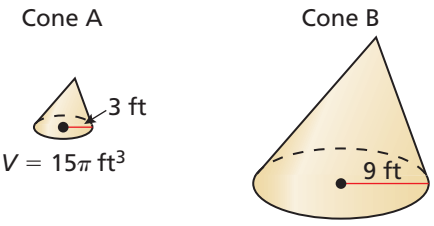
3.



Using Volumes of Cones

EXAMPLE 3 Finding the Volume of a Similar Solid

Cone A and cone B are similar. Find the volume of cone B.



SOLUTION

The scale factor is $k = \frac{\text{Radius of cone B}}{\text{Radius of cone A}} = \frac{9}{3} = 3$.

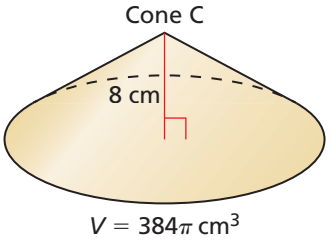
Use the scale factor to find the volume of cone B.

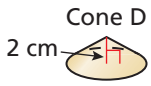
$\frac{\text{Volume of cone B}}{\text{Volume of cone A}} = k^3$ The ratio of the volumes is k^3.

$\frac{\text{Volume of cone B}}{15\pi} = 3^3$ Substitute.

$\text{Volume of cone B} = 405\pi$ Solve for volume of cone B.

▶ The volume of cone B is 405π cubic feet.



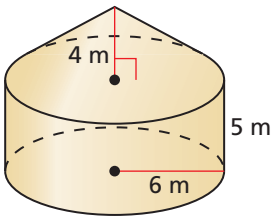


Monitoring Progress Help in English and Spanish at *BigIdeasMath.com*

4. Cone C and cone D are similar. Find the volume of cone D.

EXAMPLE 4 Finding the Volume of a Composite Solid

Find the volume of the composite solid.



SOLUTION

Let h_1 be the height of the cylinder and let h_2 be the height of the cone.

$\text{Volume of solid} = \text{Volume of cylinder} + \text{Volume of cone}$

$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$ Write formulas.

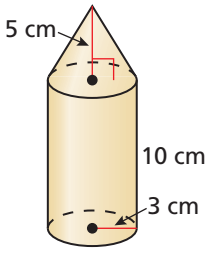
$= \pi \cdot 6^2 \cdot 5 + \frac{1}{3}\pi \cdot 6^2 \cdot 4$ Substitute.

$= 180\pi + 48\pi$ Simplify.

$= 228\pi$ Add.

≈ 716.28 Use a calculator.

▶ The volume is 228π, or about 716.28 cubic meters.



Monitoring Progress Help in English and Spanish at *BigIdeasMath.com*

5. Find the volume of the composite solid.

11.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

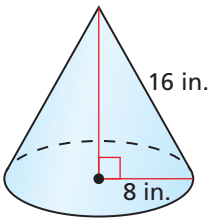
Vocabulary and Core Concept Check

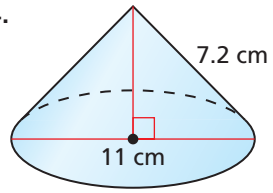
1. **WRITING** Describe the differences between pyramids and cones. Describe their similarities.

2. **COMPLETE THE SENTENCE** The volume of a cone with radius r and height h is $\frac{1}{3}$ the volume of a(n) _____ with radius r and height h.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the surface area of the right cone. *(See Example 1.)*

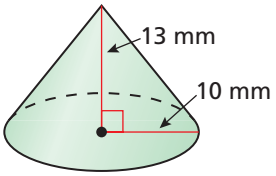
3. 

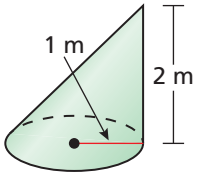
4. 

5. A right cone has a radius of 9 inches and a height of 12 inches.

6. A right cone has a diameter of 11.2 feet and a height of 9.2 feet.

In Exercises 7–10, find the volume of the cone. *(See Example 2.)*

7. 

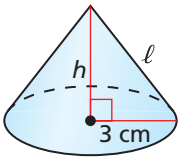
8. 

9. A cone has a diameter of 11.5 inches and a height of 15.2 inches.

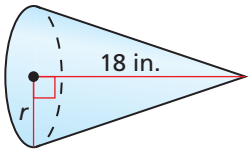
10. A right cone has a radius of 3 feet and a slant height of 6 feet.

In Exercises 11 and 12, find the missing dimension(s).

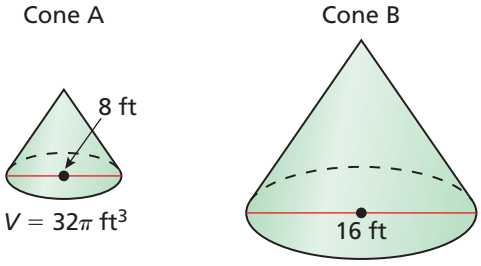
11. Surface area $= 75.4$ cm^2

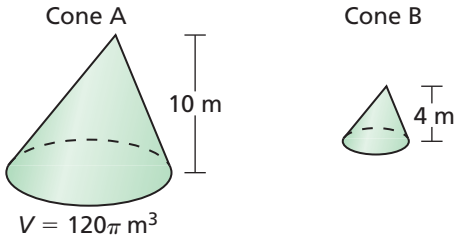


12. Volume $= 216\pi$ in.^{3}

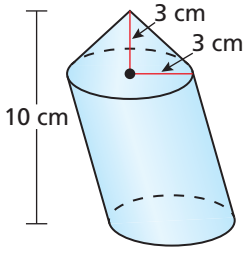


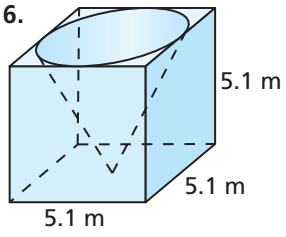
In Exercises 13 and 14, the cones are similar. Find the volume of cone B. *(See Example 3.)*

13. 

14. 

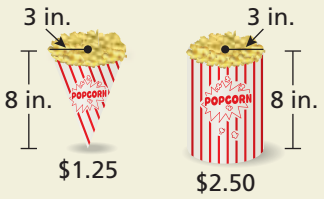
In Exercises 15 and 16, find the volume of the composite solid. *(See Example 4.)*

15. 

16. 

17. **ANALYZING RELATIONSHIPS** A cone has height h and a base with radius r. You want to change the cone so its volume is doubled. What is the new height if you change only the height? What is the new radius if you change only the radius? Explain.

18. HOW DO YOU SEE IT A snack stand serves a small order of popcorn in a cone-shaped container and a large order of popcorn in a cylindrical container. Do not perform any calculations.

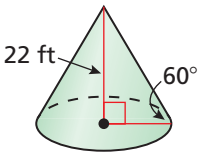


a. How many small containers of popcorn do you have to buy to equal the amount of popcorn in a large container? Explain.

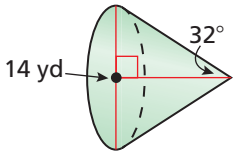
b. Which container gives you more popcorn for your money? Explain.

In Exercises 19 and 20, find the volume of the right cone.

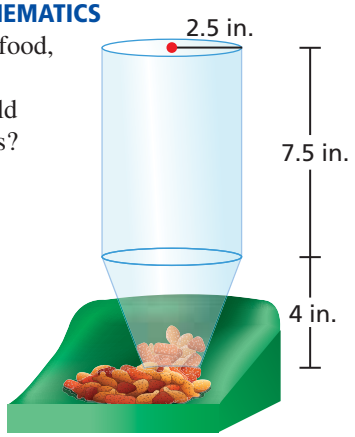
19.



20.

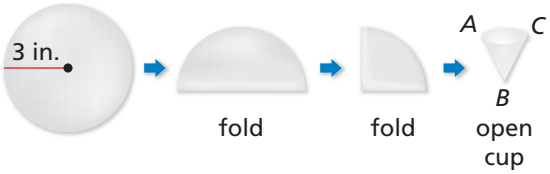


21. MODELING WITH MATHEMATICS A cat eats half a cup of food, twice per day. Will the automatic pet feeder hold enough food for 10 days? Explain your reasoning. (1 cup $\approx$ 14.4 in.^{3})

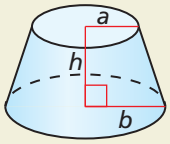


22. MODELING WITH MATHEMATICS During a chemistry lab, you use a funnel to pour a solvent into a flask. The radius of the funnel is 5 centimeters and its height is 10 centimeters. You pour the solvent into the funnel at a rate of 80 milliliters per second and the solvent flows out of the funnel at a rate of 65 milliliters per second. How long will it be before the funnel overflows? (1 mL = 1 cm^3)

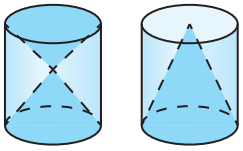
23. REASONING To make a paper drinking cup, start with a circular piece of paper that has a 3-inch radius, then follow the given steps. How does the surface area of the cup compare to the original paper circle? Find $m\angle ABC$.



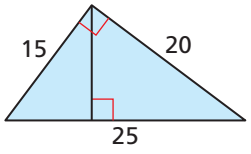
24. THOUGHT PROVOKING A *frustum* of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Write a formula for the volume of the frustum of a cone in terms of a, b, and h. (*Hint*: Consider the "missing" top of the cone and use similar triangles.)



25. MAKING AN ARGUMENT In the figure, the two cylinders are congruent. The combined height of the two smaller cones equals the height of the larger cone. Your friend claims that this means the total volume of the two smaller cones is equal to the volume of the larger cone. Is your friend correct? Justify your answer.



26. CRITICAL THINKING When the given triangle is rotated around each of its sides, solids of revolution are formed. Describe the three solids and find their volumes. Give your answers in terms of π.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the indicated measure. *(Section 11.2)*

27. area of a circle with a radius of 7 feet

28. area of a circle with a diameter of 22 centimeters

29. diameter of a circle with an area of 256π square meters

30. radius of a circle with an area of 529π square inches