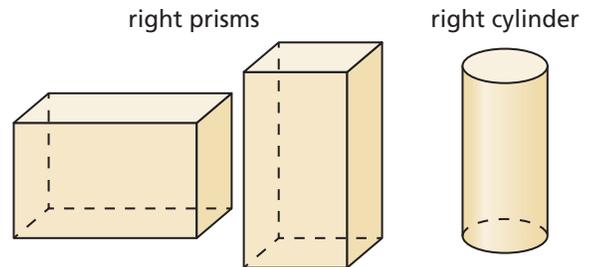


11.5 Volumes of Prisms and Cylinders

Essential Question How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

Recall that the volume V of a right prism or a right cylinder is equal to the product of the area of a base B and the height h .

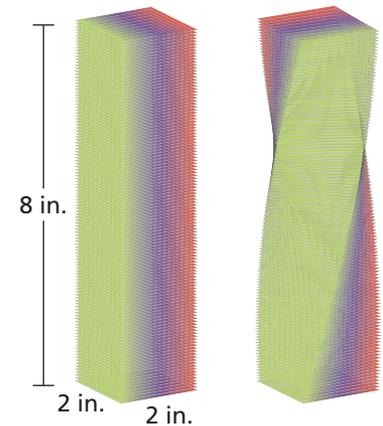
$$V = Bh$$



EXPLORATION 1 Finding Volume

Work with a partner. Consider a stack of square papers that is in the form of a right prism.

- What is the volume of the prism?
- When you twist the stack of papers, as shown at the right, do you change the volume? Explain your reasoning.
- Write a carefully worded conjecture that describes the conclusion you reached in part (b).
- Use your conjecture to find the volume of the twisted stack of papers.

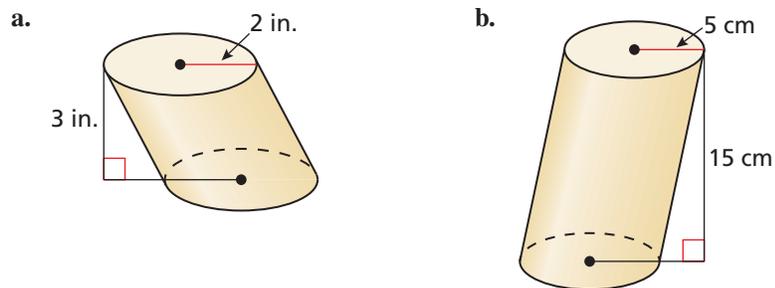


ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely to others.

EXPLORATION 2 Finding Volume

Work with a partner. Use the conjecture you wrote in Exploration 1 to find the volume of the cylinder.



Communicate Your Answer

- How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?
- In Exploration 1, would the conjecture you wrote change if the papers in each stack were not squares? Explain your reasoning.

11.5 Lesson

Core Vocabulary

volume, p. 626
Cavalieri's Principle, p. 626
density, p. 628
similar solids, p. 630

Previous

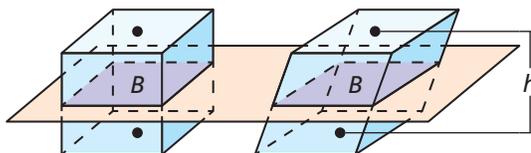
prism
cylinder
composite solid

What You Will Learn

- ▶ Find volumes of prisms and cylinders.
- ▶ Use the formula for density.
- ▶ Use volumes of prisms and cylinders.

Finding Volumes of Prisms and Cylinders

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3). **Cavalieri's Principle**, named after Bonaventura Cavalieri (1598–1647), states that if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. The prisms below have equal heights h and equal cross-sectional areas B at every level. By Cavalieri's Principle, the prisms have the same volume.



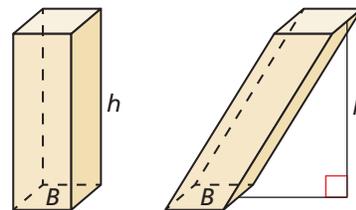
Core Concept

Volume of a Prism

The volume V of a prism is

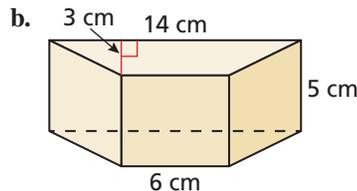
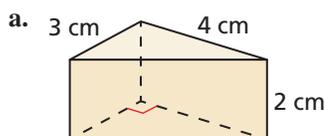
$$V = Bh$$

where B is the area of a base and h is the height.



EXAMPLE 1 Finding Volumes of Prisms

Find the volume of each prism.



SOLUTION

a. The area of a base is $B = \frac{1}{2}(3)(4) = 6 \text{ cm}^2$ and the height is $h = 2 \text{ cm}$.

$$\begin{aligned} V &= Bh && \text{Formula for volume of a prism} \\ &= 6(2) && \text{Substitute.} \\ &= 12 && \text{Simplify.} \end{aligned}$$

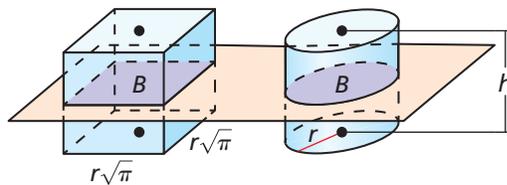
- ▶ The volume is 12 cubic centimeters.

b. The area of a base is $B = \frac{1}{2}(3)(6 + 14) = 30 \text{ cm}^2$ and the height is $h = 5 \text{ cm}$.

$$\begin{aligned} V &= Bh && \text{Formula for volume of a prism} \\ &= 30(5) && \text{Substitute.} \\ &= 150 && \text{Simplify.} \end{aligned}$$

- ▶ The volume is 150 cubic centimeters.

Consider a cylinder with height h and base radius r and a rectangular prism with the same height that has a square base with sides of length $r\sqrt{\pi}$.



The cylinder and the prism have the same cross-sectional area, πr^2 , at every level and the same height. By Cavalieri's Principle, the prism and the cylinder have the same volume. The volume of the prism is $V = Bh = \pi r^2 h$, so the volume of the cylinder is also $V = Bh = \pi r^2 h$.

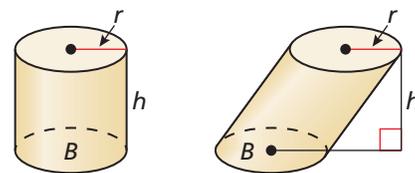
Core Concept

Volume of a Cylinder

The volume V of a cylinder is

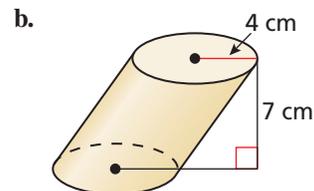
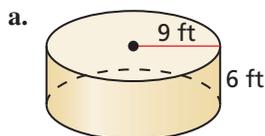
$$V = Bh = \pi r^2 h$$

where B is the area of a base, h is the height, and r is the radius of a base.



EXAMPLE 2 Finding Volumes of Cylinders

Find the volume of each cylinder.



SOLUTION

a. The dimensions of the cylinder are $r = 9$ ft and $h = 6$ ft.

$$V = \pi r^2 h = \pi(9)^2(6) = 486\pi \approx 1526.81$$

► The volume is 486π , or about 1526.81 cubic feet.

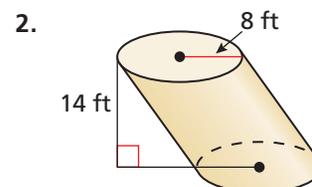
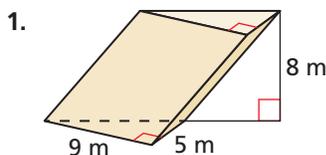
b. The dimensions of the cylinder are $r = 4$ cm and $h = 7$ cm.

$$V = \pi r^2 h = \pi(4)^2(7) = 112\pi \approx 351.86$$

► The volume is 112π , or about 351.86 cubic centimeters.

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Find the volume of the solid.



Using the Formula for Density

Density is the amount of matter that an object has in a given unit of volume. The density of an object is calculated by dividing its mass by its volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

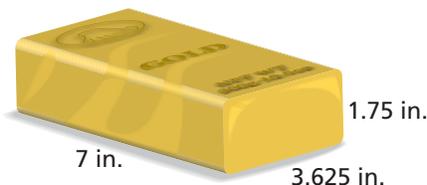
Different materials have different densities, so density can be used to distinguish between materials that look similar. For example, table salt and sugar look alike. However, table salt has a density of 2.16 grams per cubic centimeter, while sugar has a density of 1.58 grams per cubic centimeter.

EXAMPLE 3 Using the Formula for Density



According to the U.S. Mint, Fort Knox houses about 9.2 million pounds of gold.

The diagram shows the dimensions of a standard gold bar at Fort Knox. Gold has a density of 19.3 grams per cubic centimeter. Find the mass of a standard gold bar to the nearest gram.



SOLUTION

Step 1 Convert the dimensions to centimeters using 1 inch = 2.54 centimeters.

$$\text{Length } 7 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 17.78 \text{ cm}$$

$$\text{Width } 3.625 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 9.2075 \text{ cm}$$

$$\text{Height } 1.75 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 4.445 \text{ cm}$$

Step 2 Find the volume.

The area of a base is $B = 17.78(9.2075) = 163.70935 \text{ cm}^2$ and the height is $h = 4.445 \text{ cm}$.

$$V = Bh = 163.70935(4.445) \approx 727.69 \text{ cm}^3$$

Step 3 Let x represent the mass in grams. Substitute the values for the volume and the density in the formula for density and solve for x .

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{Formula for density}$$

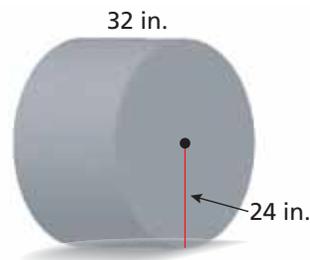
$$19.3 \approx \frac{x}{727.69} \quad \text{Substitute.}$$

$$14,044 \approx x \quad \text{Multiply each side by 727.69.}$$

► The mass of a standard gold bar is about 14,044 grams.

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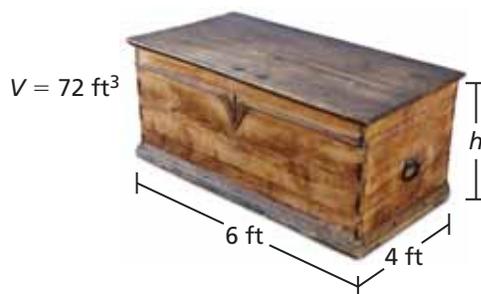
3. The diagram shows the dimensions of a concrete cylinder. Concrete has a density of 2.3 grams per cubic centimeter. Find the mass of the concrete cylinder to the nearest gram.



Using Volumes of Prisms and Cylinders

EXAMPLE 4 Modeling with Mathematics

You are building a rectangular chest. You want the length to be 6 feet, the width to be 4 feet, and the volume to be 72 cubic feet. What should the height be?



SOLUTION

- 1. Understand the Problem** You know the dimensions of the base of a rectangular prism and the volume. You are asked to find the height.
- 2. Make a Plan** Write the formula for the volume of a rectangular prism, substitute known values, and solve for the height h .
- 3. Solve the Problem** The area of a base is $B = 6(4) = 24 \text{ ft}^2$ and the volume is $V = 72 \text{ ft}^3$.

$$V = Bh \quad \text{Formula for volume of a prism}$$

$$72 = 24h \quad \text{Substitute.}$$

$$3 = h \quad \text{Divide each side by 24.}$$

▶ The height of the chest should be 3 feet.

- 4. Look Back** Check your answer.

$$V = Bh = 24(3) = 72 \quad \checkmark$$

EXAMPLE 5 Solving a Real-Life Problem

You are building a 6-foot-tall dresser. You want the volume to be 36 cubic feet. What should the area of the base be? Give a possible length and width.

SOLUTION

$$V = Bh \quad \text{Formula for volume of a prism}$$

$$36 = B \cdot 6 \quad \text{Substitute.}$$

$$6 = B \quad \text{Divide each side by 6.}$$

▶ The area of the base should be 6 square feet. The length could be 3 feet and the width could be 2 feet.



$$V = 36 \text{ ft}^3$$

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- 4. WHAT IF?** In Example 4, you want the length to be 5 meters, the width to be 3 meters, and the volume to be 60 cubic meters. What should the height be?
- 5. WHAT IF?** In Example 5, you want the height to be 5 meters and the volume to be 75 cubic meters. What should the area of the base be? Give a possible length and width.

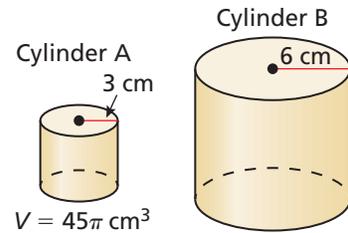
Core Concept

Similar Solids

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The ratio of the corresponding linear measures of two similar solids is called the *scale factor*. If two similar solids have a scale factor of k , then the ratio of their volumes is equal to k^3 .

EXAMPLE 6 Finding the Volume of a Similar Solid

Cylinder A and cylinder B are similar.
Find the volume of cylinder B.



SOLUTION

$$\begin{aligned} \text{The scale factor is } k &= \frac{\text{Radius of cylinder B}}{\text{Radius of cylinder A}} \\ &= \frac{6}{3} = 2. \end{aligned}$$

Use the scale factor to find the volume of cylinder B.

$$\frac{\text{Volume of cylinder B}}{\text{Volume of cylinder A}} = k^3 \quad \text{The ratio of the volumes is } k^3.$$

$$\frac{\text{Volume of cylinder B}}{45\pi} = 2^3 \quad \text{Substitute.}$$

$$\text{Volume of cylinder B} = 360\pi \quad \text{Solve for volume of cylinder B.}$$

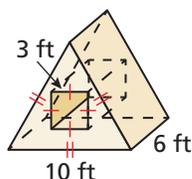
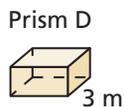
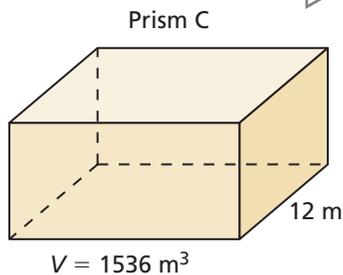
▶ The volume of cylinder B is 360π cubic centimeters.

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6. Prism C and prism D are similar. Find the volume of prism D.

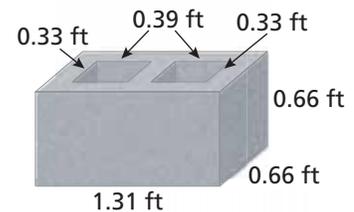
COMMON ERROR

Be sure to write the ratio of the volumes in the same order you wrote the ratio of the radii.



EXAMPLE 7 Finding the Volume of a Composite Solid

Find the volume of the concrete block.



SOLUTION

To find the area of the base, subtract two times the area of the small rectangle from the large rectangle.

$$\begin{aligned} B &= \text{Area of large rectangle} - 2 \cdot \text{Area of small rectangle} \\ &= 1.31(0.66) - 2(0.33)(0.39) \\ &= 0.6072 \end{aligned}$$

Using the formula for the volume of a prism, the volume is

$$V = Bh = 0.6072(0.66) \approx 0.40.$$

▶ The volume is about 0.40 cubic foot.

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7. Find the volume of the composite solid.

11.5 Exercises

Vocabulary and Core Concept Check

- VOCABULARY** In what type of units is the volume of a solid measured?
- COMPLETE THE SENTENCE** Density is the amount of _____ that an object has in a given unit of _____.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the volume of the prism.
(See Example 1.)

-
-
-
-

In Exercises 7–10, find the volume of the cylinder.
(See Example 2.)

-
-
-
-

In Exercises 11 and 12, make a sketch of the solid and find its volume. Round your answer to the nearest hundredth.

- A prism has a height of 11.2 centimeters and an equilateral triangle for a base, where each base edge is 8 centimeters.

- A pentagonal prism has a height of 9 feet and each base edge is 3 feet.

- PROBLEM SOLVING** A piece of copper with a volume of 8.25 cubic centimeters has a mass of 73.92 grams. A piece of iron with a volume of 5 cubic centimeters has a mass of 39.35 grams. Which metal has the greater density?



- PROBLEM SOLVING** The United States has minted one-dollar silver coins called the American Eagle Silver Bullion Coin since 1986. Each coin has a diameter of 40.6 millimeters and is 2.98 millimeters thick. The density of silver is 10.5 grams per cubic centimeter. What is the mass of an American Eagle Silver Bullion Coin to the nearest gram? (See Example 3.)



- ERROR ANALYSIS** Describe and correct the error in finding the volume of the cylinder.

X

$$\begin{aligned}
 V &= 2\pi rh \\
 &= 2\pi(4)(3) \\
 &= 24\pi
 \end{aligned}$$

So, the volume of the cylinder is 24π cubic feet.

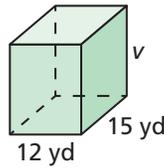
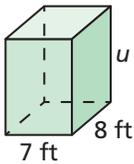
16. **ERROR ANALYSIS** Describe and correct the error in finding the density of an object that has a mass of 24 grams and a volume of 28.3 cubic centimeters.

X $\text{density} = \frac{28.3}{24} \approx 1.18$

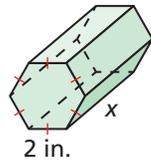
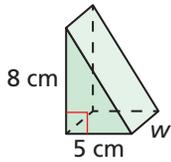
So, the density is about 1.18 cubic centimeters per gram.

In Exercises 17–22, find the missing dimension of the prism or cylinder. (See Example 4.)

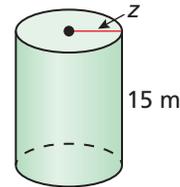
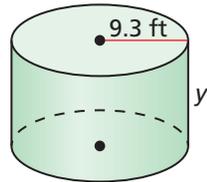
17. Volume = 560 ft³ 18. Volume = 2700 yd³



19. Volume = 80 cm³ 20. Volume = 72.66 in.³



21. Volume = 3000 ft³ 22. Volume = 1696.5 m³

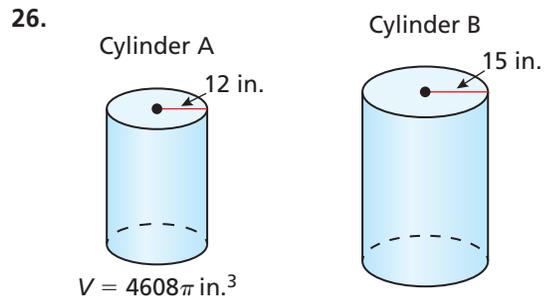
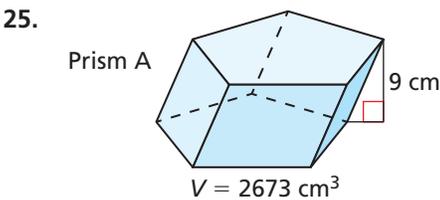


In Exercises 23 and 24, find the area of the base of the rectangular prism with the given volume and height. Then give a possible length and width. (See Example 5.)

23. $V = 154 \text{ in.}^3$, $h = 11 \text{ in.}$

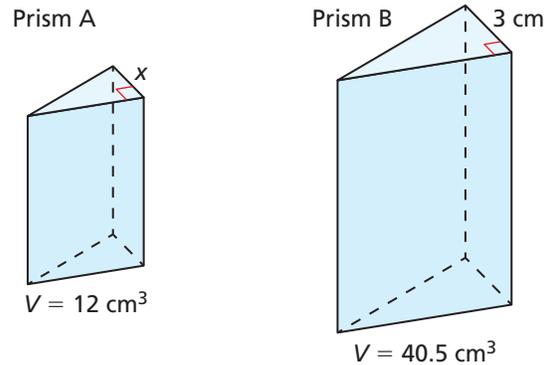
24. $V = 27 \text{ m}^3$, $h = 3 \text{ m}$

In Exercises 25 and 26, the solids are similar. Find the volume of solid B. (See Example 6.)

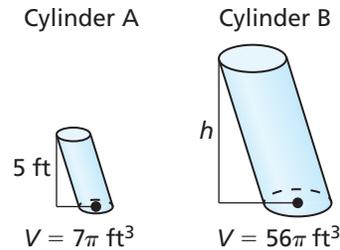


In Exercises 27 and 28, the solids are similar. Find the indicated measure.

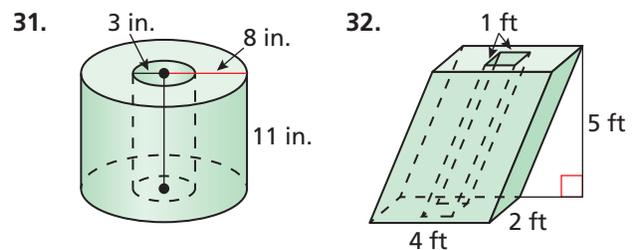
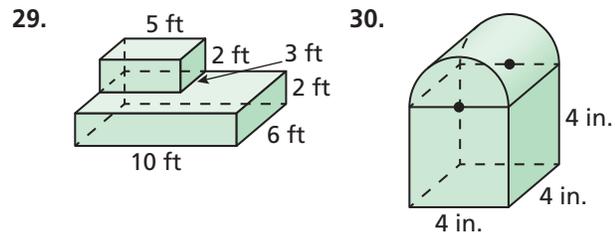
27. height x of the base of prism A



28. height h of cylinder B



In Exercises 29–32, find the volume of the composite solid. (See Example 7.)

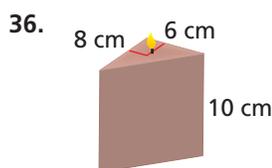
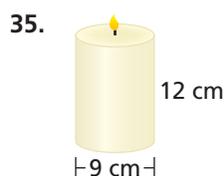


33. **MODELING WITH MATHEMATICS** The Great Blue Hole is a cylindrical trench located off the coast of Belize. It is approximately 1000 feet wide and 400 feet deep. About how many gallons of water does the Great Blue Hole contain? ($1 \text{ ft}^3 \approx 7.48$ gallons)



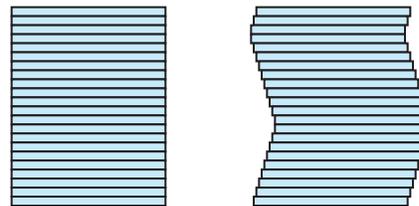
34. **COMPARING METHODS** The *Volume Addition Postulate* states that the volume of a solid is the sum of the volumes of all its nonoverlapping parts. Use this postulate to find the volume of the block of concrete in Example 7 by subtracting the volume of each hole from the volume of the large rectangular prism. Which method do you prefer? Explain your reasoning.

REASONING In Exercises 35 and 36, you are melting a rectangular block of wax to make candles. How many candles of the given shape can be made using a block that measures 10 centimeters by 9 centimeters by 20 centimeters?



37. **PROBLEM SOLVING** An aquarium shaped like a rectangular prism has a length of 30 inches, a width of 10 inches, and a height of 20 inches. You fill the aquarium $\frac{3}{4}$ full with water. When you submerge a rock in the aquarium, the water level rises 0.25 inch.
- Find the volume of the rock.
 - How many rocks of this size can you place in the aquarium before water spills out?
38. **PROBLEM SOLVING** You drop an irregular piece of metal into a container partially filled with water and measure that the water level rises 4.8 centimeters. The square base of the container has a side length of 8 centimeters. You measure the mass of the metal to be 450 grams. What is the density of the metal?

39. **WRITING** Both of the figures shown are made up of the same number of congruent rectangles. Explain how Cavalieri's Principle can be adapted to compare the areas of these figures.

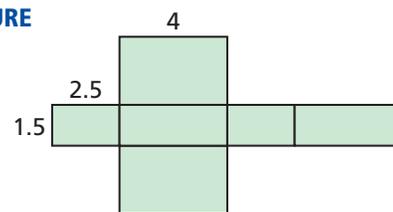


40. **HOW DO YOU SEE IT?** Each stack of memo papers contains 500 equally-sized sheets of paper. Compare their volumes. Explain your reasoning.

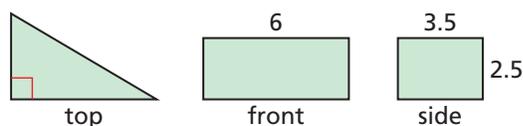


41. **USING STRUCTURE**

Sketch the solid formed by the net. Then find the volume of the solid.



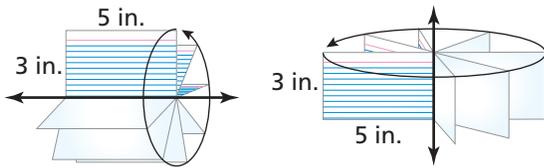
42. **USING STRUCTURE** Sketch the solid with the given views. Then find the volume of the solid.



43. **OPEN-ENDED** Sketch two rectangular prisms that have volumes of 100 cubic inches but different surface areas. Include dimensions in your sketches.
44. **MODELING WITH MATHEMATICS** Which box gives you more cereal for your money? Explain.

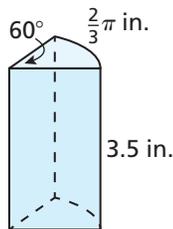


45. **CRITICAL THINKING** A 3-inch by 5-inch index card is rotated around a horizontal line and a vertical line to produce two different solids. Which solid has a greater volume? Explain your reasoning.



46. **CRITICAL THINKING** The height of cylinder X is twice the height of cylinder Y. The radius of cylinder X is half the radius of cylinder Y. Compare the volumes of cylinder X and cylinder Y. Justify your answer.

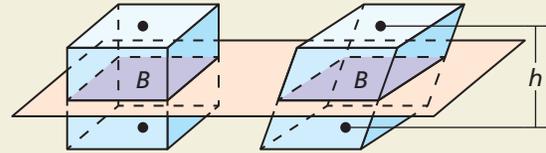
47. **USING STRUCTURE** Find the volume of the solid shown. The bases of the solid are sectors of circles.



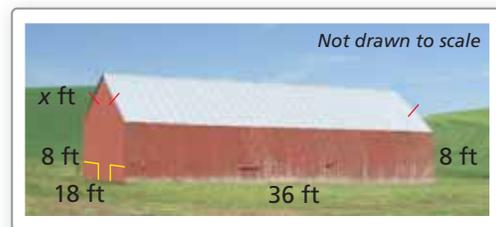
48. **MATHEMATICAL CONNECTIONS** You drill a circular hole of radius r through the base of a cylinder of radius R . Assume the hole is drilled completely through to the other base. You want the volume of the hole to be half the volume of the cylinder. Express r as a function of R .
49. **ANALYZING RELATIONSHIPS** How can you change the height of a cylinder so that the volume is increased by 25% but the radius remains the same?
50. **ANALYZING RELATIONSHIPS** How can you change the edge length of a cube so that the volume is reduced by 40%?

51. **MAKING AN ARGUMENT** You have two objects of equal volume. Your friend says you can compare the densities of the objects by comparing their mass, because the heavier object will have a greater density. Is your friend correct? Explain your reasoning.

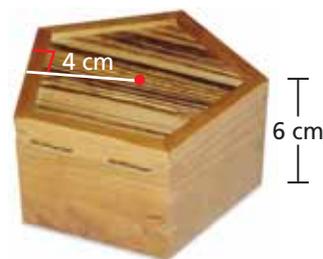
52. **THOUGHT PROVOKING** Cavalieri's Principle states that the two solids shown below have the same volume. Do they also have the same surface area? Explain your reasoning.



53. **PROBLEM SOLVING** A barn is in the shape of a pentagonal prism with the dimensions shown. The volume of the barn is 9072 cubic feet. Find the dimensions of each half of the roof.



54. **PROBLEM SOLVING** A wooden box is in the shape of a regular pentagonal prism. The sides, top, and bottom of the box are 1 centimeter thick. Approximate the volume of wood used to construct the box. Round your answer to the nearest tenth.

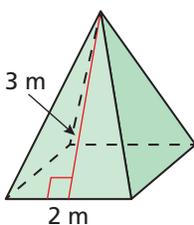


Maintaining Mathematical Proficiency

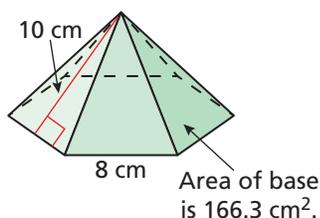
Reviewing what you learned in previous grades and lessons

Find the surface area of the regular pyramid. (*Skills Review Handbook*)

55.



56.



57.

