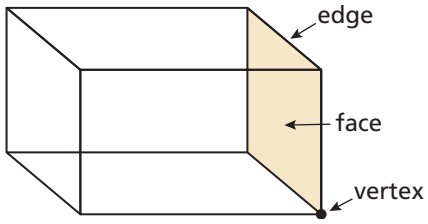
11.4 Three-Dimensional Figures

Essential Question What is the relationship between the numbers of vertices V, edges E, and faces F of a polyhedron?

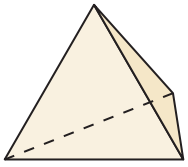
A **polyhedron** is a solid that is bounded by polygons, called **faces**.

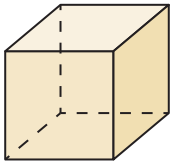
- Each *vertex* is a point.
- Each *edge* is a segment of a line.
- Each *face* is a portion of a plane.

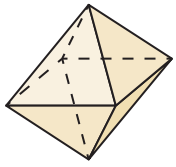


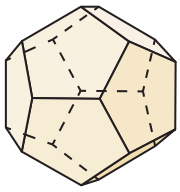
EXPLORATION 1 Analyzing a Property of Polyhedra

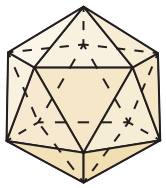
Work with a partner. The five *Platonic solids* are shown below. Each of these solids has congruent regular polygons as faces. Complete the table by listing the numbers of vertices, edges, and faces of each Platonic solid.


tetrahedron


cube


octahedron


dodecahedron


icosahedron

Solid	Vertices, V	Edges, E	Faces, F
tetrahedron			
cube			
octahedron			
dodecahedron			
icosahedron			

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to reason inductively about data.

Communicate Your Answer

2. What is the relationship between the numbers of vertices V, edges E, and faces F of a polyhedron? (*Note*: Swiss mathematician Leonhard Euler (1707–1783) discovered a formula that relates these quantities.)

3. Draw three polyhedra that are different from the Platonic solids given in Exploration 1. Count the numbers of vertices, edges, and faces of each polyhedron. Then verify that the relationship you found in Question 2 is valid for each polyhedron.

11.4 Lesson

Core Vocabulary

polyhedron, *p. 618*
face, *p. 618*
edge, *p. 618*
vertex, *p. 618*
cross section, *p. 619*
solid of revolution, *p. 620*
axis of revolution, *p. 620*

Previous

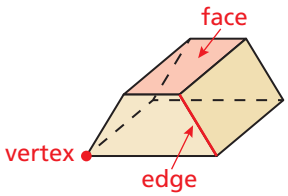
solid
prism
pyramid
cylinder
cone
sphere
base

What You Will Learn

- Classify solids.
- Describe cross sections.
- Sketch and describe solids of revolution.

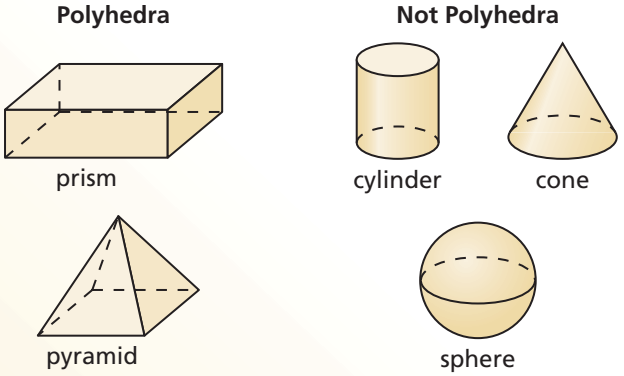
Classifying Solids

A three-dimensional figure, or solid, is bounded by flat or curved surfaces that enclose a single region of space. A **polyhedron** is a solid that is bounded by polygons, called **faces**. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.

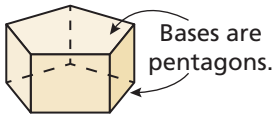


Core Concept

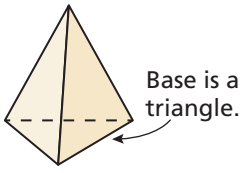
Types of Solids



Pentagonal prism



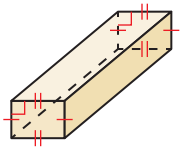
Triangular pyramid

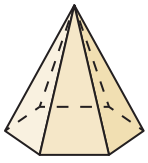


To name a prism or a pyramid, use the shape of the *base*. The two bases of a prism are congruent polygons in parallel planes. For example, the bases of a pentagonal prism are pentagons. The base of a pyramid is a polygon. For example, the base of a triangular pyramid is a triangle.

EXAMPLE 1 Classifying Solids

Tell whether each solid is a polyhedron. If it is, name the polyhedron.

a. 

b. 

c. 

SOLUTION

a. The solid is formed by polygons, so it is a polyhedron. The two bases are congruent rectangles, so it is a rectangular prism.

b. The solid is formed by polygons, so it is a polyhedron. The base is a hexagon, so it is a hexagonal pyramid.

c. The cone has a curved surface, so it is not a polyhedron.

Monitoring Progress

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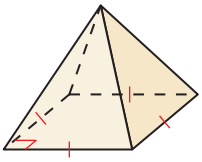
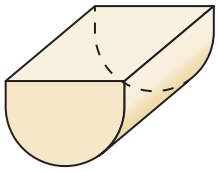
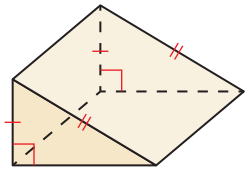
Tell whether the solid is a polyhedron. If it is, name the polyhedron.

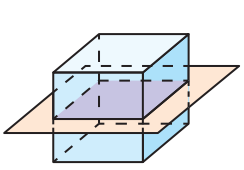
1. 

2. 

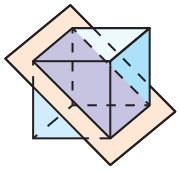
3. 

Describing Cross Sections

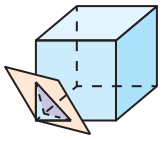
Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, three different cross sections of a cube are shown below.



square



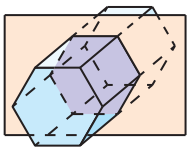
rectangle

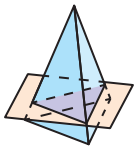


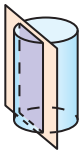
triangle

EXAMPLE 2 Describing Cross Sections

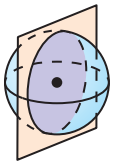
Describe the shape formed by the intersection of the plane and the solid.

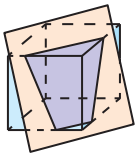
a. 

b. 

c. 

d. 

e. 

f. 

SOLUTION

a. The cross section is a hexagon.

b. The cross section is a triangle.

c. The cross section is a rectangle.

d. The cross section is a circle.

e. The cross section is a circle.

f. The cross section is a trapezoid.

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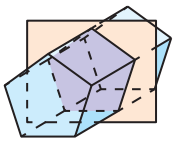
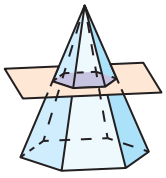
Describe the shape formed by the intersection of the plane and the solid.

4. 

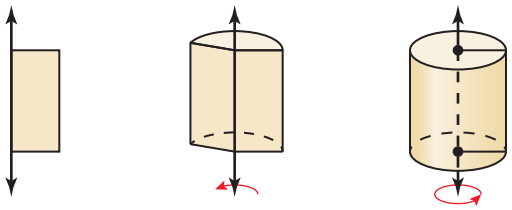
5. 

6. 

Sketching and Describing Solids of Revolution

A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis. The line around which the shape is rotated is called the **axis of revolution**.

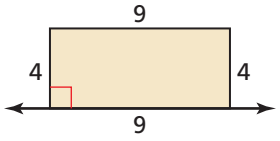
For example, when you rotate a rectangle around a line that contains one of its sides, the solid of revolution that is produced is a cylinder.



EXAMPLE 3 Sketching and Describing Solids of Revolution

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

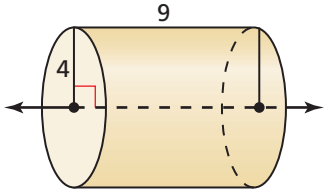
a.



b.

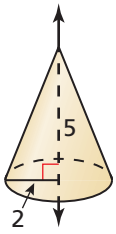
SOLUTION

a.



▶ The solid is a cylinder with a height of 9 and a base radius of 4.

b.



▶ The solid is a cone with a height of 5 and a base radius of 2.

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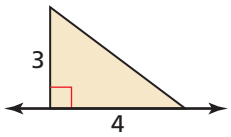
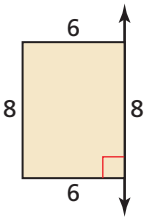
Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

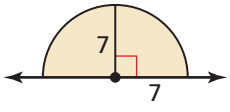
7.



8.



9.



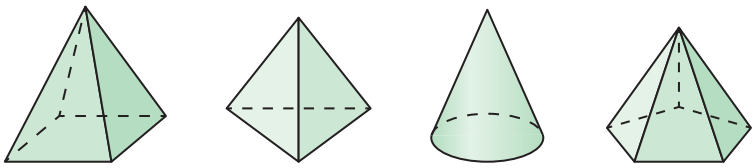
11.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

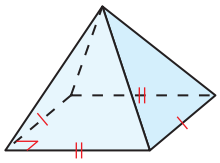
1. **VOCABULARY** A(n) _____ is a solid that is bounded by polygons.

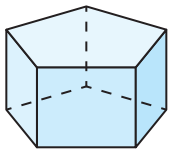
2. **WHICH ONE DOESN'T BELONG?** Which solid does *not* belong with the other three? Explain your reasoning.

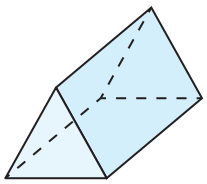


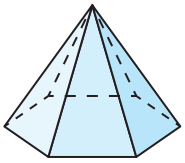
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, match the polyhedron with its name.

3. 

4. 

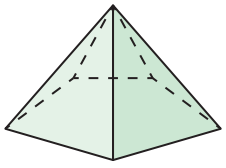
5. 

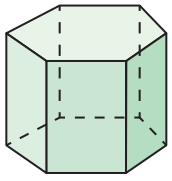
6. 

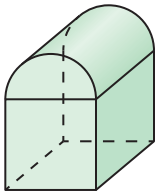
A. triangular prism **B.** rectangular pyramid

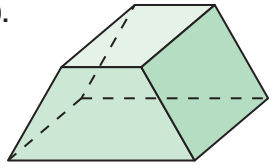
C. hexagonal pyramid **D.** pentagonal prism

In Exercises 7–10, tell whether the solid is a polyhedron. If it is, name the polyhedron. *(See Example 1.)*

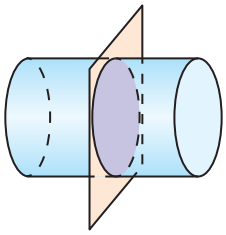
7. 

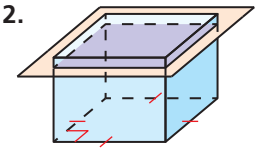
8. 

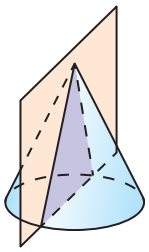
9. 

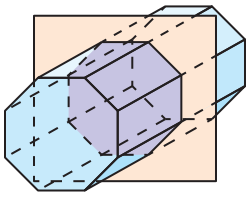
10. 

In Exercises 11–14, describe the cross section formed by the intersection of the plane and the solid. *(See Example 2.)*

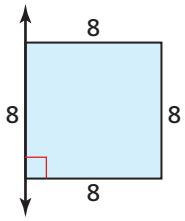
11. 

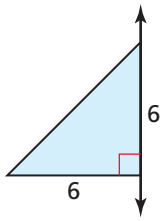
12. 

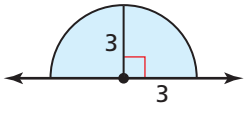
13. 

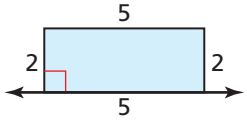
14. 

In Exercises 15–18, sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid. *(See Example 3.)*

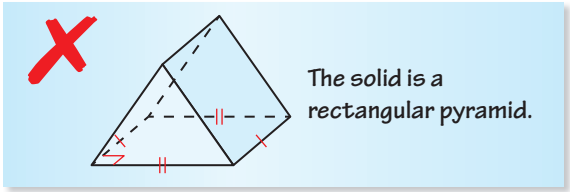
15. 

16. 

17. 

18. 

19. ERROR ANALYSIS Describe and correct the error in identifying the solid.



20. HOW DO YOU SEE IT? Is the swimming pool shown a polyhedron? If it is, name the polyhedron. If not, explain why not.

In Exercises 21–26, sketch the polyhedron.

21. triangular prism

22. rectangular prism

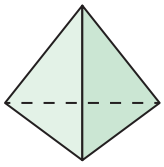
23. pentagonal prism

24. hexagonal prism

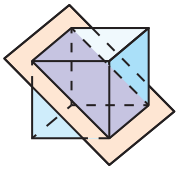
25. square pyramid

26. pentagonal pyramid

27. MAKING AN ARGUMENT Your friend says that the polyhedron shown is a triangular prism. Your cousin says that it is a triangular pyramid. Who is correct? Explain your reasoning.



28. ATTENDING TO PRECISION The figure shows a plane intersecting a cube through four of its vertices. The edge length of the cube is 6 inches.



a. Describe the shape of the cross section.

b. What is the perimeter of the cross section?

c. What is the area of the cross section?

REASONING In Exercises 29–34, tell whether it is possible for a cross section of a cube to have the given shape. If it is, describe or sketch how the plane could intersect the cube.

29. circle

30. pentagon

31. rhombus

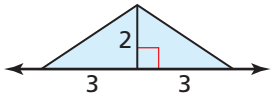
32. isosceles triangle

33. hexagon

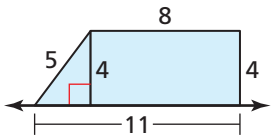
34. scalene triangle

35. REASONING Sketch the composite solid produced by rotating the figure around the given axis. Then identify and describe the composite solid.

a.



b.



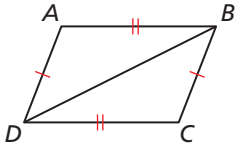
36. THOUGHT PROVOKING Describe how Plato might have argued that there are precisely five *Platonic Solids* (see page 617). (*Hint*: Consider the angles that meet at a vertex.)

Maintaining Mathematical Proficiency

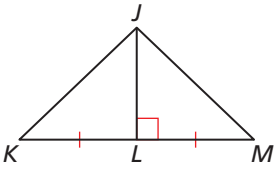
Reviewing what you learned in previous grades and lessons

Decide whether enough information is given to prove that the triangles are congruent. If so, state the theorem you would use. *(Sections 5.3, 5.5, and 5.6)*

37. $\triangle ABD$, $\triangle CDB$



38. $\triangle JLK$, $\triangle JLM$



39. $\triangle RQP$, $\triangle RTS$