10.3 Using Chords

Essential Question What are two ways to determine when a chord is a diameter of a circle?

LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

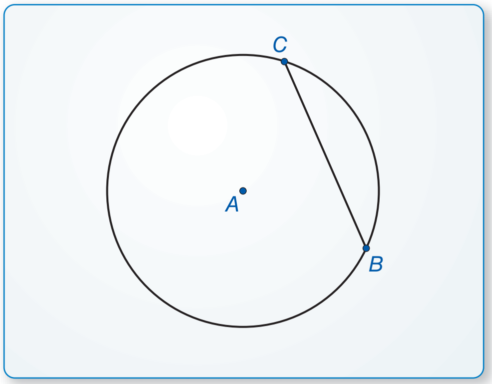
EXPLORATION 1 Drawing Diameters

Work with a partner. Use dynamic geometry software to construct a circle of radius 5 with center at the origin. Draw a diameter that has the given point as an endpoint. Explain how you know that the chord you drew is a diameter.

a. (4, 3) **b.** (0, 5) **c.** (−3, 4) **d.** (−5, 0)

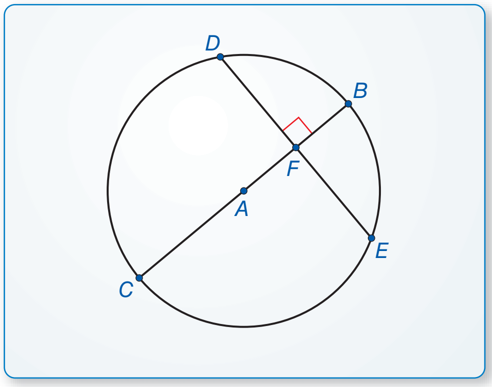
EXPLORATION 2 Writing a Conjecture about Chords

Work with a partner. Use dynamic geometry software to construct a chord $\overline{BC}$ of a circle A. Construct a chord on the perpendicular bisector of $\overline{BC}$. What do you notice? Change the original chord and the circle several times. Are your results always the same? Use your results to write a conjecture.



EXPLORATION 3 A Chord Perpendicular to a Diameter

Work with a partner. Use dynamic geometry software to construct a diameter $\overline{BC}$ of a circle A. Then construct a chord $\overline{DE}$ perpendicular to $\overline{BC}$ at point F. Find the lengths DF and EF. What do you notice? Change the chord perpendicular to $\overline{BC}$ and the circle several times. Do you always get the same results? Write a conjecture about a chord that is perpendicular to a diameter of a circle.



Communicate Your Answer

4. What are two ways to determine when a chord is a diameter of a circle?

10.3 Lesson

What You Will Learn

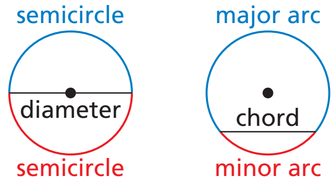
- Use chords of circles to find lengths and arc measures.

Core Vocabulary

Previous
chord
arc
diameter

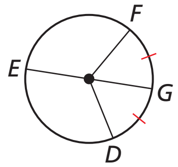
Using Chords of Circles

Recall that a *chord* is a segment with endpoints on a circle. Because its endpoints lie on the circle, any chord divides the circle into two arcs. A diameter divides a circle into two semicircles. Any other chord divides a circle into a minor arc and a major arc.



READING

If $\overset{\frown}{GD} \cong \overset{\frown}{GF}$, then the point G, and any line, segment, or ray that contains G, *bisects* $\overset{\frown}{FD}$.

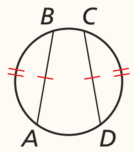


$\overline{EG}$ bisects $\overset{\frown}{FD}$.

Theorems

Theorem 10.6 Congruent Corresponding Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

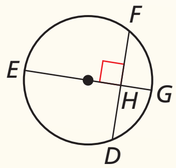


Proof Ex. 19, p. 550

$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

Theorem 10.7 Perpendicular Chord Bisector Theorem

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

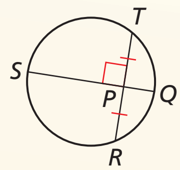


Proof Ex. 22, p. 550

If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.

Theorem 10.8 Perpendicular Chord Bisector Converse

If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.

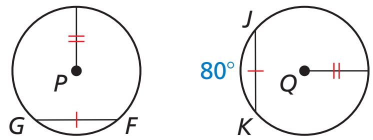


Proof Ex. 23, p. 550

If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

EXAMPLE 1 Using Congruent Chords to Find an Arc Measure

In the diagram, $\odot P \cong \odot Q$, $\overline{FG} \cong \overline{JK}$, and $m\overset{\frown}{JK} = 80°$. Find $m\overset{\frown}{FG}$.

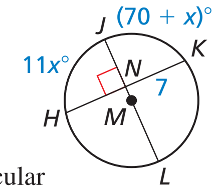


SOLUTION

Because $\overline{FG}$ and $\overline{JK}$ are congruent chords in congruent circles, the corresponding minor arcs $\overset{\frown}{FG}$ and $\overset{\frown}{JK}$ are congruent by the Congruent Corresponding Chords Theorem.

- So, $m\overset{\frown}{FG} = m\overset{\frown}{JK} = 80°$.

EXAMPLE 2 Using a Diameter



a. Find HK. **b.** Find $m\overset{\frown}{HK}$.

SOLUTION

a. Diameter $\overline{JL}$ is perpendicular to $\overline{HK}$. So, by the Perpendicular Chord Bisector Theorem, $\overline{JL}$ bisects $\overline{HK}$, and $HN = NK$.

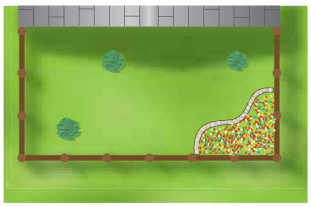
▶ So, $HK = 2(NK) = 2(7) = 14$.

b. Diameter $\overline{JL}$ is perpendicular to $\overline{HK}$. So, by the Perpendicular Chord Bisector Theorem, $\overline{JL}$ bisects $\overset{\frown}{HK}$, and $m\overset{\frown}{HJ} = m\overset{\frown}{JK}$.

$m\overset{\frown}{HJ} = m\overset{\frown}{JK}$	Perpendicular Chord Bisector Theorem
$11x° = (70 + x)°$	Substitute.
$10x = 70$	Subtract x from each side.
$x = 7$	Divide each side by 10.

▶ So, $m\overset{\frown}{HJ} = m\overset{\frown}{JK} = (70 + x)° = (70 + 7)° = 77°$, and $m\overset{\frown}{HK} = 2(m\overset{\frown}{HJ}) = 2(77°) = 154°$.

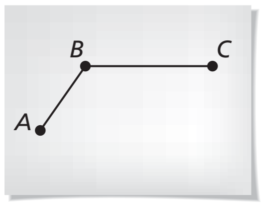
EXAMPLE 3 Using Perpendicular Bisectors



Three bushes are arranged in a garden, as shown. Where should you place a sprinkler so that it is the same distance from each bush?

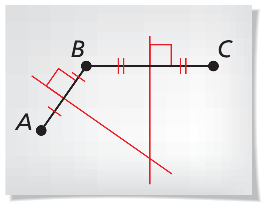
SOLUTION

Step 1



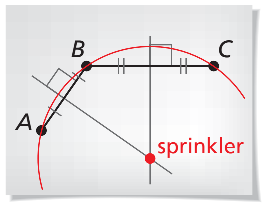
Label the bushes A, B, and C, as shown. Draw segments $\overline{AB}$ and $\overline{BC}$.

Step 2



Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$. By the Perpendicular Chord Bisector Converse, these lie on diameters of the circle containing A, B, and C.

Step 3



Find the point where the perpendicular bisectors intersect. This is the center of the circle, which is equidistant from points A, B, and C.

Monitoring Progress

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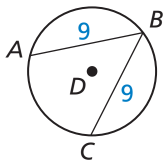
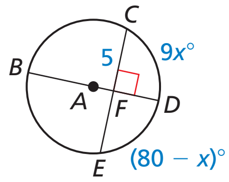
In Exercises 1 and 2, use the diagram of $\odot D$.



1. If $m\overset{\frown}{AB} = 110°$, find $m\overset{\frown}{BC}$.

2. If $m\overset{\frown}{AC} = 150°$, find $m\overset{\frown}{AB}$.

In Exercises 3 and 4, find the indicated length or arc measure.



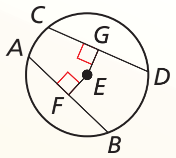
3. CE

4. $m\overset{\frown}{CE}$

Theorem

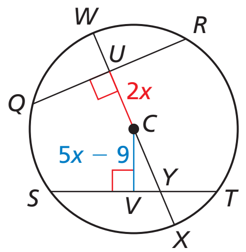
Theorem 10.9 Equidistant Chords Theorem

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



Proof Ex. 25, p. 550

$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

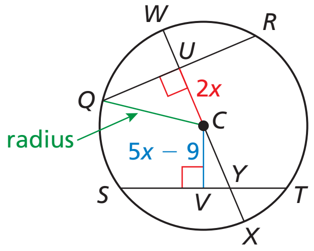


EXAMPLE 4 Using Congruent Chords to Find a Circle's Radius

In the diagram, $QR = ST = 16$, $CU = 2x$, and $CV = 5x - 9$. Find the radius of $\odot C$.

SOLUTION

Because $\overline{CQ}$ is a segment whose endpoints are the center and a point on the circle, it is a radius of $\odot C$. Because $\overline{CU} \perp \overline{QR}$, $\triangle QUC$ is a right triangle. Apply properties of chords to find the lengths of the legs of $\triangle QUC$.



Step 1 Find CU.

Because $\overline{QR}$ and $\overline{ST}$ are congruent chords, $\overline{QR}$ and $\overline{ST}$ are equidistant from C by the Equidistant Chords Theorem. So, $CU = CV$.

$CU = CV$	Equidistant Chords Theorem
$2x = 5x - 9$	Substitute.
$x = 3$	Solve for x.

So, $CU = 2x = 2(3) = 6$.

Step 2 Find QU.

Because diameter $\overline{WX} \perp \overline{QR}$, $\overline{WX}$ bisects $\overline{QR}$ by the Perpendicular Chord Bisector Theorem.

So, $QU = \frac{1}{2}(16) = 8$.

Step 3 Find CQ.

Because the lengths of the legs are $CU = 6$ and $QU = 8$, $\triangle QUC$ is a right triangle with the Pythagorean triple 6, 8, 10. So, $CQ = 10$.

▶ So, the radius of $\odot C$ is 10 units.

Monitoring Progress

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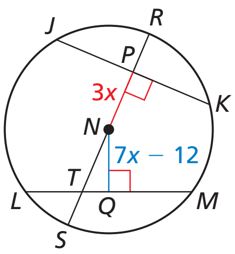


5. In the diagram, $JK = LM = 24$, $NP = 3x$, and $NQ = 7x - 12$. Find the radius of $\odot N$.

10.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

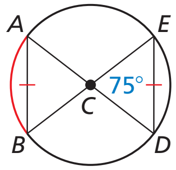
Vocabulary and Core Concept Check

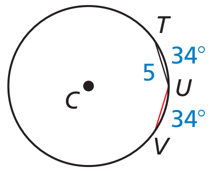
1. **WRITING** Describe what it means to bisect a chord.

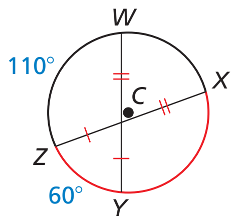
2. **WRITING** Two chords of a circle are perpendicular and congruent. Does one of them have to be a diameter? Explain your reasoning.

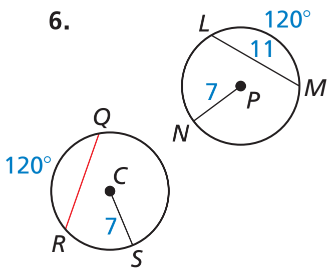
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the measure of the red arc or chord in ⊙*C*. *(See Example 1.)*

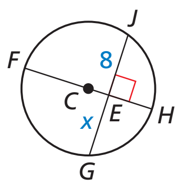
3. 

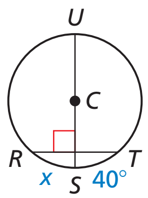
4. 

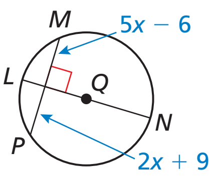
5. 

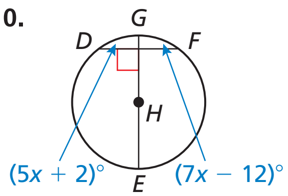
6. 

In Exercises 7–10, find the value of *x*. *(See Example 2.)*

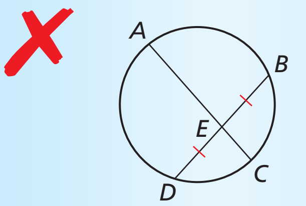
7. 

8. 

9. 

10. 

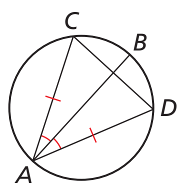
11. **ERROR ANALYSIS** Describe and correct the error in reasoning.

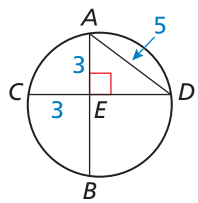


Because $\overline{AC}$ bisects $\overline{DB}$, $\overset{\frown}{BC} \cong \overset{\frown}{CD}$.

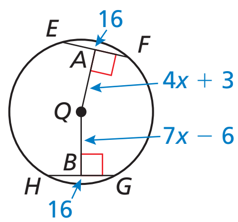
12. **PROBLEM SOLVING** In the cross section of the submarine shown, the control panels are parallel and the same length. Describe a method you can use to find the center of the cross section. Justify your method. *(See Example 3.)*

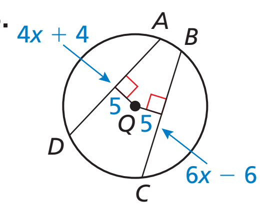
In Exercises 13 and 14, determine whether $\overline{AB}$ is a diameter of the circle. Explain your reasoning.

13. 

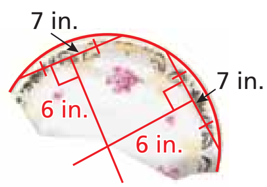
14. 

In Exercises 15 and 16, find the radius of ⊙*Q*. *(See Example 4.)*

15. 

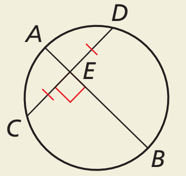
16. 

17. **PROBLEM SOLVING** An archaeologist finds part of a circular plate. What was the diameter of the plate to the nearest tenth of an inch? Justify your answer.

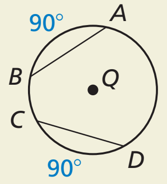


18. HOW DO YOU SEE IT? What can you conclude from each diagram? Name a theorem that justifies your answer.

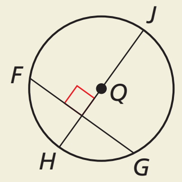
a.



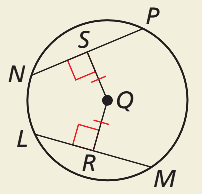
b.



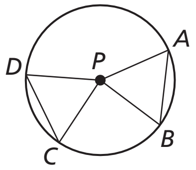
c.



d.



19. PROVING A THEOREM Use the diagram to prove each part of the biconditional in the Congruent Corresponding Chords Theorem (Theorem 10.6).



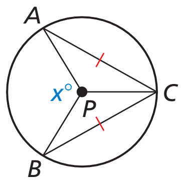
a. Given $\overline{AB}$ and $\overline{CD}$ are congruent chords.

Prove $\widehat{AB} \cong \widehat{CD}$

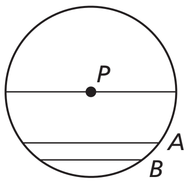
b. Given $\widehat{AB} \cong \widehat{CD}$

Prove $\overline{AB} \cong \overline{CD}$

20. MATHEMATICAL CONNECTIONS In $\odot P$, all the arcs shown have integer measures. Show that x must be even.



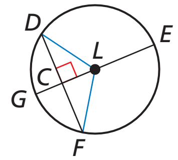
21. REASONING In $\odot P$, the lengths of the parallel chords are 20, 16, and 12. Find $m\widehat{AB}$. Explain your reasoning.



22. PROVING A THEOREM Use congruent triangles to prove the Perpendicular Chord Bisector Theorem (Theorem 10.7).

Given $\overline{EG}$ is a diameter of $\odot L$.
$\overline{EG} \perp \overline{DF}$

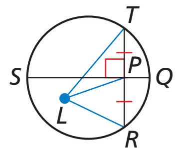
Prove $\overline{DC} \cong \overline{FC}$, $\widehat{DG} \cong \widehat{FG}$



23. PROVING A THEOREM Write a proof of the Perpendicular Chord Bisector Converse (Theorem 10.8).

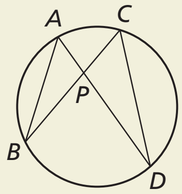
Given $\overline{QS}$ is a perpendicular bisector of $\overline{RT}$.

Prove $\overline{QS}$ is a diameter of the circle L.



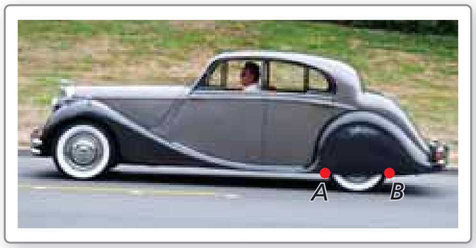
(*Hint*: Plot the center L and draw $\triangle LPT$ and $\triangle LPR$.)

24. THOUGHT PROVOKING Consider two chords that intersect at point P. Do you think that $\frac{AP}{BP} = \frac{CP}{DP}$? Justify your answer.



25. PROVING A THEOREM Use the diagram with the Equidistant Chords Theorem (Theorem 10.9) on page 548 to prove both parts of the biconditional of this theorem.

26. MAKING AN ARGUMENT A car is designed so that the rear wheel is only partially visible below the body of the car. The bottom edge of the panel is parallel to the ground. Your friend claims that the point where the tire touches the ground bisects $\widehat{AB}$. Is your friend correct? Explain your reasoning.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the missing interior angle measure. *(Section 7.1)*

27. Quadrilateral $JKLM$ has angle measures $m\angle J = 32°$, $m\angle K = 25°$, and $m\angle L = 44°$. Find $m\angle M$.

28. Pentagon $PQRST$ has angle measures $m\angle P = 85°$, $m\angle Q = 134°$, $m\angle R = 97°$, and $m\angle S = 102°$. Find $m\angle T$.