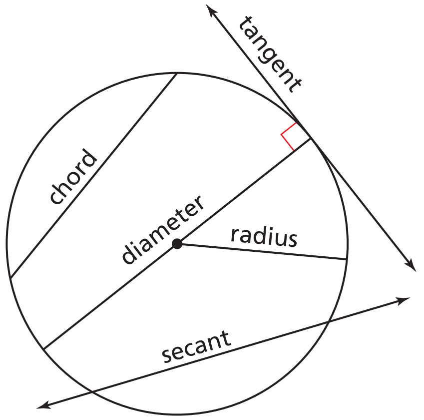
10.1 Lines and Segments That Intersect Circles

Essential Question What are the definitions of the lines and segments that intersect a circle?

EXPLORATION 1 Lines and Line Segments That Intersect Circles

Work with a partner. The drawing at the right shows five lines or segments that intersect a circle. Use the relationships shown to write a definition for each type of line or segment. Then use the Internet or some other resource to verify your definitions.



Chord:

Secant:

Tangent:

Radius:

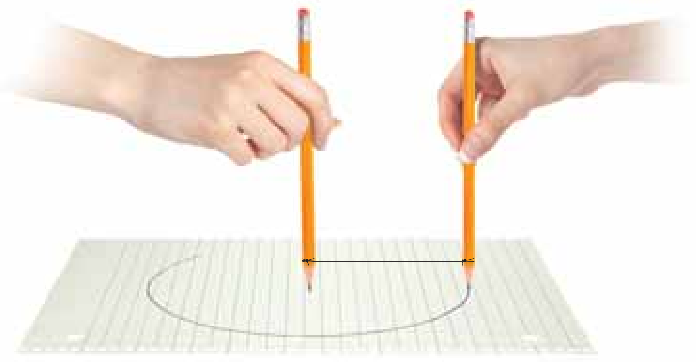
Diameter:

EXPLORATION 2 Using String to Draw a Circle

Work with a partner. Use two pencils, a piece of string, and a piece of paper.

a. Tie the two ends of the piece of string loosely around the two pencils.

b. Anchor one pencil on the paper at the center of the circle. Use the other pencil to draw a circle around the anchor point while using slight pressure to keep the string taut. Do not let the string wind around either pencil.



REASONING ABSTRACTLY

To be proficient in math, you need to know and flexibly use different properties of operations and objects.

c. Explain how the distance between the two pencil points as you draw the circle is related to two of the lines or line segments you defined in Exploration 1.

Communicate Your Answer

3. What are the definitions of the lines and segments that intersect a circle?

4. Of the five types of lines and segments in Exploration 1, which one is a subset of another? Explain.

5. Explain how to draw a circle with a diameter of 8 inches.

10.1 Lesson

Core Vocabulary

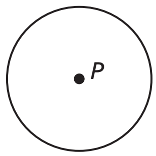
circle, *p. 530*
center, *p. 530*
radius, *p. 530*
chord, *p. 530*
diameter, *p. 530*
secant, *p. 530*
tangent, *p. 530*
point of tangency, *p. 530*
tangent circles, *p. 531*
concentric circles, *p. 531*
common tangent, *p. 531*

What You Will Learn

- Identify special segments and lines.
- Draw and identify common tangents.
- Use properties of tangents.

Identifying Special Segments and Lines

A **circle** is the set of all points in a plane that are equidistant from a given point called the **center** of the circle. A circle with center P is called "circle P" and can be written as $\odot P$.



circle P, or $\odot P$

READING

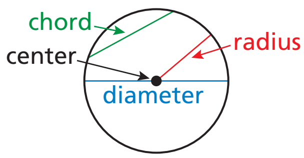
The words "radius" and "diameter" refer to lengths as well as segments. For a given circle, think of *a* radius and *a* diameter as segments and *the* radius and *the* diameter as lengths.

Core Concept

Lines and Segments That Intersect Circles

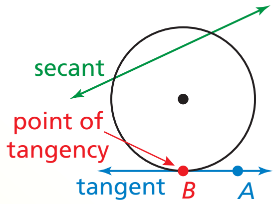
A segment whose endpoints are the center and any point on a circle is a **radius**.

A **chord** is a segment whose endpoints are on a circle. A **diameter** is a chord that contains the center of the circle.

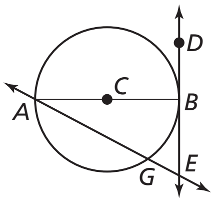


A **secant** is a line that intersects a circle in two points.

A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the **point of tangency**. The *tangent ray* $\overrightarrow{AB}$ and the *tangent segment* $\overline{AB}$ are also called tangents.



EXAMPLE 1 Identifying Special Segments and Lines



Tell whether the line, ray, or segment is best described as a *radius*, *chord*, *diameter*, *secant*, or *tangent* of $\odot C$.

a. $\overline{AC}$ **b.** $\overline{AB}$
c. $\overrightarrow{DE}$ **d.** $\overleftrightarrow{AE}$

SOLUTION

a. $\overline{AC}$ is a radius because C is the center and A is a point on the circle.

b. $\overline{AB}$ is a diameter because it is a chord that contains the center C.

c. $\overrightarrow{DE}$ is a tangent ray because it is contained in a line that intersects the circle in exactly one point.

d. $\overleftrightarrow{AE}$ is a secant because it is a line that intersects the circle in two points.

STUDY TIP

In this book, assume that all segments, rays, or lines that appear to be tangent to a circle are tangents.

Monitoring Progress

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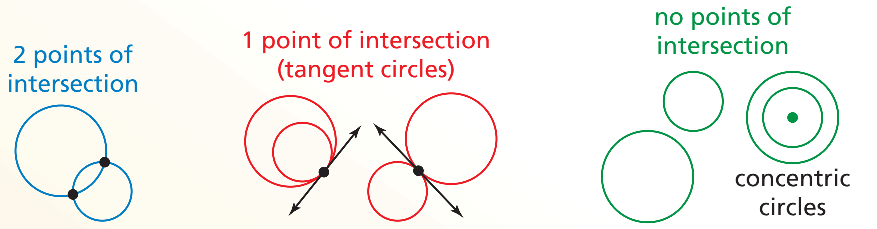
1. In Example 1, what word best describes $\overline{AG}$? $\overline{CB}$?
2. In Example 1, name a tangent and a tangent segment.

Drawing and Identifying Common Tangents

Core Concept

Coplanar Circles and Common Tangents

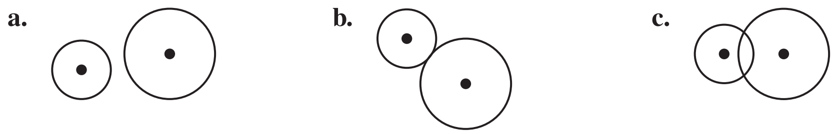
In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric circles**.



A line or segment that is tangent to two coplanar circles is called a **common tangent**. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

EXAMPLE 2 Drawing and Identifying Common Tangents

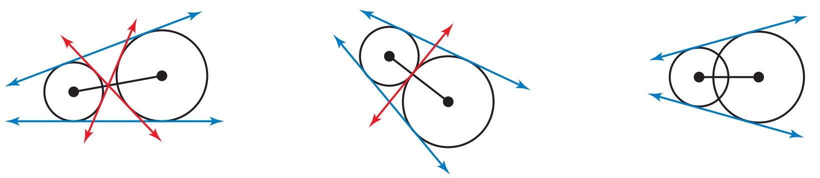
Tell how many common tangents the circles have and draw them. Use blue to indicate common external tangents and red to indicate common internal tangents.



SOLUTION

Draw the segment that joins the centers of the two circles. Then draw the common tangents. Use blue to indicate lines that do not intersect the segment joining the centers and red to indicate lines that intersect the segment joining the centers.

a. 4 common tangents **b.** 3 common tangents **c.** 2 common tangents



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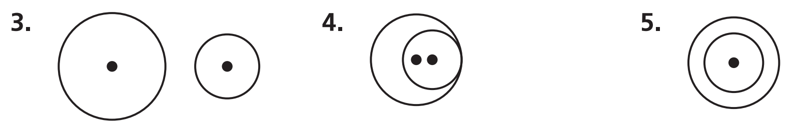
Tell how many common tangents the circles have and draw them. State whether the tangents are external tangents or internal tangents.

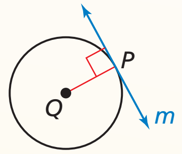


Using Properties of Tangents

Theorems

Theorem 10.1 Tangent Line to Circle Theorem

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

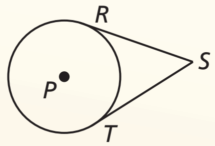


Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof Ex. 47, p. 536

Theorem 10.2 External Tangent Congruence Theorem

Tangent segments from a common external point are congruent.

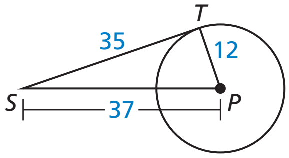


If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

Proof Ex. 46, p. 536

EXAMPLE 3 Verifying a Tangent to a Circle

Is $\overline{ST}$ tangent to $\odot P$?



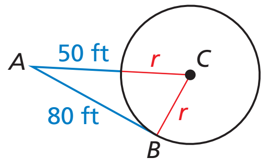
SOLUTION

Use the Converse of the Pythagorean Theorem (Theorem 9.2). Because $12^2 + 35^2 = 37^2$, $\triangle PTS$ is a right triangle and $\overline{ST} \perp \overline{PT}$. So, $\overline{ST}$ is perpendicular to a radius of $\odot P$ at its endpoint on $\odot P$.

▶ By the Tangent Line to Circle Theorem, $\overline{ST}$ is tangent to $\odot P$.

EXAMPLE 4 Finding the Radius of a Circle

In the diagram, point B is a point of tangency. Find the radius r of $\odot C$.



SOLUTION

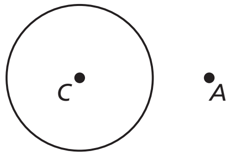
You know from the Tangent Line to Circle Theorem that $\overline{AB} \perp \overline{BC}$, so $\triangle ABC$ is a right triangle. You can use the Pythagorean Theorem (Theorem 9.1).

$AC^2 = BC^2 + AB^2$	Pythagorean Theorem
$(r + 50)^2 = r^2 + 80^2$	Substitute.
$r^2 + 100r + 2500 = r^2 + 6400$	Multiply.
$100r = 3900$	Subtract r^2 and 2500 from each side.
$r = 39$	Divide each side by 100.

▶ The radius is 39 feet.

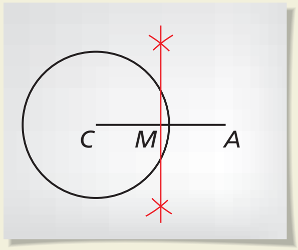
CONSTRUCTION Constructing a Tangent to a Circle

Given $\odot C$ and point A, construct a line tangent to $\odot C$ that passes through A. Use a compass and straightedge.



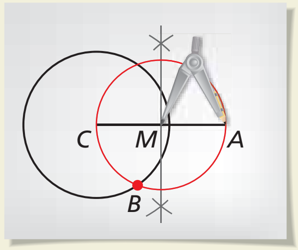
SOLUTION

Step 1



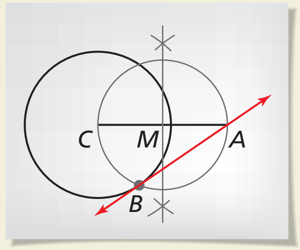
Find a midpoint
Draw $\overline{AC}$. Construct the bisector of the segment and label the midpoint M.

Step 2



Draw a circle
Construct $\odot M$ with radius MA. Label one of the points where $\odot M$ intersects $\odot C$ as point B.

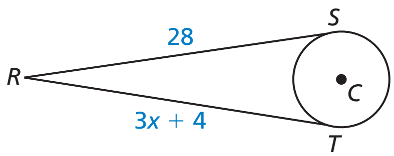
Step 3



Construct a tangent line
Draw $\overleftrightarrow{AB}$. It is a tangent to $\odot C$ that passes through A.

EXAMPLE 5 Using Properties of Tangents

$\overline{RS}$ is tangent to $\odot C$ at S, and $\overline{RT}$ is tangent to $\odot C$ at T. Find the value of x.



SOLUTION

$RS = RT$	External Tangent Congruence Theorem
$28 = 3x + 4$	Substitute.
$8 = x$	Solve for x.

▶ The value of x is 8.

Monitoring Progress

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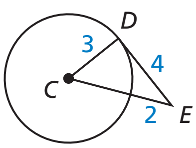
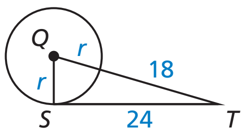
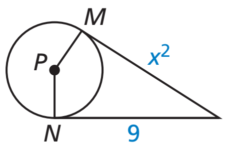
6. Is $\overline{DE}$ tangent to $\odot C$?



7. $\overline{ST}$ is tangent to $\odot Q$. Find the radius of $\odot Q$.



8. Points M and N are points of tangency. Find the value(s) of x.



10.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How are chords and secants alike? How are they different?

2. **WRITING** Explain how you can determine from the context whether the words *radius* and *diameter* are referring to segments or lengths.

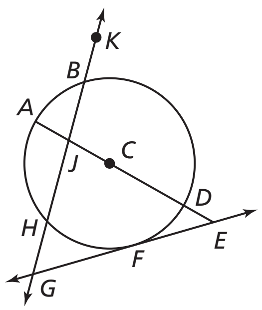
3. **COMPLETE THE SENTENCE** Coplanar circles that have a common center are called ________.

4. **WHICH ONE DOESN'T BELONG?** Which segment does *not* belong with the other three? Explain your reasoning.

chord	radius	tangent	diameter

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, use the diagram. *(See Example 1.)*



5. Name the circle.

6. Name two radii.

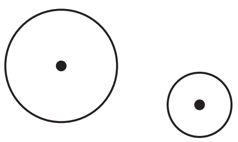
7. Name two chords.

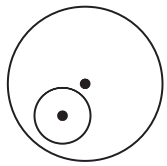
8. Name a diameter.

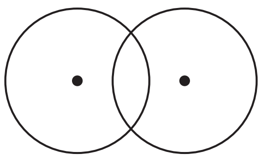
9. Name a secant.

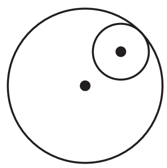
10. Name a tangent and a point of tangency.

In Exercises 11–14, copy the diagram. Tell how many common tangents the circles have and draw them. *(See Example 2.)*

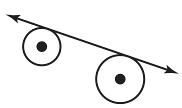
11. 

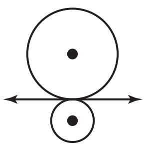
12. 

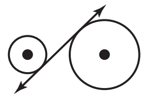
13. 

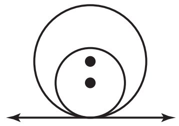
14. 

In Exercises 15–18, tell whether the common tangent is *internal* or *external*.

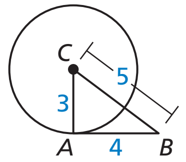
15. 

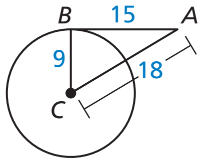
16. 

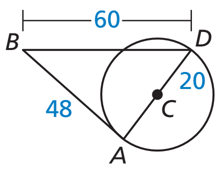
17. 

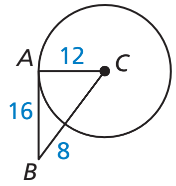
18. 

In Exercises 19–22, tell whether $\overline{AB}$ is tangent to $\odot C$. Explain your reasoning. *(See Example 3.)*

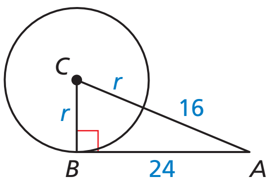
19. 

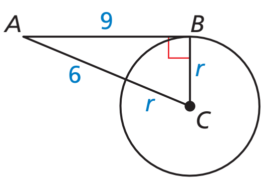
20. 

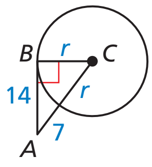
21. 

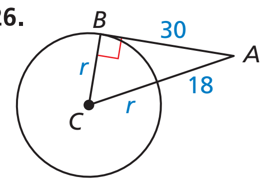
22. 

In Exercises 23–26, point B is a point of tangency. Find the radius r of $\odot C$. *(See Example 4.)*

23. 

24. 

25. 

26. 

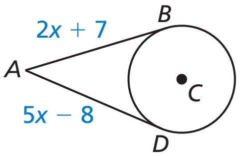
CONSTRUCTION **In Exercises 27 and 28, construct $\odot C$ with the given radius and point A outside of $\odot C$. Then construct a line tangent to $\odot C$ that passes through A.**

27. $r = 2$ in.

28. $r = 4.5$ cm

In Exercises 29–32, points B and D are points of tangency. Find the value(s) of x. *(See Example 5.)*

29.



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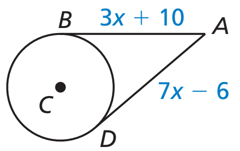

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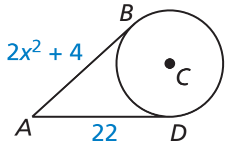

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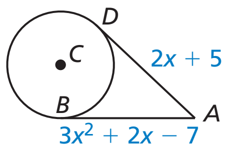
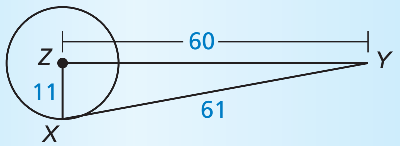
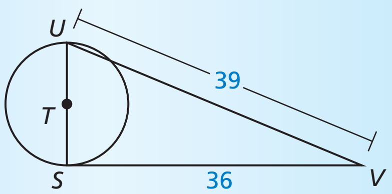


33. **ERROR ANALYSIS** Describe and correct the error in determining whether $\overline{XY}$ is tangent to $\odot Z$.



Because $11^2 + 60^2 = 61^2$, $\triangle XYZ$ is a right triangle. So, $\overline{XY}$ is tangent to $\odot Z$.

34. **ERROR ANALYSIS** Describe and correct the error in finding the radius of $\odot T$.



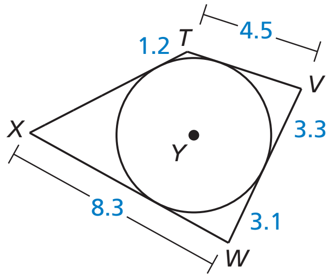
$39^2 - 36^2 = 15^2$

So, the radius is 15.

35. **ABSTRACT REASONING** For a point outside of a circle, how many lines exist tangent to the circle that pass through the point? How many such lines exist for a point on the circle? inside the circle? Explain your reasoning.

36. **CRITICAL THINKING** When will two lines tangent to the same circle not intersect? Justify your answer.

37. **USING STRUCTURE** Each side of quadrilateral $TVWX$ is tangent to $\odot Y$. Find the perimeter of the quadrilateral.

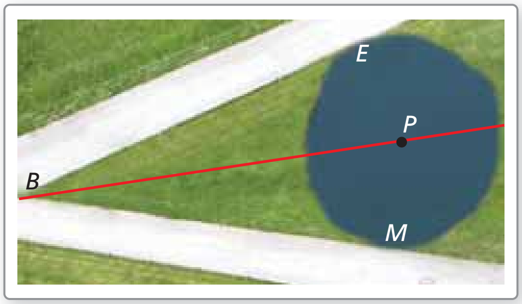


38. **LOGIC** In $\odot C$, radii $\overline{CA}$ and $\overline{CB}$ are perpendicular. $\overleftrightarrow{BD}$ and $\overleftrightarrow{AD}$ are tangent to $\odot C$.

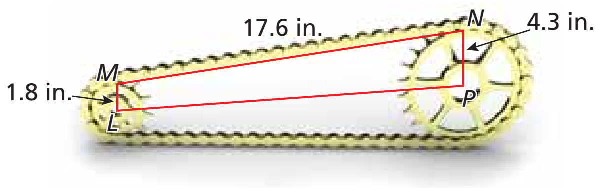
a. Sketch $\odot C$, $\overline{CA}$, $\overline{CB}$, $\overleftrightarrow{BD}$, and $\overleftrightarrow{AD}$.

b. What type of quadrilateral is $CADB$? Explain your reasoning.

39. **MAKING AN ARGUMENT** Two bike paths are tangent to an approximately circular pond. Your class is building a nature trail that begins at the intersection B of the bike paths and runs between the bike paths and over a bridge through the center P of the pond. Your classmate uses the Converse of the Angle Bisector Theorem (Theorem 6.4) to conclude that the trail must bisect the angle formed by the bike paths. Is your classmate correct? Explain your reasoning.

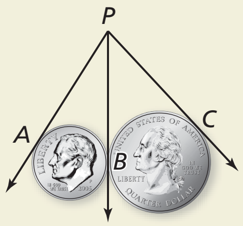


40. **MODELING WITH MATHEMATICS** A bicycle chain is pulled tightly so that $\overline{MN}$ is a common tangent of the gears. Find the distance between the centers of the gears.

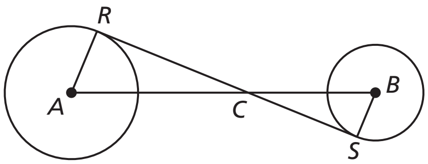


41. **WRITING** Explain why the diameter of a circle is the longest chord of the circle.

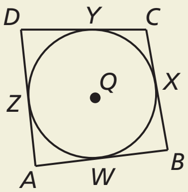
42. **HOW DO YOU SEE IT?** In the figure, $\overrightarrow{PA}$ is tangent to the dime, $\overrightarrow{PC}$ is tangent to the quarter, and $\overrightarrow{PB}$ is a common internal tangent. How do you know that $\overline{PA} \cong \overline{PB} \cong \overline{PC}$?



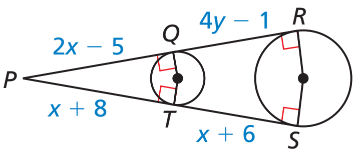
43. **PROOF** In the diagram, $\overline{RS}$ is a common internal tangent to $\odot A$ and $\odot B$. Prove that $\frac{AC}{BC} = \frac{RC}{SC}$.



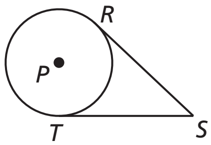
44. **THOUGHT PROVOKING** A polygon is *circumscribed* about a circle when every side of the polygon is tangent to the circle. In the diagram, quadrilateral *ABCD* is circumscribed about $\odot Q$. Is it always true that $AB + CD = AD + BC$? Justify your answer.



45. **MATHEMATICAL CONNECTIONS** Find the values of x and y. Justify your answer.



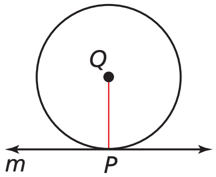
46. **PROVING A THEOREM** Prove the External Tangent Congruence Theorem (Theorem 10.2).



Given $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$.

Prove $\overline{SR} \cong \overline{ST}$

47. **PROVING A THEOREM** Use the diagram to prove each part of the biconditional in the Tangent Line to Circle Theorem (Theorem 10.1).



a. Prove indirectly that if a line is tangent to a circle, then it is perpendicular to a radius. (*Hint*: If you assume line m is not perpendicular to $\overline{QP}$, then the perpendicular segment from point Q to line m must intersect line m at some other point R.)

Given Line m is tangent to $\odot Q$ at point P.

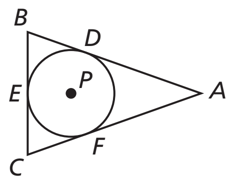
Prove $m \perp \overline{QP}$

b. Prove indirectly that if a line is perpendicular to a radius at its endpoint, then the line is tangent to the circle.

Given $m \perp \overline{QP}$

Prove Line m is tangent to $\odot Q$.

48. **REASONING** In the diagram, $AB = AC = 12$, $BC = 8$, and all three segments are tangent to $\odot P$. What is the radius of $\odot P$? Justify your answer.

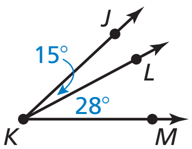


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the indicated measure. *(Section 1.2 and Section 1.5)*

49. $m\angle JKM$



50. AB