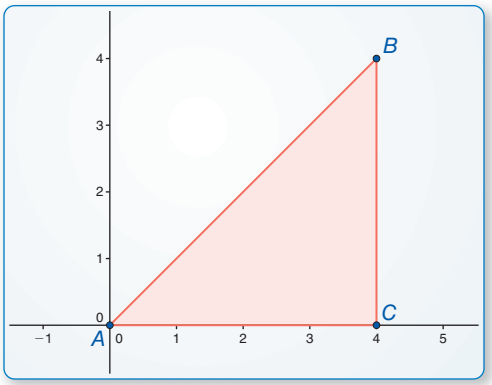
# 9.6 Solving Right Triangles

**Essential Question** When you know the lengths of the sides of a right triangle, how can you find the measures of the two acute angles?

## EXPLORATION 1 Solving Special Right Triangles

**Work with a partner.** Use the figures to find the values of the sine and cosine of $\angle A$ and $\angle B$. Use these values to find the measures of $\angle A$ and $\angle B$. Use dynamic geometry software to verify your answers.

**a.**

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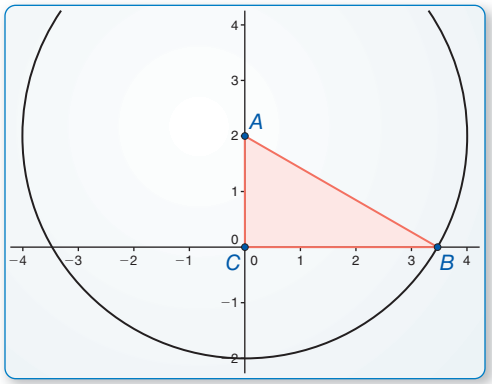


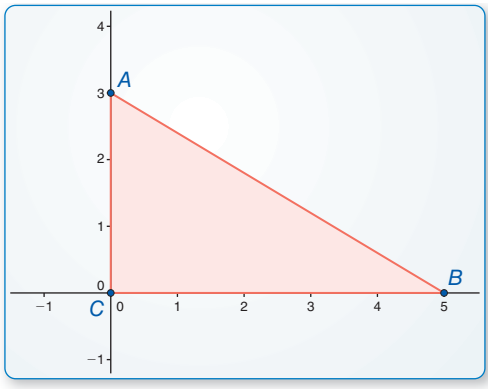

**ATTENDING TO PRECISION**

To be proficient in math, you need to calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context.

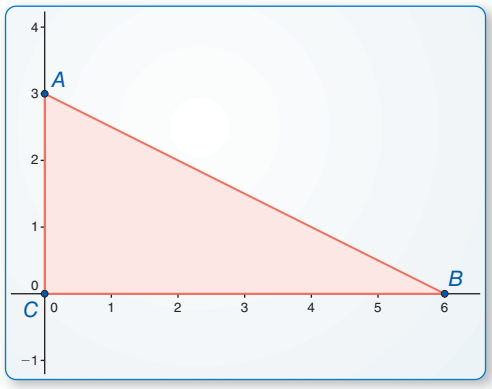
## EXPLORATION 2 Solving Right Triangles

**Work with a partner.** You can use a calculator to find the measure of an angle when you know the value of the sine, cosine, or tangent of the angle. Use the inverse sine, inverse cosine, or inverse tangent feature of your calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree. Then use dynamic geometry software to verify your answers.

**a.**




**b.**




## Communicate Your Answer

3. When you know the lengths of the sides of a right triangle, how can you find the measures of the two acute angles?

4. A ladder leaning against a building forms a right triangle with the building and the ground. The legs of the right triangle (in meters) form a 5-12-13 Pythagorean triple. Find the measures of the two acute angles to the nearest tenth of a degree.

# 9.6 Lesson

## Core Vocabulary

inverse tangent, *p. 502*
inverse sine, *p. 502*
inverse cosine, *p.502*
solve a right triangle, *p. 503*

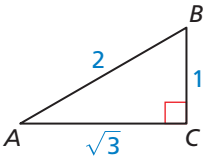
## What You Will Learn

- Use inverse trigonometric ratios.
- Solve right triangles.

## Using Inverse Trigonometric Ratios

### EXAMPLE 1 Identifying Angles from Trigonometric Ratios

Determine which of the two acute angles has a cosine of 0.5.




**SOLUTION**

Find the cosine of each acute angle.

$$\cos A = \frac{\text{adj. to } \angle A}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660 \qquad \cos B = \frac{\text{adj. to } \angle B}{\text{hyp.}} = \frac{1}{2} = 0.5$$

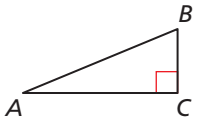
- The acute angle that has a cosine of 0.5 is $\angle B$.

If the measure of an acute angle is 60°, then its cosine is 0.5. The converse is also true. If the cosine of an acute angle is 0.5, then the measure of the angle is 60°. So, in Example 1, the measure of $\angle B$ must be 60° because its cosine is 0.5.

## Core Concept

### Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.




**Inverse Tangent** If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \tan^{-1}\frac{BC}{AC} = m\angle A$

**Inverse Sine** If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \sin^{-1}\frac{BC}{AB} = m\angle A$

**Inverse Cosine** If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \cos^{-1}\frac{AC}{AB} = m\angle A$

**READING**

The expression "$\tan^{-1} x$" is read as "the inverse tangent of $x$."

### EXAMPLE 2 Finding Angle Measures

Let $\angle A$, $\angle B$, and $\angle C$ be acute angles. Use a calculator to approximate the measures of $\angle A$, $\angle B$, and $\angle C$ to the nearest tenth of a degree.

**a.** $\tan A = 0.75$ **b.** $\sin B = 0.87$ **c.** $\cos C = 0.15$

**SOLUTION**

**a.** $m\angle A = \tan^{-1} 0.75 \approx 36.9°$

**b.** $m\angle B = \sin^{-1} 0.87 \approx 60.5°$

**c.** $m\angle C = \cos^{-1} 0.15 \approx 81.4°$

**ANOTHER WAY**

You can use the Table of Trigonometric Ratios available at *BigIdeasMath.com* to approximate $\tan^{-1} 0.75$ to the nearest degree. Find the number closest to 0.75 in the tangent column and read the angle measure at the left.

## Monitoring Progress

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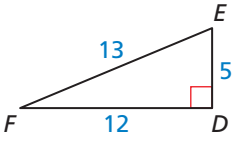
**Determine which of the two acute angles has the given trigonometric ratio.**




**1.** The sine of the angle is $\frac{12}{13}$.

**2.** The tangent of the angle is $\frac{5}{12}$.

## Monitoring Progress

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**Let $\angle G$, $\angle H$, and $\angle K$ be acute angles. Use a calculator to approximate the measures of $\angle G$, $\angle H$, and $\angle K$ to the nearest tenth of a degree.**

**3.** $\tan G = 0.43$ **4.** $\sin H = 0.68$ **5.** $\cos K = 0.94$

# Solving Right Triangles

## Core Concept

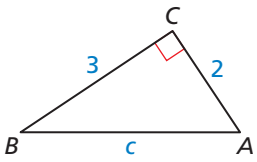
### Solving a Right Triangle

To **solve a right triangle** means to find all unknown side lengths and angle measures. You can solve a right triangle when you know either of the following.

- two side lengths
- one side length and the measure of one acute angle

## EXAMPLE 3 Solving a Right Triangle

Solve the right triangle. Round decimal answers to the nearest tenth.




### SOLUTION

**Step 1** Use the Pythagorean Theorem (Theorem 9.1) to find the length of the hypotenuse.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 3^2 + 2^2$ Substitute.

$c^2 = 13$ Simplify.

$c = \sqrt{13}$ Find the positive square root.

$c \approx 3.6$ Use a calculator.

**ANOTHER WAY**

You could also have found $m\angle A$ first by finding $\tan^{-1}\frac{3}{2} \approx 56.3°$.

**Step 2** Find $m\angle B$.

$m\angle B = \tan^{-1}\frac{2}{3} \approx 33.7°$ Use a calculator.

**Step 3** Find $m\angle A$.

Because $\angle A$ and $\angle B$ are complements, you can write

$m\angle A = 90° - m\angle B$

$\approx 90° - 33.7°$

$= 56.3°$.

▶ In $\triangle ABC$, $c \approx 3.6$, $m\angle B \approx 33.7°$, and $m\angle A \approx 56.3°$.

## Monitoring Progress

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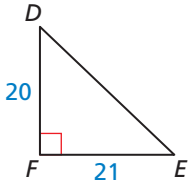
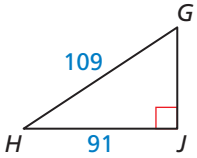
**Solve the right triangle. Round decimal answers to the nearest tenth.**

**6.**

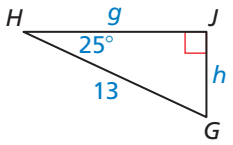



**7.**



## EXAMPLE 4 Solving a Right Triangle

Solve the right triangle. Round decimal answers to the nearest tenth.




### SOLUTION

Use trigonometric ratios to find the values of $g$ and $h$.

$$\sin H = \frac{\text{opp.}}{\text{hyp.}} \qquad \cos H = \frac{\text{adj.}}{\text{hyp.}}$$

$$\sin 25^\circ = \frac{h}{13} \qquad \cos 25^\circ = \frac{g}{13}$$

$$13 \cdot \sin 25^\circ = h \qquad 13 \cdot \cos 25^\circ = g$$

$$5.5 \approx h \qquad 11.8 \approx g$$

Because $\angle H$ and $\angle G$ are complements, you can write

$$m\angle G = 90^\circ - m\angle H = 90^\circ - 25^\circ = 65^\circ.$$

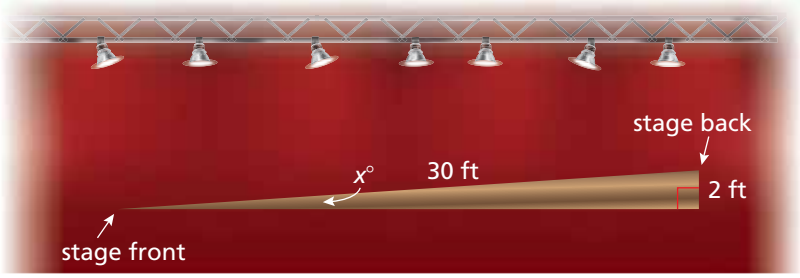
▶ In $\triangle GHJ$, $h \approx 5.5$, $g \approx 11.8$, and $m\angle G = 65^\circ$.

**READING**

A *raked stage* slants upward from front to back to give the audience a better view.

## EXAMPLE 5 Solving a Real-Life Problem

Your school is building a *raked stage*. The stage will be 30 feet long from front to back, with a total rise of 2 feet. You want the rake (angle of elevation) to be 5° or less for safety. Is the raked stage within your desired range?




### SOLUTION

Use the inverse sine ratio to find the degree measure $x$ of the rake.

$$x \approx \sin^{-1}\frac{2}{30} \approx 3.8$$

▶ The rake is about 3.8°, so it is within your desired range of 5° or less.

## Monitoring Progress

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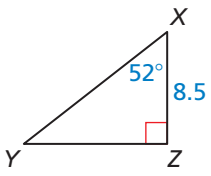
**8.** Solve the right triangle. Round decimal answers to the nearest tenth.




**9. WHAT IF?** In Example 5, suppose another raked stage is 20 feet long from front to back with a total rise of 2 feet. Is the raked stage within your desired range?

# 9.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

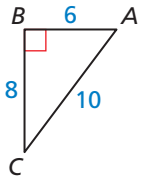
1. **COMPLETE THE SENTENCE** To solve a right triangle means to find the measures of all its _____ and _____.

2. **WRITING** Explain when you can use a trigonometric ratio to find a side length of a right triangle and when you can use the Pythagorean Theorem (Theorem 9.1).

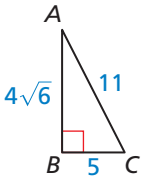
## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–6, determine which of the two acute angles has the given trigonometric ratio.** *(See Example 1.)*

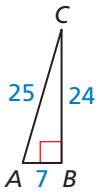
3. The cosine of the angle is $\frac{4}{5}$.



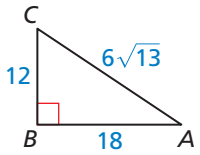

4. The sine of the angle is $\frac{5}{11}$.




5. The sine of the angle is 0.96.




6. The tangent of the angle is 1.5.

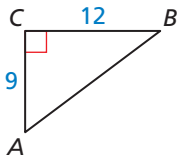



**In Exercises 7–12, let $\angle D$ be an acute angle. Use a calculator to approximate the measure of $\angle D$ to the nearest tenth of a degree.** *(See Example 2.)*

7. $\sin D = 0.75$
8. $\sin D = 0.19$
9. $\cos D = 0.33$
10. $\cos D = 0.64$
11. $\tan D = 0.28$
12. $\tan D = 0.72$

**In Exercises 13–18, solve the right triangle. Round decimal answers to the nearest tenth.** *(See Examples 3 and 4.)*

13.

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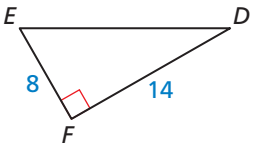


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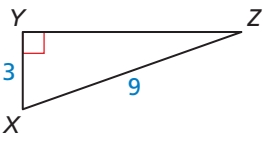


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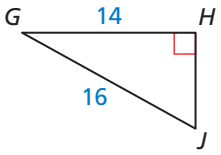


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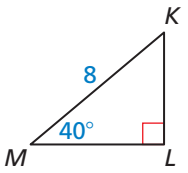


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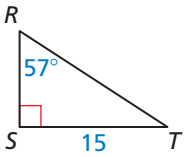
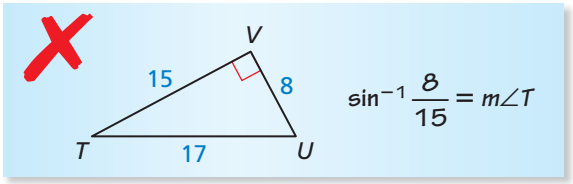
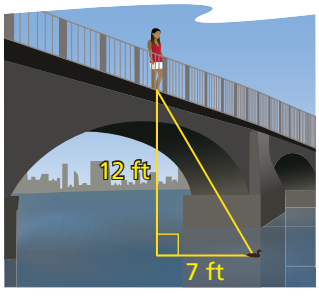



19. **ERROR ANALYSIS** Describe and correct the error in using an inverse trigonometric ratio.

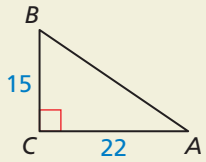



20. **PROBLEM SOLVING** In order to unload clay easily, the body of a dump truck must be elevated to at least 45°. The body of a dump truck that is 14 feet long has been raised 8 feet. Will the clay pour out easily? Explain your reasoning. *(See Example 5.)*

21. **PROBLEM SOLVING** You are standing on a footbridge that is 12 feet above a lake. You look down and see a duck in the water. The duck is 7 feet away from the footbridge. What is the angle of elevation from the duck to you?



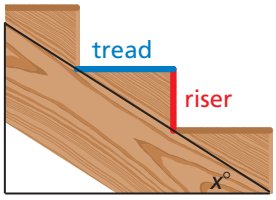
**22. HOW DO YOU SEE IT?** Write three expressions that can be used to approximate the measure of $\angle A$. Which expression would you choose? Explain your choice.




**23. MODELING WITH MATHEMATICS** The Uniform Federal Accessibility Standards specify that a wheelchair ramp may not have an incline greater than 4.76°. You want to build a ramp with a vertical rise of 8 inches. You want to minimize the horizontal distance taken up by the ramp. Draw a diagram showing the approximate dimensions of your ramp.

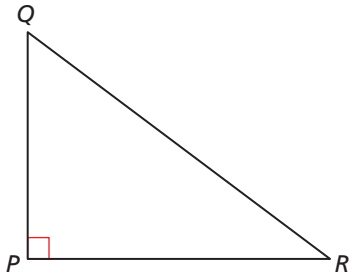
**24. MODELING WITH MATHEMATICS** The horizontal part of a step is called the *tread*. The vertical part is called the *riser*. The recommended riser-to-tread ratio is 7 inches : 11 inches.

**a.** Find the value of $x$ for stairs built using the recommended riser-to-tread ratio.




**b.** You want to build stairs that are less steep than the stairs in part (a). Give an example of a riser-to-tread ratio that you could use. Find the value of $x$ for your stairs.

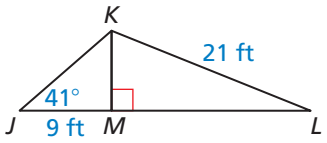
**25. USING TOOLS** Find the measure of $\angle R$ without using a protractor. Justify your technique.



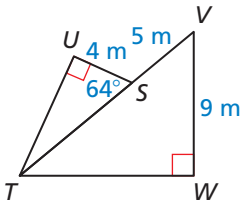

**26. MAKING AN ARGUMENT** Your friend claims that $\tan^{-1} x = \frac{1}{\tan x}$. Is your friend correct? Explain your reasoning.

**USING STRUCTURE In Exercises 27 and 28, solve each triangle.**

**27.** $\triangle JKM$ and $\triangle LKM$




**28.** $\triangle TUS$ and $\triangle VTW$




**29. MATHEMATICAL CONNECTIONS** Write an expression that can be used to find the measure of the acute angle formed by each line and the $x$-axis. Then approximate the angle measure to the nearest tenth of a degree.

**a.** $y = 3x$

**b.** $y = \frac{4}{3}x + 4$

**30. THOUGHT PROVOKING** Simplify each expression. Justify your answer.

**a.** $\sin^{-1}(\sin x)$

**b.** $\tan(\tan^{-1} y)$

**c.** $\cos(\cos^{-1} z)$

**31. REASONING** Explain why the expression $\sin^{-1}(1.2)$ does not make sense.

**32. USING STRUCTURE** The perimeter of rectangle $ABCD$ is 16 centimeters, and the ratio of its width to its length is 1 : 3. Segment $BD$ divides the rectangle into two congruent triangles. Find the side lengths and angle measures of these two triangles.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Solve the equation.** *(Skills Review Handbook)*

**33.** $\frac{12}{x} = \frac{3}{2}$ **34.** $\frac{13}{9} = \frac{x}{18}$ **35.** $\frac{x}{2.1} = \frac{4.1}{3.5}$ **36.** $\frac{5.6}{12.7} = \frac{4.9}{x}$