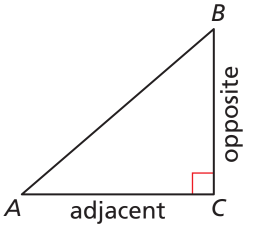
9.4 The Tangent Ratio

Essential Question

How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The *tangent* of $\angle A$ (written as tan A) is defined as follows.

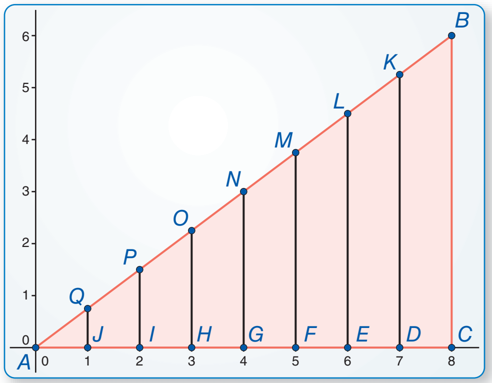
$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



EXPLORATION 1 Calculating a Tangent Ratio

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$, as shown. Construct segments perpendicular to $\overline{AC}$ to form right triangles that share vertex A and are similar to $\triangle ABC$ with vertices, as shown.



Sample
Points
$A(0, 0)$
$B(8, 6)$
$C(8, 0)$
Angle
$m\angle BAC = 36.87°$

b. Calculate each given ratio to complete the table for the decimal value of tan A for each right triangle. What can you conclude?

Ratio	$\frac{BC}{AC}$	$\frac{KD}{AD}$	$\frac{LE}{AE}$	$\frac{MF}{AF}$	$\frac{NG}{AG}$	$\frac{OH}{AH}$	$\frac{PI}{AI}$	$\frac{QJ}{AJ}$
tan *A*								

ATTENDING TO PRECISION

To be proficient in math, you need to express numerical answers with a degree of precision appropriate for the problem context.

EXPLORATION 2 Using a Calculator

Work with a partner. Use a calculator that has a tangent key to calculate the tangent of 36.87°. Do you get the same result as in Exploration 1? Explain.

Communicate Your Answer

3. Repeat Exploration 1 for $\triangle ABC$ with vertices $A(0, 0)$, $B(8, 5)$, and $C(8, 0)$. Construct the seven perpendicular segments so that not all of them intersect $\overline{AC}$ at integer values of x. Discuss your results.

4. How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?

9.4 Lesson

What You Will Learn

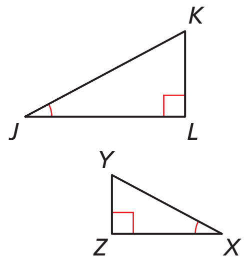
- Use the tangent ratio.
- Solve real-life problems involving the tangent ratio.

Core Vocabulary

trigonometric ratio, *p. 488*
tangent, *p. 488*
angle of elevation, *p. 490*

Using the Tangent Ratio

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. All right triangles with a given acute angle are similar by the AA Similarity Theorem (Theorem 8.3). So, $\triangle JKL \sim \triangle XYZ$, and you can write $\frac{KL}{YZ} = \frac{JL}{XZ}$. This can be rewritten as $\frac{KL}{JL} = \frac{YZ}{XZ}$, which is a trigonometric ratio. So, trigonometric ratios are constant for a given angle measure.



The **tangent** ratio is a trigonometric ratio for acute angles that involves the lengths of the legs of a right triangle.

READING

Remember the following abbreviations.

tangent → tan
opposite → opp.
adjacent → adj.

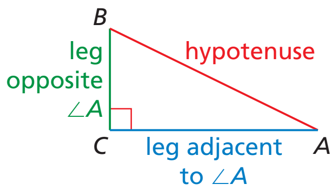
Core Concept

Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$.

The tangent of $\angle A$ (written as $\tan A$) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



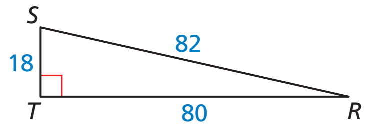
In the right triangle above, $\angle A$ and $\angle B$ are complementary. So, $\angle B$ is acute. You can use the same diagram to find the tangent of $\angle B$. Notice that the leg adjacent to $\angle A$ is the leg *opposite* $\angle B$ and the leg opposite $\angle A$ is the leg *adjacent* to $\angle B$.

ATTENDING TO PRECISION

Unless told otherwise, you should round the values of trigonometric ratios to four decimal places and round lengths to the nearest tenth.

EXAMPLE 1 Finding Tangent Ratios

Find $\tan S$ and $\tan R$. Write each answer as a fraction and as a decimal rounded to four places.



SOLUTION

$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{80}{18} = \frac{40}{9} \approx 4.4444$$

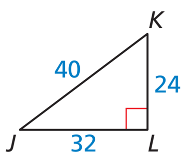
$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{18}{80} = \frac{9}{40} = 0.2250$$

Monitoring Progress

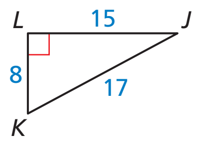
Help in English and Spanish at *BigIdeasMath.com*

Find tan *J* and tan *K*. Write each answer as a fraction and as a decimal rounded to four places.

1.

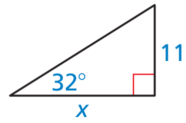


2.



EXAMPLE 2 Finding a Leg Length

Find the value of x. Round your answer to the nearest tenth.



SOLUTION

Use the tangent of an acute angle to find a leg length.

USING TOOLS STRATEGICALLY

You can also use the Table of Trigonometric Ratios available at *BigIdeasMath.com* to find the decimal approximations of trigonometric ratios.

$\tan 32^\circ = \dfrac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 32°.
$\tan 32^\circ = \dfrac{11}{x}$	Substitute.
$x \cdot \tan 32^\circ = 11$	Multiply each side by x.
$x = \dfrac{11}{\tan 32^\circ}$	Divide each side by tan 32°.
$x \approx 17.6$	Use a calculator.

▶ The value of x is about 17.6.

You can find the tangent of an acute angle measuring 30°, 45°, or 60° by applying what you know about special right triangles.

STUDY TIP

The tangents of all 60° angles are the same constant ratio. Any right triangle with a 60° angle can be used to determine this value.

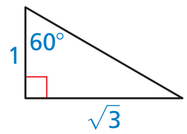
EXAMPLE 3 Using a Special Right Triangle to Find a Tangent

Use a special right triangle to find the tangent of a 60° angle.

SOLUTION

Step 1 Because all 30°-60°-90° triangles are similar, you can simplify your calculations by choosing 1 as the length of the shorter leg. Use the 30°-60°-90° Triangle Theorem (Theorem 9.5) to find the length of the longer leg.

longer leg $=$ shorter leg $\cdot \sqrt{3}$	30°-60°-90° Triangle Theorem
$= 1 \cdot \sqrt{3}$	Substitute.
$= \sqrt{3}$	Simplify.



Step 2 Find tan 60°.

$\tan 60^\circ = \dfrac{\text{opp.}}{\text{adj.}}$	Write ratio for tangent of 60°.
$\tan 60^\circ = \dfrac{\sqrt{3}}{1}$	Substitute.
$\tan 60^\circ = \sqrt{3}$	Simplify.

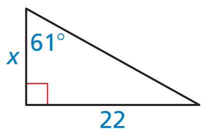
▶ The tangent of any 60° angle is $\sqrt{3} \approx 1.7321$.

Monitoring Progress

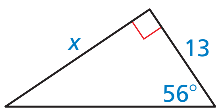
Help in English and Spanish at *BigIdeasMath.com*

Find the value of x. Round your answer to the nearest tenth.

3.



4.



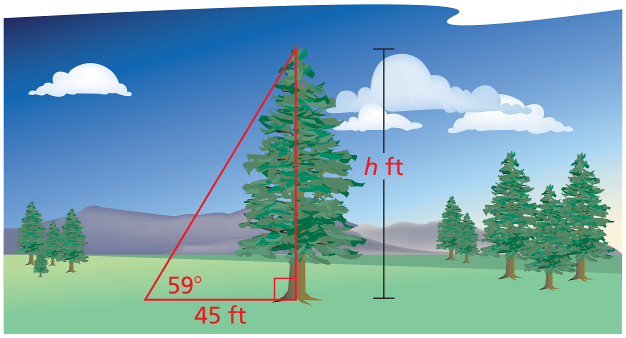
5. **WHAT IF?** In Example 3, the length of the shorter leg is 5 instead of 1. Show that the tangent of 60° is still equal to $\sqrt{3}$.

Solving Real-Life Problems

The angle that an upward line of sight makes with a horizontal line is called the **angle of elevation**.

EXAMPLE 4 **Modeling with Mathematics**

You are measuring the height of a spruce tree. You stand 45 feet from the base of the tree. You measure the angle of elevation from the ground to the top of the tree to be 59°. Find the height h of the tree to the nearest foot.



SOLUTION

1. **Understand the Problem** You are given the angle of elevation and the distance from the tree. You need to find the height of the tree to the nearest foot.
2. **Make a Plan** Write a trigonometric ratio for the tangent of the angle of elevation involving the height h. Then solve for h.
3. **Solve the Problem**

$$\tan 59^\circ = \frac{\text{opp.}}{\text{adj.}}$$ Write ratio for tangent of 59°.

$$\tan 59^\circ = \frac{h}{45}$$ Substitute.

$$45 \cdot \tan 59^\circ = h$$ Multiply each side by 45.

$$74.9 \approx h$$ Use a calculator.

▶ The tree is about 75 feet tall.

4. **Look Back** Check your answer. Because 59° is close to 60°, the value of h should be close to the length of the longer leg of a 30°-60°-90° triangle, where the length of the shorter leg is 45 feet.

$$\text{longer leg} = \text{shorter leg} \cdot \sqrt{3}$$ 30°-60°-90° Triangle Theorem

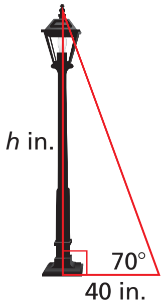
$$= 45 \cdot \sqrt{3}$$ Substitute.

$$\approx 77.9$$ Use a calculator.

The value of 77.9 feet is close to the value of h. ✓

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*



6. You are measuring the height of a lamppost. You stand 40 inches from the base of the lamppost. You measure the angle of elevation from the ground to the top of the lamppost to be 70°. Find the height h of the lamppost to the nearest inch.

9.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

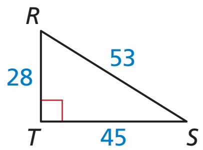
Vocabulary and Core Concept Check

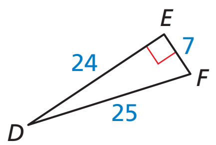
1. **COMPLETE THE SENTENCE** The tangent ratio compares the length of _______ to the length of ________.

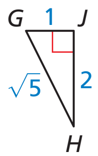
2. **WRITING** Explain how you know the tangent ratio is constant for a given angle measure.

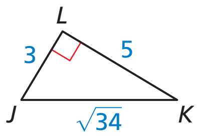
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the tangents of the acute angles in the right triangle. Write each answer as a fraction and as a decimal rounded to four decimal places. *(See Example 1.)*

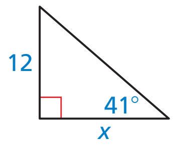
3. 

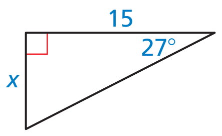
4. 

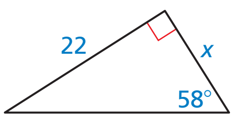
5. 

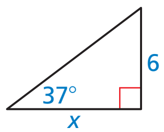
6. 

In Exercises 7–10, find the value of *x*. Round your answer to the nearest tenth. *(See Example 2.)*

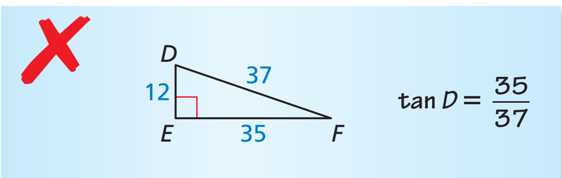
7. 

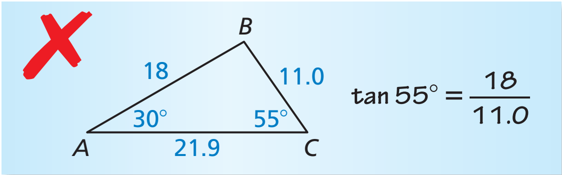
8. 

9. 

10. 

ERROR ANALYSIS In Exercises 11 and 12, describe the error in the statement of the tangent ratio. Correct the error if possible. Otherwise, write not possible.

11. 

12. 

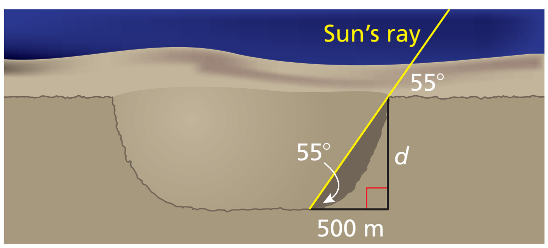
In Exercises 13 and 14, use a special right triangle to find the tangent of the given angle measure. *(See Example 3.)*

13. 45°

14. 30°

15. **MODELING WITH MATHEMATICS** A surveyor is standing 118 feet from the base of the Washington Monument. The surveyor measures the angle of elevation from the ground to the top of the monument to be 78°. Find the height *h* of the Washington Monument to the nearest foot. *(See Example 4.)*

16. **MODELING WITH MATHEMATICS** Scientists can measure the depths of craters on the moon by looking at photos of shadows. The length of the shadow cast by the edge of a crater is 500 meters. The angle of elevation of the rays of the Sun is 55°. Estimate the depth *d* of the crater.



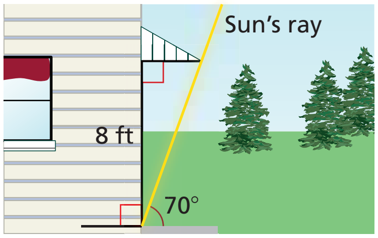
17. **USING STRUCTURE** Find the tangent of the smaller acute angle in a right triangle with side lengths 5, 12, and 13.

18. **USING STRUCTURE** Find the tangent of the larger acute angle in a right triangle with side lengths 3, 4, and 5.

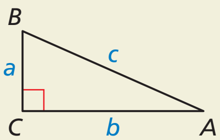
19. **REASONING** How does the tangent of an acute angle in a right triangle change as the angle measure increases? Justify your answer.

20. **CRITICAL THINKING** For what angle measure(s) is the tangent of an acute angle in a right triangle equal to 1? greater than 1? less than 1? Justify your answer.

21. **MAKING AN ARGUMENT** Your family room has a sliding-glass door. You want to buy an awning for the door that will be just long enough to keep the Sun out when it is at its highest point in the sky. The angle of elevation of the rays of the Sun at this point is 70°, and the height of the door is 8 feet. Your sister claims you can determine how far the overhang should extend by multiplying 8 by tan 70°. Is your sister correct? Explain.

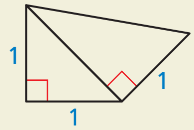


22. **HOW DO YOU SEE IT?** Write expressions for the tangent of each acute angle in the right triangle. Explain how the tangent of one acute angle is related to the tangent of the other acute angle. What kind of angle pair is $\angle A$ and $\angle B$?

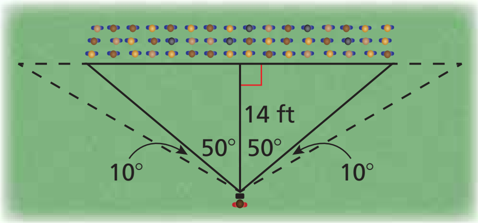


23. **REASONING** Explain why it is not possible to find the tangent of a right angle or an obtuse angle.

24. **THOUGHT PROVOKING** To create the diagram below, you begin with an isosceles right triangle with legs 1 unit long. Then the hypotenuse of the first triangle becomes the leg of a second triangle, whose remaining leg is 1 unit long. Continue the diagram until you have constructed an angle whose tangent is $\frac{1}{\sqrt{6}}$. Approximate the measure of this angle.



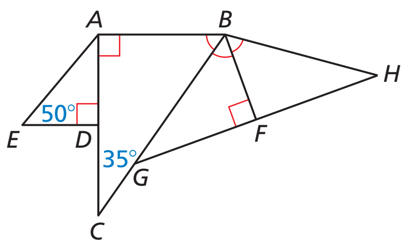
25. **PROBLEM SOLVING** Your class is having a class picture taken on the lawn. The photographer is positioned 14 feet away from the center of the class. The photographer turns 50° to look at either end of the class.



a. What is the distance between the ends of the class?

b. The photographer turns another 10° either way to see the end of the camera range. If each student needs 2 feet of space, about how many more students can fit at the end of each row? Explain.

26. **PROBLEM SOLVING** Find the perimeter of the figure, where $AC = 26$, $AD = BF$, and D is the midpoint of $\overline{AC}$.

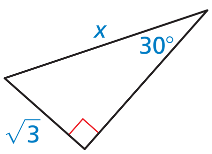


Maintaining Mathematical Proficiency

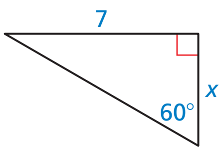
Reviewing what you learned in previous grades and lessons

Find the value of x. *(Section 9.2)*

27.



28.



29.