9.2 Special Right Triangles

Essential Question What is the relationship among the side lengths of 45°-45°-90° triangles? 30°-60°-90° triangles?

EXPLORATION 1 Side Ratios of an Isosceles Right Triangle

Work with a partner.

a. Use dynamic geometry software to construct an isosceles right triangle with a leg length of 4 units.

b. Find the acute angle measures. Explain why this triangle is called a 45°-45°-90° triangle.
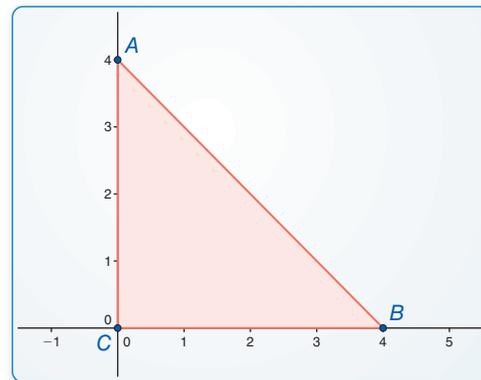
ATTENDING TO PRECISION

To be proficient in math, you need to express numerical answers with a degree of precision appropriate for the problem context.

c. Find the exact ratios of the side lengths (using square roots).

$$\frac{AB}{AC} = \boxed{}$$

$$\frac{AB}{BC} = \boxed{}$$

$$\frac{AC}{BC} = \boxed{}$$

Sample

Points
$A(0, 4)$
$B(4, 0)$
$C(0, 0)$
Segments
$AB = 5.66$
$BC = 4$
$AC = 4$
Angles
$m\angle A = 45°$
$m\angle B = 45°$

d. Repeat parts (a) and (c) for several other isosceles right triangles. Use your results to write a conjecture about the ratios of the side lengths of an isosceles right triangle.

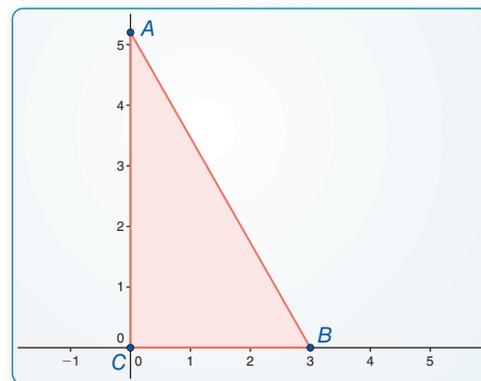### EXPLORATION 2 Side Ratios of a 30°-60°-90° Triangle

Work with a partner.

a. Use dynamic geometry software to construct a right triangle with acute angle measures of 30° and 60° (a 30°-60°-90° triangle), where the shorter leg length is 3 units.

b. Find the exact ratios of the side lengths (using square roots).

$$\frac{AB}{AC} = \boxed{}$$

$$\frac{AB}{BC} = \boxed{}$$

$$\frac{AC}{BC} = \boxed{}$$

Sample

Points
$A(0, 5.20)$
$B(3, 0)$
$C(0, 0)$
Segments
$AB = 6$
$BC = 3$
$AC = 5.20$
Angles
$m\angle A = 30°$
$m\angle B = 60°$

c. Repeat parts (a) and (b) for several other 30°-60°-90° triangles. Use your results to write a conjecture about the ratios of the side lengths of a 30°-60°-90° triangle.

Communicate Your Answer

3. What is the relationship among the side lengths of 45°-45°-90° triangles? 30°-60°-90° triangles?
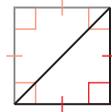
What You Will Learn

▶ Find side lengths in special right triangles.

▶ Solve real-life problems involving special right triangles.

Core Vocabulary

Previous
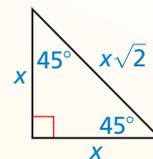isosceles triangle

Finding Side Lengths in Special Right Triangles

A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half diagonally.

🔵 Theorem

REMEMBER

An expression involving a radical with index 2 is in simplest form when no radicands have perfect squares as factors other than 1, no radicands contain fractions, and no radicals appear in the denominator of a fraction.

> **Theorem 9.4   45°-45°-90° Triangle Theorem**
>
> In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.
>
>
> *Proof* Ex. 19, p. 476
>
> hypotenuse = leg $\cdot \sqrt{2}$

EXAMPLE 1   Finding Side Lengths in 45°-45°-90° Triangles

Find the value of x. Write your answer in simplest form.

a.

b.
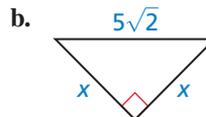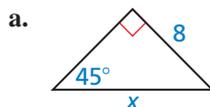
SOLUTION

a. By the Triangle Sum Theorem (Theorem 5.1), the measure of the third angle must be 45°, so the triangle is a 45°-45°-90° triangle.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2} \qquad \text{45°-45°-90° Triangle Theorem}$$

$$x = 8 \cdot \sqrt{2} \qquad \text{Substitute.}$$

$$x = 8\sqrt{2} \qquad \text{Simplify.}$$

▶ The value of x is $8\sqrt{2}$.

b. By the Base Angles Theorem (Theorem 5.6) and the Corollary to the Triangle Sum Theorem (Corollary 5.1), the triangle is a 45°-45°-90° triangle.

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2} \qquad \text{45°-45°-90° Triangle Theorem}$$

$$5\sqrt{2} = x \cdot \sqrt{2} \qquad \text{Substitute.}$$

$$\frac{5\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}} \qquad \text{Divide each side by } \sqrt{2}.$$
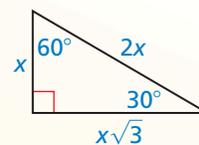
$$5 = x \qquad \text{Simplify.}$$

▶ The value of x is 5.

🌀 Theorem

Theorem 9.5   30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse = shorter leg • 2
longer leg = shorter leg • $\sqrt{3}$

Proof Ex. 21, p. 476

EXAMPLE 2   Finding Side Lengths in a 30°-60°-90° Triangle

Find the values of x and y. Write your answer in simplest form.
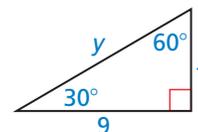
SOLUTION

Step 1 Find the value of x.

longer leg = shorter leg • $\sqrt{3}$	30°-60°-90° Triangle Theorem
$9 = x \cdot \sqrt{3}$	Substitute.
$\dfrac{9}{\sqrt{3}} = x$	Divide each side by $\sqrt{3}$.
$\dfrac{9}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = x$	Multiply by $\dfrac{\sqrt{3}}{\sqrt{3}}$.
$\dfrac{9\sqrt{3}}{3} = x$	Multiply fractions.
$3\sqrt{3} = x$	Simplify.

▶ The value of x is $3\sqrt{3}$.

Step 2 Find the value of y.

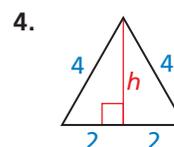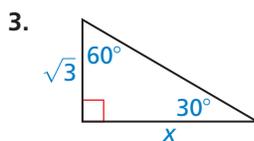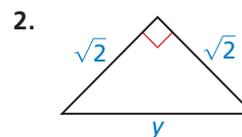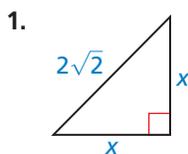|	
hypotenuse = shorter leg • 2	30°-60°-90° Triangle Theorem
$y = 3\sqrt{3} \cdot 2$	Substitute.
$y = 6\sqrt{3}$	Simplify.

▶ The value of y is $6\sqrt{3}$.

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Find the value of the variable. Write your answer in simplest form.

1.

2.

3.

4.

Solving Real-Life Problems

EXAMPLE 3   Modeling with Mathematics

The road sign is shaped like an equilateral triangle.
Estimate the area of the sign by finding the area of the
equilateral triangle.

SOLUTION

First find the height h of the triangle by dividing it into
two 30°-60°-90° triangles. The length of the longer leg
of one of these triangles is h. The length of the shorter leg
is 18 inches.

$$h = 18 \cdot \sqrt{3} = 18\sqrt{3} \qquad \text{30°-60°-90° Triangle Theorem}$$

Use $h = 18\sqrt{3}$ to find the area of the equilateral triangle.

$$\text{Area} = \tfrac{1}{2}bh = \tfrac{1}{2}(36)\left(18\sqrt{3}\right) \approx 561.18$$

▶   The area of the sign is about 561 square inches.
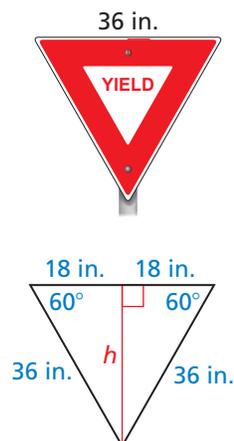
EXAMPLE 4   Finding the Height of a Ramp

A tipping platform is a ramp used to unload trucks. How high is the end of an
80-foot ramp when the tipping angle is 30°? 45°?
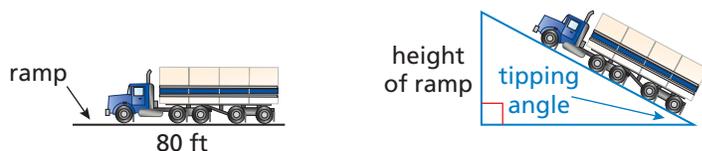
SOLUTION

When the tipping angle is 30°, the height h of the ramp is the length of the shorter leg
of a 30°-60°-90° triangle. The length of the hypotenuse is 80 feet.

$$80 = 2h \qquad \text{30°-60°-90° Triangle Theorem}$$

$$40 = h \qquad \text{Divide each side by 2.}$$

When the tipping angle is 45°, the height h of the ramp is the length of a leg of a
45°-45°-90° triangle. The length of the hypotenuse is 80 feet.

$$80 = h \cdot \sqrt{2} \qquad \text{45°-45°-90° Triangle Theorem}$$

$$\frac{80}{\sqrt{2}} = h \qquad \text{Divide each side by } \sqrt{2}.$$

$$56.6 \approx h \qquad \text{Use a calculator.}$$

▶   When the tipping angle is 30°, the ramp height is 40 feet. When the tipping angle
is 45°, the ramp height is about 56 feet 7 inches.

Monitoring Progress   ◀)) Help in English and Spanish at *BigIdeasMath.com*

5. The logo on a recycling bin resembles an equilateral triangle with side lengths of
6 centimeters. Approximate the area of the logo.

6. The body of a dump truck is raised to empty a load of sand. How high is the
14-foot-long body from the frame when it is tipped upward by a 60° angle?
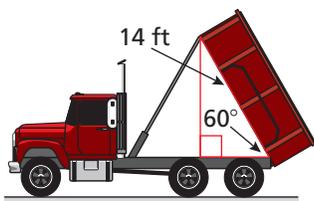
Vocabulary and Core Concept Check

1. **VOCABULARY** Name two special right triangles by their angle measures.

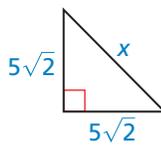2. **WRITING** Explain why the acute angles in an isosceles right triangle always measure 45°.

Monitoring Progress and Modeling with Mathematics

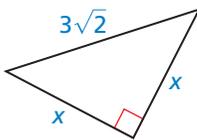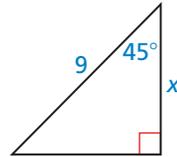**In Exercises 3–6, find the value of x. Write your answer in simplest form.** *(See Example 1.)*
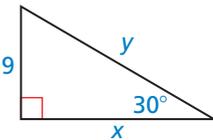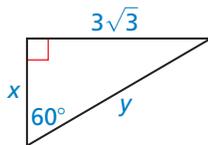
3.
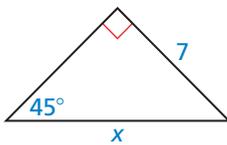
4.

5.

6.

In Exercises 7–10, find the values of x and y. Write your answers in simplest form. *(See Example 2.)*
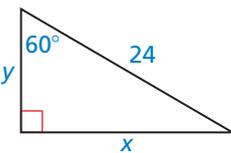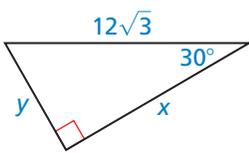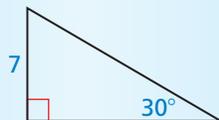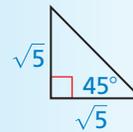
7.

8.

9.

10.

ERROR ANALYSIS **In Exercises 11 and 12, describe and correct the error in finding the length of the hypotenuse.**

11.

By the Triangle Sum Theorem (Theorem 5.1), the measure of the third angle must be 60°. So, the triangle is a 30°-60°-90° triangle.

hypotenuse = shorter leg • $\sqrt{3}$ = $7\sqrt{3}$

So, the length of the hypotenuse is $7\sqrt{3}$ units.

12.

By the Triangle Sum Theorem (Theorem 5.1), the measure of the third angle must be 45°. So, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • leg • $\sqrt{2}$ = $5\sqrt{2}$

So, the length of the hypotenuse is $5\sqrt{2}$ units.

In Exercises 13 and 14, sketch the figure that is described. Find the indicated length. Round decimal answers to the nearest tenth.
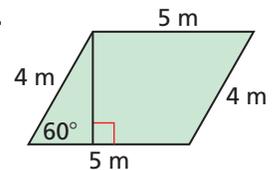
13. The side length of an equilateral triangle is 5 centimeters. Find the length of an altitude.

14. The perimeter of a square is 36 inches. Find the length of a diagonal.

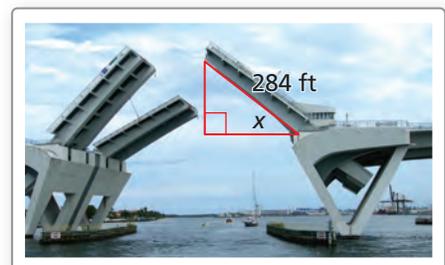**In Exercises 15 and 16, find the area of the figure. Round decimal answers to the nearest tenth.** *(See Example 3.)*

15.

16.
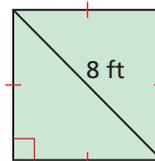
17. **PROBLEM SOLVING** Each half of the drawbridge is about 284 feet long. How high does the drawbridge rise when x is 30°? 45°? 60°? *(See Example 4.)*
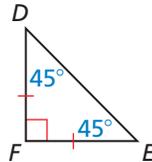
18. MODELING WITH MATHEMATICS A nut is shaped like a regular hexagon with side lengths of 1 centimeter. Find the value of x. (*Hint*: A regular hexagon can be divided into six congruent triangles.)
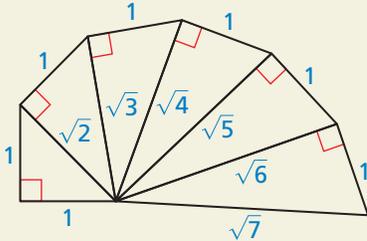
1 cm

x
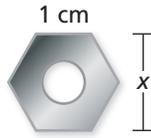
19. PROVING A THEOREM Write a paragraph proof of the 45°-45°-90° Triangle Theorem (Theorem 9.4).

Given $\triangle DEF$ is a 45°-45°-90° triangle.

Prove The hypotenuse is $\sqrt{2}$ times as long as each leg.

D
45°
45°
F   E
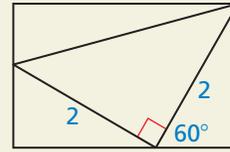
21. PROVING A THEOREM Write a paragraph proof of the 30°-60°-90° Triangle Theorem (Theorem 9.5). (*Hint*: Construct $\triangle JML$ congruent to $\triangle JKL$.)
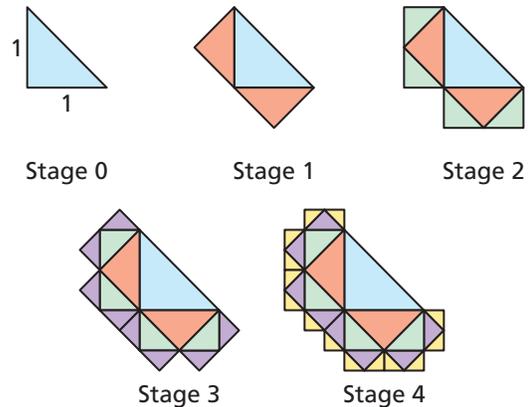
Given $\triangle JKL$ is a 30°-60°-90° triangle.

Prove The hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

K
60° x
J   30°
L
x
M

23. WRITING Describe two ways to show that all isosceles right triangles are similar to each other.

24. MAKING AN ARGUMENT Each triangle in the diagram is a 45°-45°-90° triangle. At Stage 0, the legs of the triangle are each 1 unit long. Your brother claims the lengths of the legs of the triangles added are halved at each stage. So, the length of a leg of a triangle added in Stage 8 will be $\frac{1}{256}$ unit. Is your brother correct? Explain your reasoning.
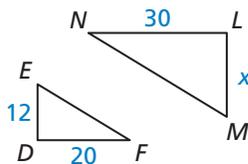
1
1

Stage 0   Stage 1   Stage 2

Stage 3   Stage 4

25. USING STRUCTURE $\triangle TUV$ is a 30°-60°-90° triangle, where two vertices are $U(3, -1)$ and $V(-3, -1)$, $\overline{UV}$ is the hypotenuse, and point T is in Quadrant I. Find the coordinates of T.

Maintaining Mathematical Proficiency   Reviewing what you learned in previous grades and lessons

Find the value of x. *(Section 8.1)*

26. $\triangle DEF \sim \triangle LMN$

N   30   L
E
x
12
M
D   20   F

27. $\triangle ABC \sim \triangle QRS$

S   4   Q
B x
3.5   R
A   7   C