

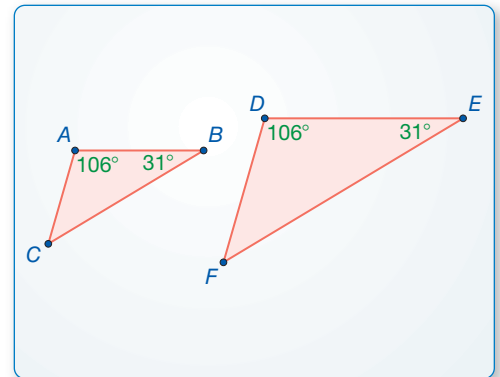
8.2 Proving Triangle Similarity by AA

Essential Question What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?

EXPLORATION 1 Comparing Triangles

Work with a partner. Use dynamic geometry software.

- a. Construct $\triangle ABC$ and $\triangle DEF$ so that $m\angle A = m\angle D = 106^\circ$, $m\angle B = m\angle E = 31^\circ$, and $\triangle DEF$ is not congruent to $\triangle ABC$.



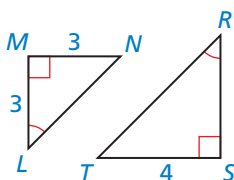
- b. Find the third angle measure and the side lengths of each triangle. Copy the table below and record your results in column 1.

	1.	2.	3.	4.	5.	6.
$m\angle A, m\angle D$	106°	88°	40°			
$m\angle B, m\angle E$	31°	42°	65°			
$m\angle C$						
$m\angle F$						
AB						
DE						
BC						
EF						
AC						
DF						

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results in constructing arguments.

- c. Are the two triangles similar? Explain.
- d. Repeat parts (a)–(c) to complete columns 2 and 3 of the table for the given angle measures.
- e. Complete each remaining column of the table using your own choice of two pairs of equal corresponding angle measures. Can you construct two triangles in this way that are *not* similar?
- f. Make a conjecture about any two triangles with two pairs of congruent corresponding angles.



Communicate Your Answer

2. What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?
3. Find RS in the figure at the left.

8.2 Lesson

Core Vocabulary

Previous
similar figures
similarity transformation

What You Will Learn

- ▶ Use the Angle-Angle Similarity Theorem.
- ▶ Solve real-life problems.

Using the Angle-Angle Similarity Theorem

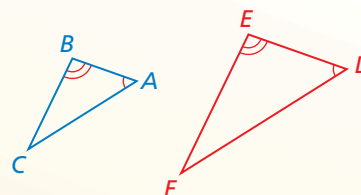
Theorem

Theorem 8.3 Angle-Angle (AA) Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$,
then $\triangle ABC \sim \triangle DEF$.

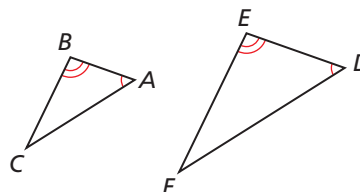
Proof p. 428



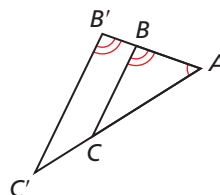
PROOF Angle-Angle (AA) Similarity Theorem

Given $\angle A \cong \angle D$, $\angle B \cong \angle E$

Prove $\triangle ABC \sim \triangle DEF$



Dilate $\triangle ABC$ using a scale factor of $k = \frac{DE}{AB}$ and center A . The image of $\triangle ABC$ is $\triangle AB'C'$.



Because a dilation is a similarity transformation, $\triangle ABC \sim \triangle AB'C'$. Because the ratio of corresponding lengths of similar polygons equals the scale factor, $\frac{AB'}{AB} = \frac{DE}{AB}$.

Multiplying each side by AB yields $AB' = DE$. By the definition of congruent segments, $\overline{AB'} \cong \overline{DE}$.

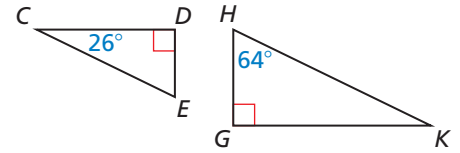
By the Reflexive Property of Congruence (Theorem 2.2), $\angle A \cong \angle A$. Because corresponding angles of similar polygons are congruent, $\angle B' \cong \angle B$. Because $\angle B' \cong \angle B$ and $\angle B \cong \angle E$, $\angle B' \cong \angle E$ by the Transitive Property of Congruence (Theorem 2.2).

Because $\angle A \cong \angle D$, $\angle B' \cong \angle E$, and $\overline{AB'} \cong \overline{DE}$, $\triangle AB'C' \cong \triangle DEF$ by the ASA Congruence Theorem (Theorem 5.10). So, a composition of rigid motions maps $\triangle AB'C'$ to $\triangle DEF$.

Because a dilation followed by a composition of rigid motions maps $\triangle ABC$ to $\triangle DEF$, $\triangle ABC \sim \triangle DEF$.

EXAMPLE 1 Using the AA Similarity Theorem

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



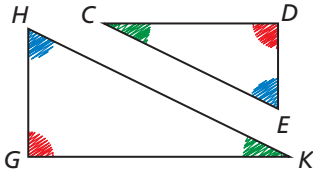
SOLUTION

Because they are both right angles, $\angle D$ and $\angle G$ are congruent.

By the Triangle Sum Theorem (Theorem 5.1), $26^\circ + 90^\circ + m\angle E = 180^\circ$, so $m\angle E = 64^\circ$. So, $\angle E$ and $\angle H$ are congruent.

► So, $\triangle CDE \sim \triangle KGH$ by the AA Similarity Theorem.

VISUAL REASONING

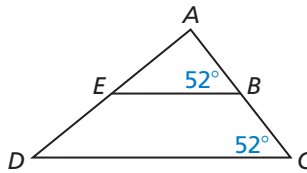


Use colored pencils to show congruent angles. This will help you write similarity statements.

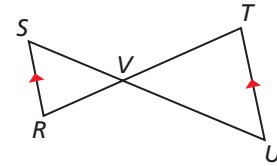
EXAMPLE 2 Using the AA Similarity Theorem

Show that the two triangles are similar.

a. $\triangle ABE \sim \triangle ACD$



b. $\triangle SVR \sim \triangle UVT$

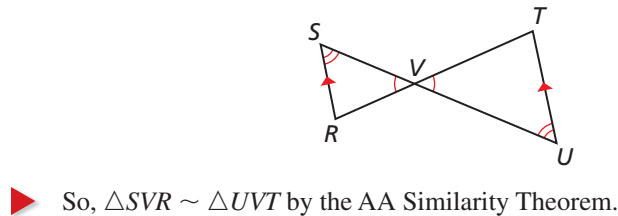


SOLUTION

a. Because $m\angle ABE$ and $m\angle C$ both equal 52° , $\angle ABE \cong \angle C$. By the Reflexive Property of Congruence (Theorem 2.2), $\angle A \cong \angle A$.

► So, $\triangle ABE \sim \triangle ACD$ by the AA Similarity Theorem.

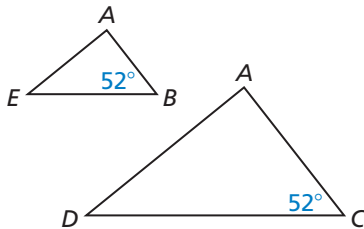
b. You know $\angle SVR \cong \angle UVT$ by the Vertical Angles Congruence Theorem (Theorem 2.6). The diagram shows $\overline{SR} \parallel \overline{UT}$, so $\angle S \cong \angle U$ by the Alternate Interior Angles Theorem (Theorem 3.2).



► So, $\triangle SVR \sim \triangle UVT$ by the AA Similarity Theorem.

VISUAL REASONING

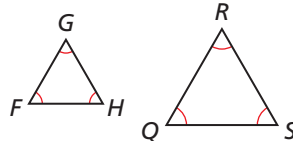
You may find it helpful to redraw the triangles separately.



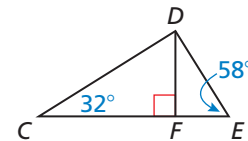
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Show that the triangles are similar. Write a similarity statement.

1. $\triangle FGH$ and $\triangle RQS$



2. $\triangle CDF$ and $\triangle DEF$



3. **WHAT IF?** Suppose that $\overline{SR} \not\parallel \overline{UT}$ in Example 2 part (b). Could the triangles still be similar? Explain.

Solving Real-Life Problems

Previously, you learned a way to use congruent triangles to find measurements indirectly. Another useful way to find measurements indirectly is by using similar triangles.

EXAMPLE 3 Modeling with Mathematics



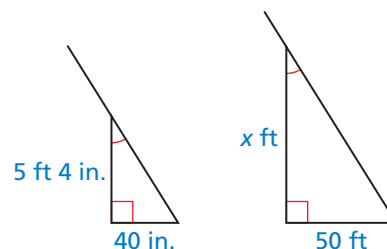
A flagpole casts a shadow that is 50 feet long. At the same time, a woman standing nearby who is 5 feet 4 inches tall casts a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?

SOLUTION

1. Understand the Problem You are given the length of a flagpole's shadow, the height of a woman, and the length of the woman's shadow. You need to find the height of the flagpole.

2. Make a Plan Use similar triangles to write a proportion and solve for the height of the flagpole.

3. Solve the Problem The flagpole and the woman form sides of two right triangles with the ground. The Sun's rays hit the flagpole and the woman at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Theorem.



You can use a proportion to find the height x . Write 5 feet 4 inches as 64 inches so that you can form two ratios of feet to inches.

$$\frac{x \text{ ft}}{64 \text{ in.}} = \frac{50 \text{ ft}}{40 \text{ in.}}$$

Write proportion of side lengths.

$$40x = 3200$$

Cross Products Property

$$x = 80$$

Solve for x .

► The flagpole is 80 feet tall.

4. Look Back Attend to precision by checking that your answer has the correct units. The problem asks for the height of the flagpole to the nearest *foot*. Because your answer is 80 feet, the units match.

Also, check that your answer is reasonable in the context of the problem. A height of 80 feet makes sense for a flagpole. You can estimate that an eight-story building would be about $8(10 \text{ feet}) = 80$ feet, so it is reasonable that a flagpole could be that tall.

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- WHAT IF?** A child who is 58 inches tall is standing next to the woman in Example 3. How long is the child's shadow?
- You are standing outside, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree.

8.2 Exercises

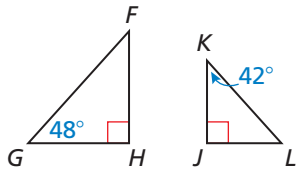
Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** If two angles of one triangle are congruent to two angles of another triangle, then the triangles are _____.
- WRITING** Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent? Explain.

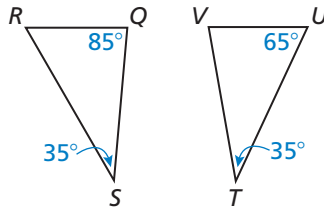
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning. (See Example 1.)

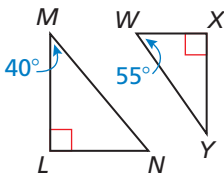
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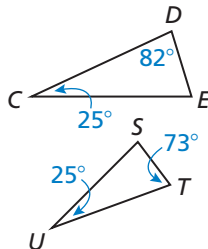
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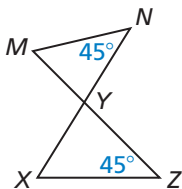


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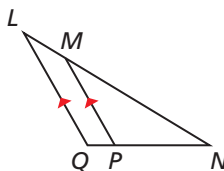


In Exercises 7–10, show that the two triangles are similar. (See Example 2.)

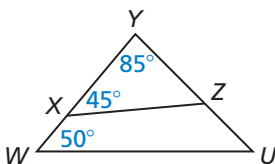
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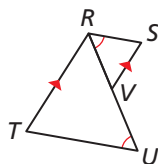
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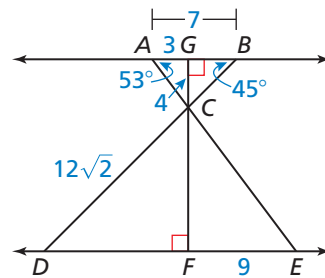
9.



10.



In Exercises 11–18, use the diagram to copy and complete the statement.



- | | |
|---|---|
| 11. $\triangle CAG \sim$ <input type="text"/> | 12. $\triangle DCF \sim$ <input type="text"/> |
| 13. $\triangle ACB \sim$ <input type="text"/> | 14. $m\angle ECF =$ <input type="text"/> |
| 15. $m\angle ECD =$ <input type="text"/> | 16. $CF =$ <input type="text"/> |
| 17. $BC =$ <input type="text"/> | 18. $DE =$ <input type="text"/> |

19. **ERROR ANALYSIS** Describe and correct the error in using the AA Similarity Theorem (Theorem 8.3).

X

Quadrilateral $ABCD \sim$ quadrilateral $EFGH$ by the AA Similarity Theorem.

20. **ERROR ANALYSIS** Describe and correct the error in finding the value of x .

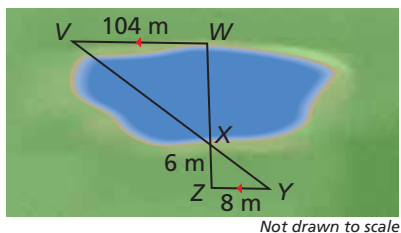
X

$$\frac{4}{6} = \frac{5}{x}$$

$$4x = 30$$

$$x = 7.5$$

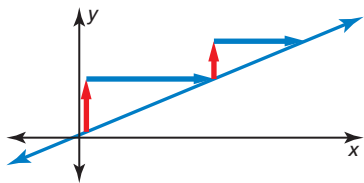
21. **MODELING WITH MATHEMATICS** You can measure the width of the lake using a surveying technique, as shown in the diagram. Find the width of the lake, WX . Justify your answer.



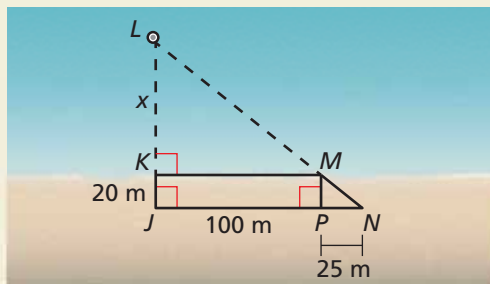
22. **MAKING AN ARGUMENT** You and your cousin are trying to determine the height of a telephone pole. Your cousin tells you to stand in the pole's shadow so that the tip of your shadow coincides with the tip of the pole's shadow. Your cousin claims to be able to use the distance between the tips of the shadows and you, the distance between you and the pole, and your height to estimate the height of the telephone pole. Is this possible? Explain. Include a diagram in your answer.

REASONING In Exercises 23–26, is it possible for $\triangle JKL$ and $\triangle XYZ$ to be similar? Explain your reasoning.

23. $m\angle J = 71^\circ$, $m\angle K = 52^\circ$, $m\angle X = 71^\circ$, and $m\angle Z = 57^\circ$
24. $\triangle JKL$ is a right triangle and $m\angle X + m\angle Y = 150^\circ$.
25. $m\angle L = 87^\circ$ and $m\angle Y = 94^\circ$
26. $m\angle J + m\angle K = 85^\circ$ and $m\angle Y + m\angle Z = 80^\circ$
27. **MATHEMATICAL CONNECTIONS** Explain how you can use similar triangles to show that any two points on a line can be used to find its slope.



28. **HOW DO YOU SEE IT?** In the diagram, which triangles would you use to find the distance x between the shoreline and the buoy? Explain your reasoning.

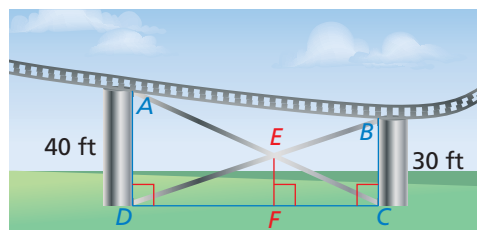


29. **WRITING** Explain why all equilateral triangles are similar.

30. **THOUGHT PROVOKING** Decide whether each is a valid method of showing that two quadrilaterals are similar. Justify your answer.

- a. AAA b. AAAA

31. **PROOF** Without using corresponding lengths in similar polygons, prove that the ratio of two corresponding angle bisectors in similar triangles is equal to the scale factor.
32. **PROOF** Prove that if the lengths of two sides of a triangle are a and b , respectively, then the lengths of the corresponding altitudes to those sides are in the ratio $\frac{b}{a}$.
33. **MODELING WITH MATHEMATICS** A portion of an amusement park ride is shown. Find EF . Justify your answer.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether there is enough information to prove that the triangles are congruent. Explain your reasoning. (*Section 5.3, Section 5.5, and Section 5.6*)

