# 7.1 Angles of Polygons

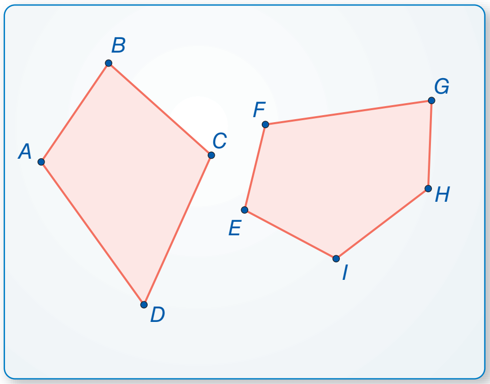
**Essential Question** What is the sum of the measures of the interior angles of a polygon?

## EXPLORATION 1 The Sum of the Angle Measures of a Polygon

**Work with a partner.** Use dynamic geometry software.

**a.** Draw a quadrilateral and a pentagon. Find the sum of the measures of the interior angles of each polygon.

**Sample**




**b.** Draw other polygons and find the sums of the measures of their interior angles. Record your results in the table below.

| Number of sides, $n$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| Sum of angle measures, $S$ | | | | | | | |

**c.** Plot the data from your table in a coordinate plane.

**d.** Write a function that fits the data. Explain what the function represents.

**CONSTRUCTING VIABLE ARGUMENTS**

To be proficient in math, you need to reason inductively about data.

## EXPLORATION 2 Measure of One Angle in a Regular Polygon

**Work with a partner.**

**a.** Use the function you found in Exploration 1 to write a new function that gives the measure of one interior angle in a regular polygon with $n$ sides.

**b.** Use the function in part (a) to find the measure of one interior angle of a regular pentagon. Use dynamic geometry software to check your result by constructing a regular pentagon and finding the measure of one of its interior angles.

**c.** Copy your table from Exploration 1 and add a row for the measure of one interior angle in a regular polygon with $n$ sides. Complete the table. Use dynamic geometry software to check your results.

## Communicate Your Answer

**3.** What is the sum of the measures of the interior angles of a polygon?

**4.** Find the measure of one interior angle in a regular dodecagon (a polygon with 12 sides).

# 7.1 Lesson

## Core Vocabulary

diagonal, *p. 360*
equilateral polygon, *p. 361*
equiangular polygon, *p. 361*
regular polygon, *p. 361*

***Previous***
polygon
convex
interior angles
exterior angles

## What You Will Learn

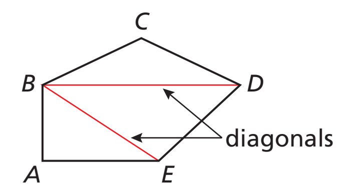
- Use the interior angle measures of polygons.
- Use the exterior angle measures of polygons.

## Using Interior Angle Measures of Polygons

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*. A **diagonal** of a polygon is a segment that joins two nonconsecutive vertices.

As you can see, the diagonals from one vertex divide a polygon into triangles. Dividing a polygon with $n$ sides into $(n - 2)$ triangles shows that the sum of the measures of the interior angles of a polygon is a multiple of 180°.

Polygon *ABCDE*



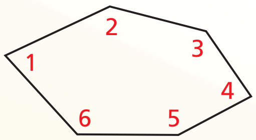

*A* and *B* are consecutive vertices.
Vertex *B* has two diagonals, $\overline{BD}$ and $\overline{BE}$.

## Theorem

**Theorem 7.1 Polygon Interior Angles Theorem**

The sum of the measures of the interior angles of a convex $n$-gon is $(n - 2) \cdot 180°$.

$$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180°$$




$n = 6$

*Proof* Ex. 42 (for pentagons), p. 365

**REMEMBER**

A polygon is *convex* when no line that contains a side of the polygon contains a point in the interior of the polygon.

### EXAMPLE 1 Finding the Sum of Angle Measures in a Polygon

Find the sum of the measures of the interior angles of the figure.

**SOLUTION**

The figure is a convex octagon. It has 8 sides. Use the Polygon Interior Angles Theorem.

| | | |
|---|---|---|
| $(n - 2) \cdot 180°$ | $= (8 - 2) \cdot 180°$ | Substitute 8 for *n*. |
| | $= 6 \cdot 180°$ | Subtract. |
| | $= 1080°$ | Multiply. |

▶ The sum of the measures of the interior angles of the figure is 1080°.

## Monitoring Progress

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**1.** The coin shown is in the shape of an 11-gon. Find the sum of the measures of the interior angles.

## EXAMPLE 2 Finding the Number of Sides of a Polygon

The sum of the measures of the interior angles of a convex polygon is 900°. Classify the polygon by the number of sides.

### SOLUTION

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides $n$. Then solve the equation to find the number of sides.

| | |
|---|---|
| $(n - 2) \cdot 180° = 900°$ | Polygon Interior Angles Theorem |
| $n - 2 = 5$ | Divide each side by 180°. |
| $n = 7$ | Add 2 to each side. |

▶ The polygon has 7 sides. It is a heptagon.

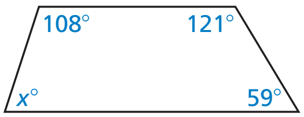
## Corollary

**Corollary 7.1 Corollary to the Polygon Interior Angles Theorem**

The sum of the measures of the interior angles of a quadrilateral is 360°.

*Proof* Ex. 43, p. 366

## EXAMPLE 3 Finding an Unknown Interior Angle Measure




Find the value of $x$ in the diagram.

### SOLUTION

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving $x$. Then solve the equation.

| | |
|---|---|
| $x° + 108° + 121° + 59° = 360°$ | Corollary to the Polygon Interior Angles Theorem |
| $x + 288 = 360$ | Combine like terms. |
| $x = 72$ | Subtract 288 from each side. |

▶ The value of $x$ is 72.

## Monitoring Progress

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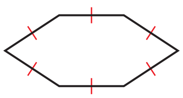
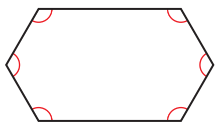
**2.** The sum of the measures of the interior angles of a convex polygon is 1440°. Classify the polygon by the number of sides.

**3.** The measures of the interior angles of a quadrilateral are $x°$, $3x°$, $5x°$, and $7x°$. Find the measures of all the interior angles.

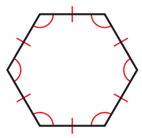
In an **equilateral polygon**, all sides are congruent.



In an **equiangular polygon**, all angles in the interior of the polygon are congruent.

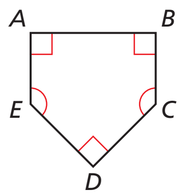


A **regular polygon** is a convex polygon that is both equilateral and equiangular.



## EXAMPLE 4 Finding Angle Measures in Polygons

A home plate for a baseball field is shown.




**a.** Is the polygon regular? Explain your reasoning.

**b.** Find the measures of $\angle C$ and $\angle E$.

### SOLUTION

**a.** The polygon is not equilateral or equiangular. So, the polygon is not regular.

**b.** Find the sum of the measures of the interior angles.

$(n - 2) \cdot 180^\circ = (5 - 2) \cdot 180^\circ = 540^\circ$ — Polygon Interior Angles Theorem

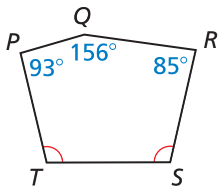
Then write an equation involving $x$ and solve the equation.

$x^\circ + x^\circ + 90^\circ + 90^\circ + 90^\circ = 540^\circ$ — Write an equation.

$2x + 270 = 540$ — Combine like terms.

$x = 135$ — Solve for $x$.

▶ So, $m\angle C = m\angle E = 135^\circ$.




### Monitoring Progress

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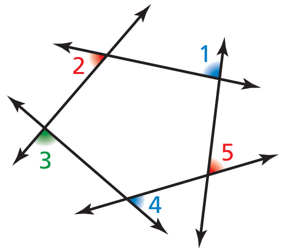
**4.** Find $m\angle S$ and $m\angle T$ in the diagram.

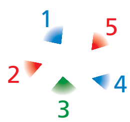
**5.** Sketch a pentagon that is equilateral but not equiangular.

## Using Exterior Angle Measures of Polygons

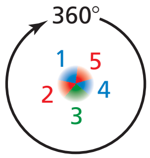
Unlike the sum of the interior angle measures of a convex polygon, the sum of the exterior angle measures does *not* depend on the number of sides of the polygon. The diagrams suggest that the sum of the measures of the exterior angles, one angle at each vertex, of a pentagon is 360°. In general, this sum is 360° for any convex polygon.




**Step 1** Shade one exterior angle at each vertex.



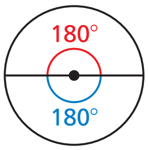

**Step 2** Cut out the exterior angles.




**Step 3** Arrange the exterior angles to form 360°.

### JUSTIFYING STEPS

To help justify this conclusion, you can visualize a circle containing two straight angles. So, there are 180° + 180°, or 360°, in a circle.



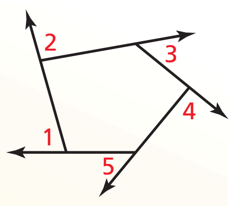

### Theorem

**Theorem 7.2 Polygon Exterior Angles Theorem**

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

$$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360^\circ$$

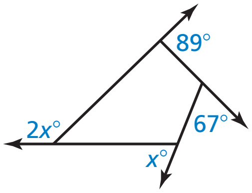
*Proof* Ex. 51, p. 366




$n = 5$

## EXAMPLE 5 Finding an Unknown Exterior Angle Measure

Find the value of $x$ in the diagram.




### SOLUTION

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$$x^\circ + 2x^\circ + 89^\circ + 67^\circ = 360^\circ$$ Polygon Exterior Angles Theorem

$$3x + 156 = 360$$ Combine like terms.

$$x = 68$$ Solve for $x$.

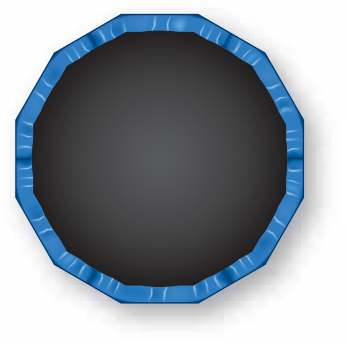
▶ The value of $x$ is 68.

**REMEMBER**

A *dodecagon* is a polygon with 12 sides and 12 vertices.

## EXAMPLE 6 Finding Angle Measures in Regular Polygons

The trampoline shown is shaped like a regular dodecagon.



**a.** Find the measure of each interior angle.

**b.** Find the measure of each exterior angle.

### SOLUTION

**a.** Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$$(n - 2) \cdot 180^\circ = (12 - 2) \cdot 180^\circ$$
$$= 1800^\circ$$

Then find the measure of one interior angle. A regular dodecagon has 12 congruent interior angles. Divide 1800° by 12.

$$\frac{1800^\circ}{12} = 150^\circ$$

▶ The measure of each interior angle in the dodecagon is 150°.

**b.** By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is 360°. Divide 360° by 12 to find the measure of one of the 12 congruent exterior angles.

$$\frac{360^\circ}{12} = 30^\circ$$

▶ The measure of each exterior angle in the dodecagon is 30°.

## Monitoring Progress

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**6.** A convex hexagon has exterior angles with measures 34°, 49°, 58°, 67°, and 75°. What is the measure of an exterior angle at the sixth vertex?

**7.** An interior angle and an adjacent exterior angle of a polygon form a linear pair. How can you use this fact as another method to find the measure of each exterior angle in Example 6?

# 7.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **VOCABULARY** Why do vertices connected by a diagonal of a polygon have to be nonconsecutive?

2. **WHICH ONE DOESN'T BELONG?** Which sum does *not* belong with the other three? Explain your reasoning.

| | |
|---|---|
| the sum of the measures of the interior angles of a quadrilateral | the sum of the measures of the exterior angles of a quadrilateral |
| the sum of the measures of the interior angles of a pentagon | the sum of the measures of the exterior angles of a pentagon |

## Monitoring Progress and Modeling with Mathematics

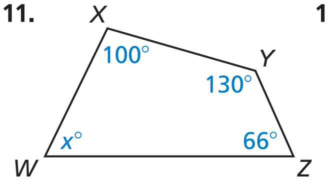
**In Exercises 3–6, find the sum of the measures of the interior angles of the indicated convex polygon.** *(See Example 1.)*

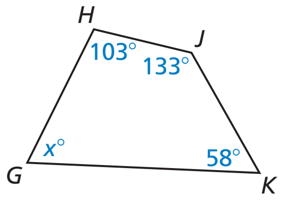
3. nonagon
4. 14-gon
5. 16-gon
6. 20-gon

**In Exercises 7–10, the sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.** *(See Example 2.)*

7. 720°
8. 1080°
9. 2520°
10. 3240°

**In Exercises 11–14, find the value of *x*.** *(See Example 3.)*

11.



12.
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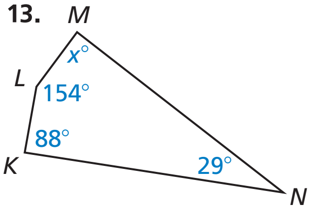
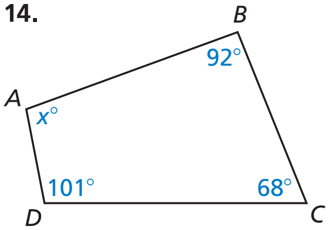
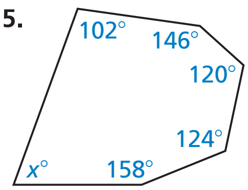



14.



**In Exercises 15–18, find the value of *x*.**

15.
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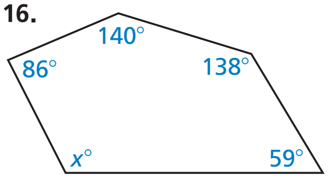


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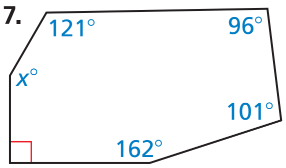
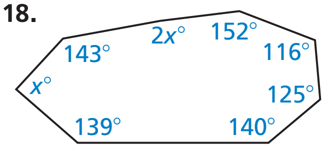
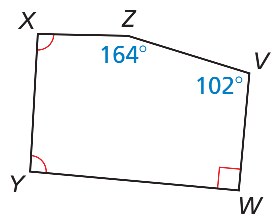



18.



**In Exercises 19–22, find the measures of ∠*X* and ∠*Y*.** *(See Example 4.)*

19.
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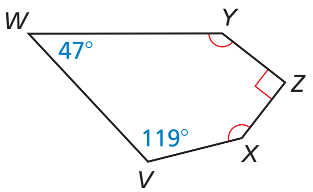


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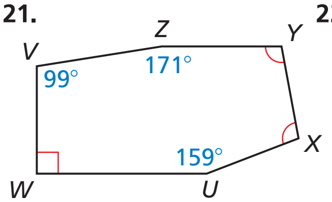


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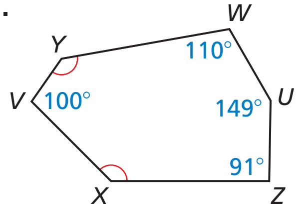
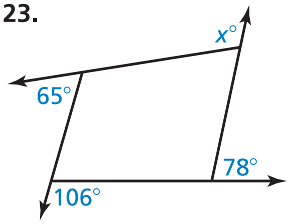
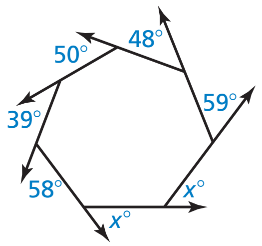


**In Exercises 23–26, find the value of $x$.** *(See Example 5.)*

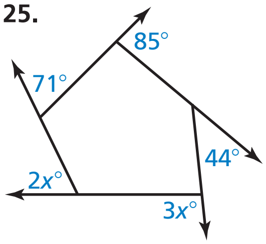
**23.**



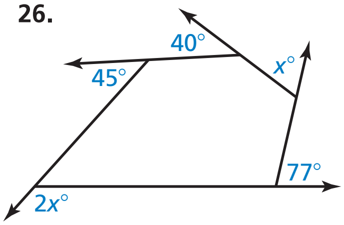

**24.**




**25.**




**26.**




**In Exercises 27–30, find the measure of each interior angle and each exterior angle of the indicated regular polygon.** *(See Example 6.)*

**27.** pentagon

**28.** 18-gon

**29.** 45-gon

**30.** 90-gon

**ERROR ANALYSIS** **In Exercises 31 and 32, describe and correct the error in finding the measure of one exterior angle of a regular pentagon.**

**31.**

$$(n-2)\cdot 180^\circ = (5-2)\cdot 180^\circ$$
$$= 3 \cdot 180^\circ$$
$$= 540^\circ$$

The sum of the measures of the angles is 540°. There are five angles, so the measure of one exterior angle is $\frac{540^\circ}{5} = 108^\circ$.

**32.**

There are a total of 10 exterior angles, two at each vertex, so the measure of one exterior angle is $\frac{360^\circ}{10} = 36^\circ$.

**33. MODELING WITH MATHEMATICS** The base of a jewelry box is shaped like a regular hexagon. What is the measure of each interior angle of the jewelry box base?

**34. MODELING WITH MATHEMATICS** The floor of the gazebo shown is shaped like a regular decagon. Find the measure of each interior angle of the regular decagon. Then find the measure of each exterior angle.

**35. WRITING A FORMULA** Write a formula to find the number of sides $n$ in a regular polygon given that the measure of one interior angle is $x°$.

**36. WRITING A FORMULA** Write a formula to find the number of sides $n$ in a regular polygon given that the measure of one exterior angle is $x°$.

**REASONING** **In Exercises 37–40, find the number of sides for the regular polygon described.**

**37.** Each interior angle has a measure of 156°.

**38.** Each interior angle has a measure of 165°.

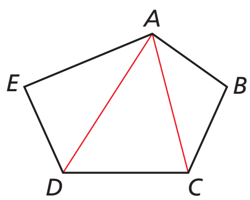
**39.** Each exterior angle has a measure of 9°.

**40.** Each exterior angle has a measure of 6°.

**41. DRAWING CONCLUSIONS** Which of the following angle measures are possible interior angle measures of a regular polygon? Explain your reasoning. Select all that apply.

Ⓐ 162° Ⓑ 171° Ⓒ 75° Ⓓ 40°

**42. PROVING A THEOREM** The Polygon Interior Angles Theorem (Theorem 7.1) states that the sum of the measures of the interior angles of a convex $n$-gon is $(n-2)\cdot 180°$. Write a paragraph proof of this theorem for the case when $n = 5$.



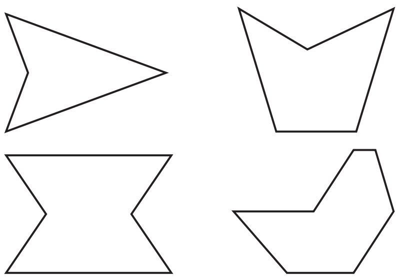
**43. PROVING A COROLLARY** Write a paragraph proof of the Corollary to the Polygon Interior Angles Theorem (Corollary 7.1).

**44. MAKING AN ARGUMENT** Your friend claims that to find the interior angle measures of a regular polygon, you do not have to use the Polygon Interior Angles Theorem (Theorem 7.1). You instead can use the Polygon Exterior Angles Theorem (Theorem 7.2) and then the Linear Pair Postulate (Postulate 2.8). Is your friend correct? Explain your reasoning.

**45. MATHEMATICAL CONNECTIONS** In an equilateral hexagon, four of the exterior angles each have a measure of $x°$. The other two exterior angles each have a measure of twice the sum of $x$ and 48. Find the measure of each exterior angle.

**46. THOUGHT PROVOKING** For a concave polygon, is it true that at least one of the interior angle measures must be greater than 180°? If not, give an example. If so, explain your reasoning.

**47. WRITING EXPRESSIONS** Write an expression to find the sum of the measures of the interior angles for a concave polygon. Explain your reasoning.



**48. ANALYZING RELATIONSHIPS** Polygon $ABCDEFGH$ is a regular octagon. Suppose sides $\overline{AB}$ and $\overline{CD}$ are extended to meet at a point $P$. Find $m\angle BPC$. Explain your reasoning. Include a diagram with your answer.

**49. MULTIPLE REPRESENTATIONS** The formula for the measure of each interior angle in a regular polygon can be written in function notation.

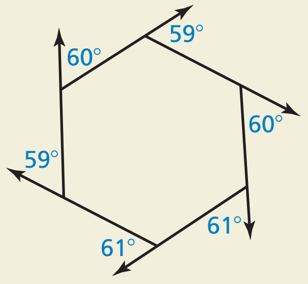
**a.** Write a function $h(n)$, where $n$ is the number of sides in a regular polygon and $h(n)$ is the measure of any interior angle in the regular polygon.

**b.** Use the function to find $h(9)$.

**c.** Use the function to find $n$ when $h(n) = 150°$.

**d.** Plot the points for $n = 3, 4, 5, 6, 7$, and 8. What happens to the value of $h(n)$ as $n$ gets larger?

**50. HOW DO YOU SEE IT?** Is the hexagon a regular hexagon? Explain your reasoning.




**51. PROVING A THEOREM** Write a paragraph proof of the Polygon Exterior Angles Theorem (Theorem 7.2). (*Hint:* In a convex $n$-gon, the sum of the measures of an interior angle and an adjacent exterior angle at any vertex is 180°.)

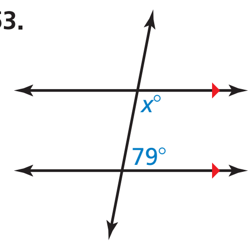
**52. ABSTRACT REASONING** You are given a convex polygon. You are asked to draw a new polygon by increasing the sum of the interior angle measures by 540°. How many more sides does your new polygon have? Explain your reasoning.

## Maintaining Mathematical Proficiency

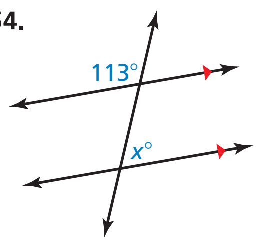
Reviewing what you learned in previous grades and lessons

**Find the value of $x$.** *(Section 3.2)*

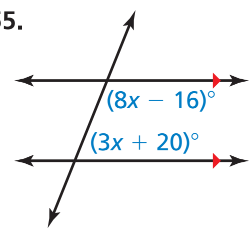
**53.**




**54.**




**55.**




**56.**