6.4 The Triangle Midsegment Theorem

**Essential Question** How are the midsegments of a triangle related to the sides of the triangle?

**Exploration 1** Midsegments of a Triangle

**Work with a partner.** Use dynamic geometry software. Draw any \( \triangle ABC \).

**a.** Plot midpoint \( D \) of \( AB \) and midpoint \( E \) of \( BC \). Draw \( DE \), which is a midsegment of \( \triangle ABC \).

![Diagram of triangleABC with midsegments DE and AC.]

**b.** Compare the slope and length of \( DE \) with the slope and length of \( AC \).

c. Write a conjecture about the relationships between the midsegments and sides of a triangle. Test your conjecture by drawing the other midsegments of \( \triangle ABC \), dragging vertices to change \( \triangle ABC \), and noting whether the relationships hold.

**Exploration 2** Midsegments of a Triangle

**Work with a partner.** Use dynamic geometry software. Draw any \( \triangle ABC \).

**a.** Draw all three midsegments of \( \triangle ABC \).

**b.** Use the drawing to write a conjecture about the triangle formed by the midsegments of the original triangle.

![Diagram of triangleABC with all midsegments drawn.]

**Communicate Your Answer**

3. How are the midsegments of a triangle related to the sides of the triangle?

4. In \( \triangle RST \), \( \overline{UV} \) is the midsegment connecting the midpoints of \( \overline{RS} \) and \( \overline{ST} \). Given \( UV = 12 \), find \( RT \).
What You Will Learn

- Use midsegments of triangles in the coordinate plane.
- Use the Triangle Midsegment Theorem to find distances.

### Using the Midsegment of a Triangle

A **midsegment of a triangle** is a segment that connects the midpoints of two sides of the triangle. Every triangle has three midsegments, which form the **midsegment triangle**.

The midsegments of \( \triangle ABC \) at the right are \( \overline{MP} \), \( \overline{MN} \), and \( \overline{NP} \). The **midsegment triangle** is \( \triangle MNP \).

### Using Midsegments in the Coordinate Plane

In \( \triangle JKL \), show that midsegment \( \overline{MN} \) is parallel to \( \overline{JL} \) and that \( MN = \frac{1}{2} JL \).

**SOLUTION**

**Step 1** Find the coordinates of \( M \) and \( N \) by finding the midpoints of \( \overline{JK} \) and \( \overline{KL} \).

\[
M \left( \frac{-6 + (-2)}{2}, \frac{1 + 5}{2} \right) = M \left( \frac{-8}{2}, \frac{6}{2} \right) = M(-4, 3)
\]

\[
N \left( \frac{-2 + 2}{2}, \frac{5 + (-1)}{2} \right) = N \left( \frac{0}{2}, \frac{4}{2} \right) = N(0, 2)
\]

**Step 2** Find and compare the slopes of \( \overline{MN} \) and \( \overline{JL} \).

\[
slope \overline{MN} = \frac{2 - 3}{0 - (-4)} = \frac{-1}{4}
\]

\[
slope \overline{JL} = \frac{-1 - 1}{2 - (-6)} = \frac{-2}{8} = \frac{-1}{4}
\]

Because the slopes are the same, \( MN \) is parallel to \( JL \).

**Step 3** Find and compare the lengths of \( MN \) and \( JL \).

\[
MN = \sqrt{(0 - (-4))^2 + (2 - 3)^2} = \sqrt{16 + 1} = \sqrt{17}
\]

\[
JL = \sqrt{(2 - (-6))^2 + (-1 - 1)^2} = \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17}
\]

Because \( \sqrt{17} = \frac{1}{2}(2\sqrt{17}) \), \( MN = \frac{1}{2} JL \).

### Monitoring Progress

Use the graph of \( \triangle ABC \).

1. In \( \triangle ABC \), show that midsegment \( \overline{DE} \) is parallel to \( \overline{AC} \) and that \( DE = \frac{1}{2} AC \).

2. Find the coordinates of the endpoints of midsegment \( \overline{EF} \), which is opposite \( \overline{AB} \). Show that \( \overline{EF} \) is parallel to \( \overline{AB} \) and that \( EF = \frac{1}{2} AB \).
Using the Triangle Midsegment Theorem

**Theorem**

**Theorem 6.8 Triangle Midsegment Theorem**

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

DE is a midsegment of \( \triangle ABC \), \( DE \parallel AC \), and \( DE = \frac{1}{2} AC \).

**Proof** Example 2, p. 331; Monitoring Progress Question 3, p. 331; Ex. 22, p. 334

![Diagram of Triangle Midsegment Theorem]

**STUDY TIP**

When assigning coordinates, try to choose coordinates that make some of the computations easier. In Example 2, you can avoid fractions by using \( 2p, 2q, \) and \( 2r \).

**Example 2** Proving the Triangle Midsegment Theorem

Write a coordinate proof of the Triangle Midsegment Theorem for one midsegment.

**Given** \( DE \) is a midsegment of \( \triangle OBC \).

**Prove** \( DE \parallel OC \) and \( DE = \frac{1}{2} OC \)

**SOLUTION**

Step 1 Place \( \triangle OBC \) in a coordinate plane and assign coordinates. Because you are finding midpoints, use \( 2p, 2q, \) and \( 2r \). Then find the coordinates of \( D \) and \( E \).

\[
D\left( \frac{2q + 0}{2}, \frac{2r + 0}{2} \right) = D(q, r) \quad E\left( \frac{2q + 2p}{2}, \frac{2r + 0}{2} \right) = E(q + p, r)
\]

Step 2 Prove \( DE \parallel OC \). The \( y \)-coordinates of \( D \) and \( E \) are the same, so \( DE \) has a slope of 0. \( OC \) is on the \( x \)-axis, so its slope is 0.

- Because their slopes are the same, \( DE \parallel OC \).

Step 3 Prove \( DE = \frac{1}{2} OC \). Use the Ruler Postulate (Post. 1.1) to find \( DE \) and \( OC \).

\[
DE = \left| (q + p) - q \right| = p \quad OC = \left| 2p - 0 \right| = 2p
\]

- Because \( p = \frac{1}{2}(2p) \), \( DE = \frac{1}{2} OC \).

**Monitoring Progress** Help in English and Spanish at BigIdeasMath.com

3. In Example 2, find the coordinates of \( F \), the midpoint of \( OC \). Show that \( FE \parallel OB \) and \( FE = \frac{1}{2} OB \).

**Example 3** Using the Triangle Midsegment Theorem

Triangles are used for strength in roof trusses. In the diagram, \( UV \) and \( VW \) are midsegments of \( \triangle RST \). Find \( UV \) and \( RS \).

**SOLUTION**

\[
UV = \frac{1}{2} \cdot RT = \frac{1}{2} (90 \text{ in.}) = 45 \text{ in.}
\]

\[
RS = 2 \cdot VW = 2(57 \text{ in.}) = 114 \text{ in.}
\]

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**EXAMPLE 4** Using the Triangle Midsegment Theorem

In the kaleidoscope image, \( \overline{AE} \equiv \overline{BE} \) and \( \overline{AD} \equiv \overline{CD} \).

Show that \( \overline{CB} \parallel \overline{DE} \).

**SOLUTION**

Because \( \overline{AE} \equiv \overline{BE} \) and \( \overline{AD} \equiv \overline{CD} \), \( E \) is the midpoint of \( \overline{AB} \) and \( D \) is the midpoint of \( \overline{AC} \) by definition. Then \( \overline{DE} \) is a midsegment of \( \triangle ABC \) by definition and \( \overline{CB} \parallel \overline{DE} \) by the Triangle Midsegment Theorem.

**EXAMPLE 5** Modeling with Mathematics

Pear Street intersects Cherry Street and Peach Street at their midpoints. Your home is at point \( P \). You leave your home and jog down Cherry Street to Plum Street, over Plum Street to Peach Street, up Peach Street to Pear Street, over Pear Street to Cherry Street, and then back home up Cherry Street. About how many miles do you jog?

**SOLUTION**

1. **Understand the Problem** You know the distances from your home to Plum Street along Peach Street, from Peach Street to Cherry Street along Plum Street, and from Pear Street to your home along Cherry Street. You need to find the other distances on your route, then find the total number of miles you jog.

2. **Make a Plan** By definition, you know that Pear Street is a midsegment of the triangle formed by the other three streets. Use the Triangle Midsegment Theorem to find the length of Pear Street and the definition of midsegment to find the length of Cherry Street. Then add the distances along your route.

3. **Solve the Problem**

   - length of Pear Street = \( \frac{1}{2} \times \) (length of Plum St.) = \( \frac{1}{2} \times 1.4 \) mi = 0.7 mi
   - length of Cherry Street = 2 \times \) (length from \( P \) to Pear St.) = 2(1.3 mi) = 2.6 mi
   - distance along your route: 2.6 + 1.4 + \( \frac{1}{2} \times 2.25 \) + 0.7 + 1.3 = 7.125

So, you jog about 7 miles.

4. **Look Back** Use compatible numbers to check that your answer is reasonable.

   total distance: 
   
   \[
   2.6 + 1.4 + \frac{1}{2} \times 2.25 + 0.7 + 1.3 = 2.5 + 1.5 + 1 + 0.5 + 1.5 = 7 \] ✓

**Monitoring Progress**

4. Copy the diagram in Example 3. Draw and name the third midsegment. Then find the length of \( \overline{VS} \) when the length of the third midsegment is 81 inches.

5. In Example 4, if \( F \) is the midpoint of \( \overline{CB} \), what do you know about \( \overline{DF} \)?

6. **WHAT IF?** In Example 5, you jog down Peach Street to Plum Street, over Plum Street to Cherry Street, up Cherry Street to Pear Street, over Pear Street to Peach Street, and then back home up Peach Street. Do you jog more miles in Example 5? Explain.
6.4 Exercises

Vocabulary and Core Concept Check

1. **VOCABULARY** The __________ of a triangle is a segment that connects the midpoints of two sides of the triangle.

2. **COMPLETE THE SENTENCE** If $DE$ is the midsegment opposite $AC$ in $\triangle ABC$, then $DE \parallel AC$ and $DE = \frac{1}{2}AC$ by the Triangle Midsegment Theorem (Thm. 6.8).

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, use the graph of $\triangle ABC$ with midsegments $DE, EF,$ and $DF$. (See Example 1.)

3. Find the coordinates of points $D, E,$ and $F$.

4. Show that $DE$ is parallel to $CB$ and that $DE = \frac{1}{2}CB$.

5. Show that $EF$ is parallel to $AC$ and that $EF = \frac{1}{2}AC$.

6. Show that $DF$ is parallel to $AB$ and that $DF = \frac{1}{2}AB$.

In Exercises 7–10, $\overline{DE}$ is a midsegment of $\triangle ABC$. Find the value of $x$. (See Example 3.)

7.

8.

9.

10.

In Exercises 11–16, $\overline{DE} \equiv \overline{JY}, \overline{YL} \equiv \overline{LZ},$ and $\overline{XK} \equiv \overline{KZ}$. Copy and complete the statement. (See Example 4.)

11. $\overline{JK} \parallel ___$

12. $\overline{JL} \parallel ___$

13. $\overline{XY} \parallel ___$

14. $\overline{JY} \equiv ___ \equiv ___$

15. $\overline{JL} \equiv ___ \equiv ___$

16. $\overline{JK} \equiv ___ \equiv ___$

MATHEMATICAL CONNECTIONS In Exercises 17–19, use $\triangle GHJ$, where $A, B,$ and $C$ are midpoints of the sides.

17. When $AB = 3x + 8$ and $GJ = 2x + 24$, what is $AB$?

18. When $AC = 3y - 5$ and $HJ = 4y + 2$, what is $HB$?

19. When $GH = 7z - 1$ and $CB = 4z - 3$, what is $GA$?

20. **ERROR ANALYSIS** Describe and correct the error.

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21. **MODELING WITH MATHEMATICS**  The distance between consecutive bases on a baseball field is 90 feet. A second baseman stands halfway between first base and second base, a shortstop stands halfway between second base and third base, and a pitcher stands halfway between first base and third base. Find the distance between the shortstop and the pitcher.  
*(See Example 5.)*

![Baseball Diagram]

22. **PROVING A THEOREM**  Use the figure from Example 2 to prove the Triangle Midsegment Theorem (Theorem 6.8) for midsegment $DF$, where $F$ is the midpoint of $OC$.  
*(See Example 2.)*

23. **CRITICAL THINKING**  $XY$ is a midsegment of $\triangle LMN$. Suppose $DE$ is called a “quarter segment” of $\triangle LMN$. What do you think an “eighth segment” would be? Make conjectures about the properties of a quarter segment and an eighth segment. Use variable coordinates to verify your conjectures.

![Coordinate Diagram]

24. **THOUGHT PROVOKING**  Find a real-life object that uses midsegments as part of its structure. Print a photograph of the object and identify the midsegments of one of the triangles in the structure.

25. **ABSTRACT REASONING**  To create the design shown, shade the triangle formed by the three midssegments of the triangle. Then repeat the process for each unshaded triangle.

![Design Diagram]

- a. What is the perimeter of the shaded triangle in Stage 1?
- b. What is the total perimeter of all the shaded triangles in Stage 2?
- c. What is the total perimeter of all the shaded triangles in Stage 3?

26. **HOW DO YOU SEE IT?**  Explain how you know that the yellow triangle is the midsegment triangle of the red triangle in the pattern of floor tiles shown.

![Floor Tile Diagram]

27. **ATTENDING TO PRECISION**  The points $P(2, 1)$, $Q(4, 5)$, and $R(7, 4)$ are the midpoints of the sides of a triangle. Graph the three midssegments. Then show how to use your graph and the properties of midsegments to draw the original triangle. Give the coordinates of each vertex.

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**Maintaining Mathematical Proficiency**

Reviewing what you learned in previous grades and lessons

Find a counterexample to show that the conjecture is false.  

(Section 2.2)

28. The difference of two numbers is always less than the greater number.

29. An isosceles triangle is always equilateral.