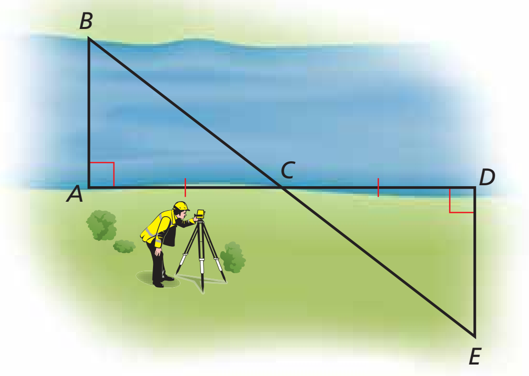
# 5.7 Using Congruent Triangles

## Essential Question

How can you use congruent triangles to make an indirect measurement?

### EXPLORATION 1 Measuring the Width of a River

**Work with a partner.** The figure shows how a surveyor can measure the width of a river by making measurements on only one side of the river.




**a.** Study the figure. Then explain how the surveyor can find the width of the river.

**b.** Write a proof to verify that the method you described in part (a) is valid.

**Given** $\angle A$ is a right angle, $\angle D$ is a right angle, $\overline{AC} \cong \overline{CD}$

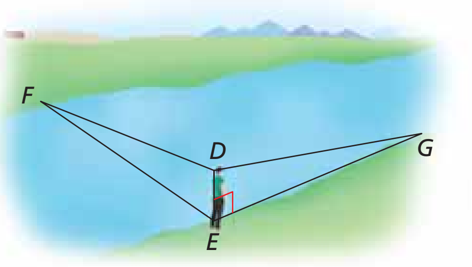
**c.** Exchange proofs with your partner and discuss the reasoning used.

**CRITIQUING THE REASONING OF OTHERS**

To be proficient in math, you need to listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### EXPLORATION 2 Measuring the Width of a River

**Work with a partner.** It was reported that one of Napoleon's officers estimated the width of a river as follows. The officer stood on the bank of the river and lowered the visor on his cap until the farthest thing visible was the edge of the bank on the other side. He then turned and noted the point on his side that was in line with the tip of his visor and his eye. The officer then paced the distance to this point and concluded that distance was the width of the river.




**a.** Study the figure. Then explain how the officer concluded that the width of the river is $EG$.

**b.** Write a proof to verify that the conclusion the officer made is correct.

**Given** $\angle DEG$ is a right angle, $\angle DEF$ is a right angle, $\angle EDG \cong \angle EDF$

**c.** Exchange proofs with your partner and discuss the reasoning used.

## Communicate Your Answer

**3.** How can you use congruent triangles to make an indirect measurement?

**4.** Why do you think the types of measurements described in Explorations 1 and 2 are called *indirect* measurements?

# 5.7 Lesson

## Core Vocabulary

***Previous***
congruent figures
corresponding parts
construction

## What You Will Learn

- Use congruent triangles.
- Prove constructions.

## Using Congruent Triangles

Congruent triangles have congruent corresponding parts. So, if you can prove that two triangles are congruent, then you know that their corresponding parts must be congruent as well.

### EXAMPLE 1 Using Congruent Triangles

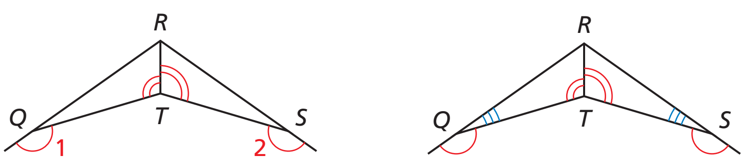
Explain how you can use the given information to prove that the hang glider parts are congruent.


**Given** $\angle 1 \cong \angle 2$, $\angle RTQ \cong \angle RTS$

**Prove** $\overline{QT} \cong \overline{ST}$

**SOLUTION**

If you can show that $\triangle QRT \cong \triangle SRT$, then you will know that $\overline{QT} \cong \overline{ST}$. First, copy the diagram and mark the given information. Then mark the information that you can deduce. In this case, $\angle RQT$ and $\angle RST$ are supplementary to congruent angles, so $\angle RQT \cong \angle RST$. Also, $\overline{RT} \cong \overline{RT}$ by the Reflexive Property of Congruence (Theorem 2.1).



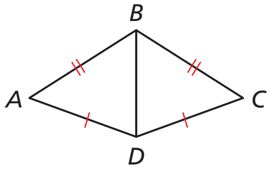

Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem (Theorem 5.11), $\triangle QRT \cong \triangle SRT$.

- Because corresponding parts of congruent triangles are congruent, $\overline{QT} \cong \overline{ST}$.

## Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

**1.** Explain how you can prove that $\angle A \cong \angle C$.



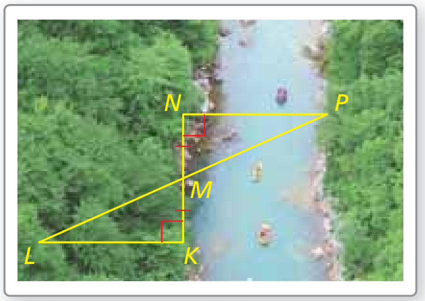
## EXAMPLE 2 Using Congruent Triangles for Measurement

### MAKING SENSE OF PROBLEMS

When you cannot easily measure a length directly, you can make conclusions about the length *indirectly*, usually by calculations based on known lengths.

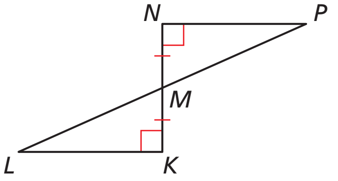
Use the following method to find the distance across a river, from point $N$ to point $P$.

- Place a stake at $K$ on the near side so that $\overline{NK} \perp \overline{NP}$.
- Find $M$, the midpoint of $\overline{NK}$.
- Locate the point $L$ so that $\overline{NK} \perp \overline{KL}$ and $L$, $P$, and $M$ are collinear.




Explain how this plan allows you to find the distance.

**SOLUTION**



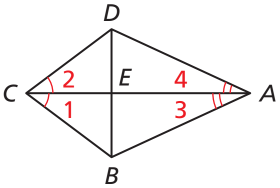

Because $\overline{NK} \perp \overline{NP}$ and $\overline{NK} \perp \overline{KL}$, $\angle N$ and $\angle K$ are congruent right angles. Because $M$ is the midpoint of $\overline{NK}$, $\overline{NM} \cong \overline{KM}$. The vertical angles $\angle KML$ and $\angle NMP$ are congruent. So, $\triangle MLK \cong \triangle MPN$ by the ASA Congruence Theorem (Theorem 5.10). Then because corresponding parts of congruent triangles are congruent, $\overline{KL} \cong \overline{NP}$. So, you can find the distance $NP$ across the river by measuring $KL$.

## EXAMPLE 3 Planning a Proof Involving Pairs of Triangles

Use the given information to write a plan for proof.

**Given** $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

**Prove** $\triangle BCE \cong \triangle DCE$




**SOLUTION**

In $\triangle BCE$ and $\triangle DCE$, you know that $\angle 1 \cong \angle 2$ and $\overline{CE} \cong \overline{CE}$. If you can show that $\overline{CB} \cong \overline{CD}$, then you can use the SAS Congruence Theorem (Theorem 5.5).

To prove that $\overline{CB} \cong \overline{CD}$, you can first prove that $\triangle CBA \cong \triangle CDA$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{CA} \cong \overline{CA}$ by the Reflexive Property of Congruence (Theorem 2.1). You can use the ASA Congruence Theorem (Theorem 5.10) to prove that $\triangle CBA \cong \triangle CDA$.

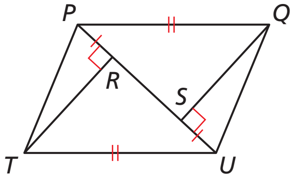
▶ **Plan for Proof** Use the ASA Congruence Theorem (Theorem 5.10) to prove that $\triangle CBA \cong \triangle CDA$. Then state that $\overline{CB} \cong \overline{CD}$. Use the SAS Congruence Theorem (Theorem 5.5) to prove that $\triangle BCE \cong \triangle DCE$.

### Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

**2.** In Example 2, does it matter how far from point $N$ you place a stake at point $K$? Explain.

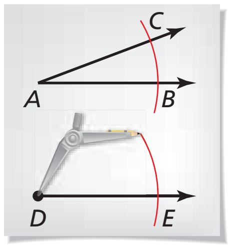
**3.** Write a plan to prove that $\triangle PTU \cong \triangle UQP$.



## Proving Constructions

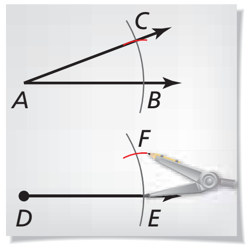
Recall that you can use a compass and a straightedge to copy an angle. The construction is shown below. You can use congruent triangles to prove that this construction is valid.

**Step 1**



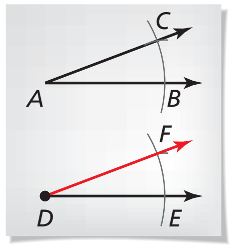

**Draw a segment and arcs**
To copy $\angle A$, draw a segment with initial point $D$. Draw an arc with center $A$. Using the same radius, draw an arc with center $D$. Label points $B$, $C$, and $E$.

**Step 2**




**Draw an arc**
Draw an arc with radius $BC$ and center $E$. Label the intersection $F$.

**Step 3**



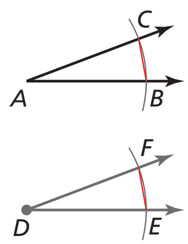

**Draw a ray**
Draw $\overrightarrow{DF}$. In Example 4, you will prove that $\angle D \cong \angle A$.

**EXAMPLE 4** **Proving a Construction**

Write a proof to verify that the construction for copying an angle is valid.

**SOLUTION**

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, one compass setting determines $\overline{AB}$, $\overline{DE}$, $\overline{AC}$, and $\overline{DF}$, and another compass setting determines $\overline{BC}$ and $\overline{EF}$. So, you can assume the following as given statements.




**Given** $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

**Prove** $\angle D \cong \angle A$

**Plan for Proof** Show that $\triangle DEF \cong \triangle ABC$, so you can conclude that the corresponding parts $\angle D$ and $\angle A$ are congruent.

**Plan in Action**

| STATEMENTS | REASONS |
|---|---|
| **1.** $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$ | **1.** Given |
| **2.** $\triangle DEF \cong \triangle ABC$ | **2.** SSS Congruence Theorem (Theorem 5.8) |
| **3.** $\angle D \cong \angle A$ | **3.** Corresponding parts of congruent triangles are congruent. |

## Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

**4.** Use the construction of an angle bisector on page 42. What segments can you assume are congruent?

# 5.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

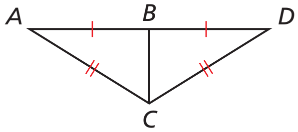
1. **COMPLETE THE SENTENCE** ______________ parts of congruent triangles are congruent.

2. **WRITING** Describe a situation in which you might choose to use indirect measurement with congruent triangles to find a measure rather than measuring directly.

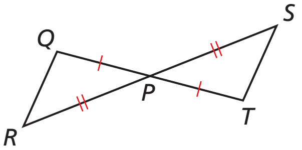
## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–8, explain how to prove that the statement is true.** *(See Example 1.)*

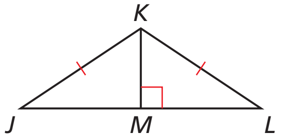
3. $\angle A \cong \angle D$



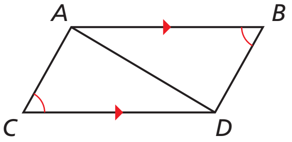

4. $\angle Q \cong \angle T$



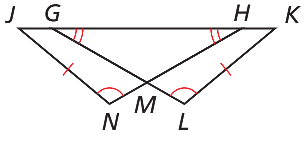

5. $\overline{JM} \cong \overline{LM}$



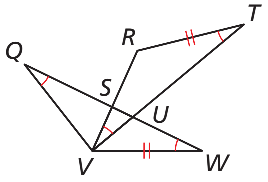

6. $\overline{AC} \cong \overline{DB}$




7. $\overline{GK} \cong \overline{HJ}$

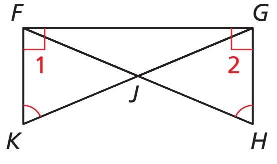



8. $\overline{QW} \cong \overline{VT}$

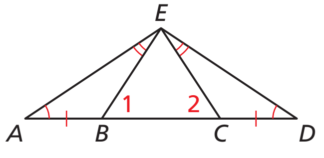



**In Exercises 9–12, write a plan to prove that $\angle 1 \cong \angle 2$.** *(See Example 3.)*

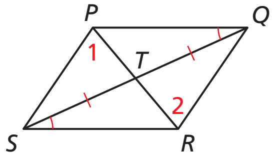
9.



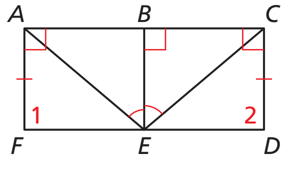

10.




11.

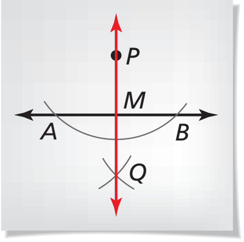



12.



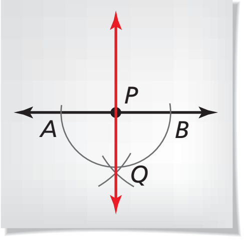

**In Exercises 13 and 14, write a proof to verify that the construction is valid.** *(See Example 4.)*

13. Line perpendicular to a line through a point not on the line




**Plan for Proof** Show that $\triangle APQ \cong \triangle BPQ$ by the SSS Congruence Theorem (Theorem 5.8). Then show that $\triangle APM \cong \triangle BPM$ using the SAS Congruence Theorem (Theorem 5.5). Use corresponding parts of congruent triangles to show that $\angle AMP$ and $\angle BMP$ are right angles.

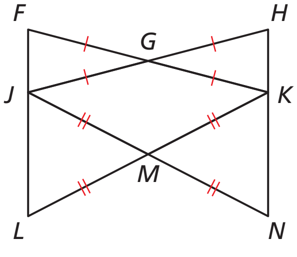
14. Line perpendicular to a line through a point on the line



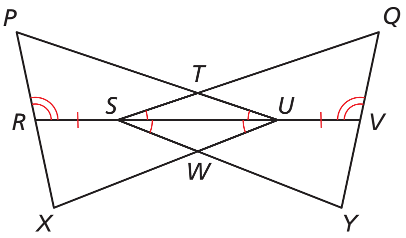

**Plan for Proof** Show that $\triangle APQ \cong \triangle BPQ$ by the SSS Congruence Theorem (Theorem 5.8). Use corresponding parts of congruent triangles to show that $\angle QPA$ and $\angle QPB$ are right angles.

**In Exercises 15 and 16, use the information given in the diagram to write a proof.**

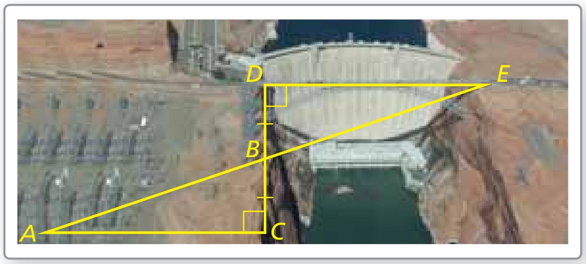
15. **Prove** $\overline{FL} \cong \overline{HN}$



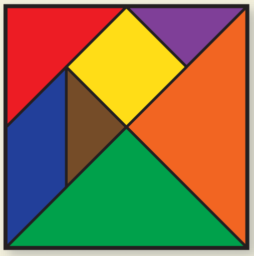
**16.** **Prove** $\triangle PUX \cong \triangle QSY$




**17.** **MODELING WITH MATHEMATICS** Explain how to find the distance across the canyon. *(See Example 2.)*



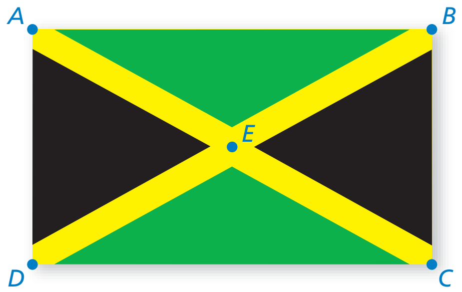

**18.** **HOW DO YOU SEE IT?** Use the tangram puzzle.



**a.** Which triangle(s) have an area that is twice the area of the purple triangle?

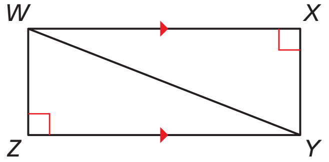
**b.** How many times greater is the area of the orange triangle than the area of the purple triangle?

**19.** **PROOF** Prove that the green triangles in the Jamaican flag are congruent if $\overline{AD} \parallel \overline{BC}$ and $E$ is the midpoint of $\overline{AC}$.




**20.** **THOUGHT PROVOKING** The Bermuda Triangle is a region in the Atlantic Ocean in which many ships and planes have mysteriously disappeared. The vertices are Miami, San Juan, and Bermuda. Use the Internet or some other resource to find the side lengths, the perimeter, and the area of this triangle (in miles). Then create a congruent triangle on land using cities as vertices.


**21.** **MAKING AN ARGUMENT** Your friend claims that $\triangle WZY$ can be proven congruent to $\triangle YXW$ using the HL Congruence Theorem (Thm. 5.9). Is your friend correct? Explain your reasoning.

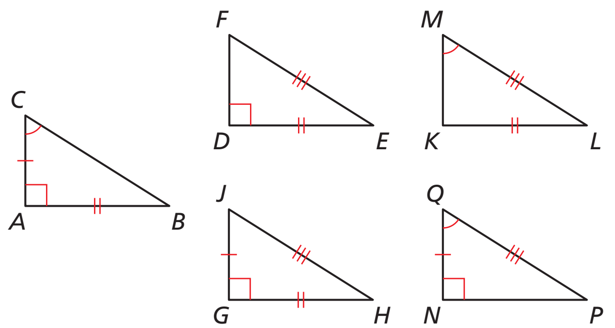



**22.** **CRITICAL THINKING** Determine whether each conditional statement is true or false. If the statement is false, rewrite it as a true statement using the converse, inverse, or contrapositive.

**a.** If two triangles have the same perimeter, then they are congruent.

**b.** If two triangles are congruent, then they have the same area.

**23.** **ATTENDING TO PRECISION** Which triangles are congruent to $\triangle ABC$? Select all that apply.




## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Find the perimeter of the polygon with the given vertices.** *(Section 1.4)*

**24.** $A(-1, 1)$, $B(4, 1)$, $C(4, -2)$, $D(-1, -2)$

**25.** $J(-5, 3)$, $K(-2, 1)$, $L(3, 4)$