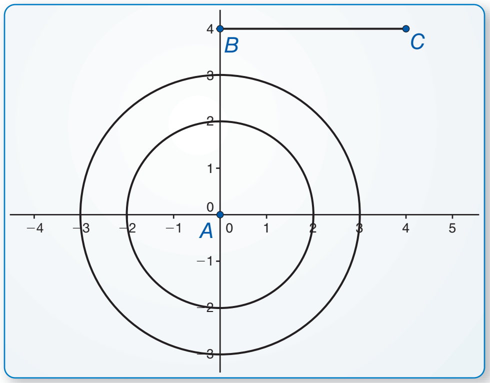
5.5 Proving Triangle Congruence by SSS

Essential Question What can you conclude about two triangles when you know the corresponding sides are congruent?

EXPLORATION 1 **Drawing Triangles**

Work with a partner. Use dynamic geometry software.

a. Construct circles with radii of 2 units and 3 units centered at the origin. Label the origin A. Then draw $\overline{BC}$ of length 4 units.

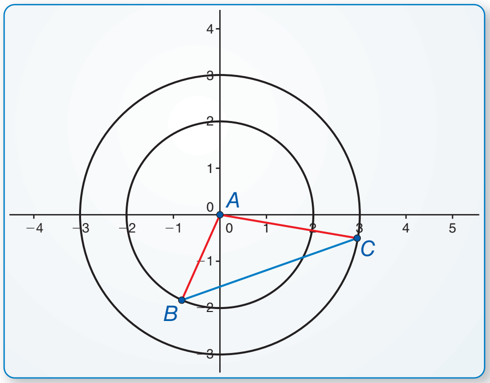


b. Move $\overline{BC}$ so that B is on the smaller circle and C is on the larger circle. Then draw $\triangle ABC$.

c. Explain why the side lengths of $\triangle ABC$ are 2, 3, and 4 units.

d. Find $m\angle A$, $m\angle B$, and $m\angle C$.

e. Repeat parts (b) and (d) several times, moving $\overline{BC}$ to different locations. Keep track of your results by copying and completing the table below. What can you conclude?



USING TOOLS STRATEGICALLY

To be proficient in math, you need to use technology to help visualize the results of varying assumptions, explore consequences, and compare predictions with data.

	A	*B*	*C*	*AB*	*AC*	*BC*	$m\angle A$	$m\angle B$	$m\angle C$
1.	(0, 0)			2	3	4			
2.	(0, 0)			2	3	4			
3.	(0, 0)			2	3	4			
4.	(0, 0)			2	3	4			
5.	(0, 0)			2	3	4			

Communicate Your Answer

2. What can you conclude about two triangles when you know the corresponding sides are congruent?

3. How would you prove your conclusion in Exploration 1(e)?

5.5 Lesson

Core Vocabulary

legs, *p. 264*
hypotenuse, *p. 264*

Previous
congruent figures
rigid motion

What You Will Learn

- Use the Side-Side-Side (SSS) Congruence Theorem.
- Use the Hypotenuse-Leg (HL) Congruence Theorem.

Using the Side-Side-Side Congruence Theorem

Theorem

Theorem 5.8 Side-Side-Side (SSS) Congruence Theorem

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

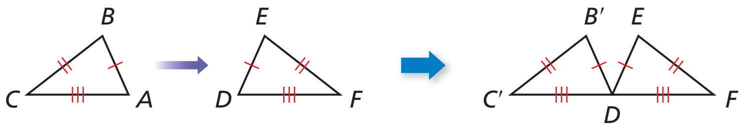
If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

PROOF Side-Side-Side (SSS) Congruence Theorem

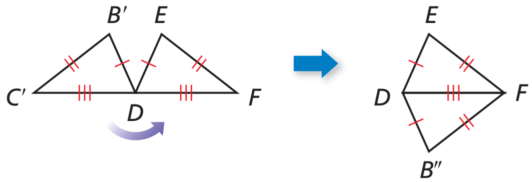
Given $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$

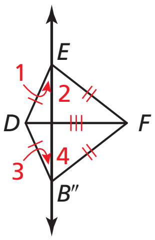
Prove $\triangle ABC \cong \triangle DEF$

First, translate $\triangle ABC$ so that point A maps to point D, as shown below.



This translation maps $\triangle ABC$ to $\triangle DB'C'$. Next, rotate $\triangle DB'C'$ counterclockwise through $\angle C'DF$ so that the image of $\overrightarrow{DC'}$ coincides with $\overrightarrow{DF}$, as shown below.





Because $\overline{DC'} \cong \overline{DF}$, the rotation maps point C' to point F. So, this rotation maps $\triangle DB'C'$ to $\triangle DB''F$. Draw an auxiliary line through points E and B''. This line creates $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$, as shown at the left.

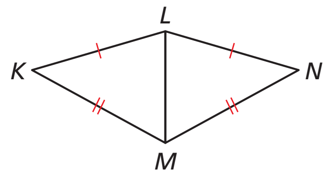
Because $\overline{DE} \cong \overline{DB''}$, $\triangle DEB''$ is an isosceles triangle. Because $\overline{FE} \cong \overline{FB''}$, $\triangle FEB''$ is an isosceles triangle. By the Base Angles Theorem (Thm. 5.6), $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. By the definition of congruence, $m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 4$. By construction, $m\angle DEF = m\angle 1 + m\angle 2$ and $m\angle DB''F = m\angle 3 + m\angle 4$. You can now use the Substitution Property of Equality to show $m\angle DEF = m\angle DB''F$.

$m\angle DEF = m\angle 1 + m\angle 2$	Angle Addition Postulate (Postulate 1.4)
$= m\angle 3 + m\angle 4$	Substitute $m\angle 3$ for $m\angle 1$ and $m\angle 4$ for $m\angle 2$.
$= m\angle DB''F$	Angle Addition Postulate (Postulate 1.4)

By the definition of congruence, $\angle DEF \cong \angle DB''F$. So, two pairs of sides and their included angles are congruent. By the SAS Congruence Theorem (Thm. 5.5), $\triangle DB''F \cong \triangle DEF$. So, a composition of rigid motions maps $\triangle DB''F$ to $\triangle DEF$. Because a composition of rigid motions maps $\triangle ABC$ to $\triangle DB''F$ and a composition of rigid motions maps $\triangle DB''F$ to $\triangle DEF$, a composition of rigid motions maps $\triangle ABC$ to $\triangle DEF$. So, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 1 Using the SSS Congruence Theorem

Write a proof.



Given $\overline{KL} \cong \overline{NL}$, $\overline{KM} \cong \overline{NM}$

Prove $\triangle KLM \cong \triangle NLM$

SOLUTION

	STATEMENTS	REASONS
S	**1.** $\overline{KL} \cong \overline{NL}$	**1.** Given
S	**2.** $\overline{KM} \cong \overline{NM}$	**2.** Given
S	**3.** $\overline{LM} \cong \overline{LM}$	**3.** Reflexive Property of Congruence (Thm. 2.1)
	4. $\triangle KLM \cong \triangle NLM$	**4.** SSS Congruence Theorem

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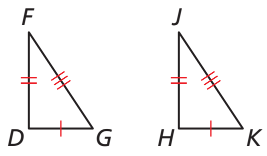
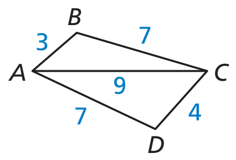
Decide whether the congruence statement is true. Explain your reasoning.

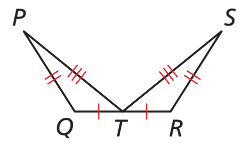
1. $\triangle DFG \cong \triangle HJK$



2. $\triangle ACB \cong \triangle CAD$

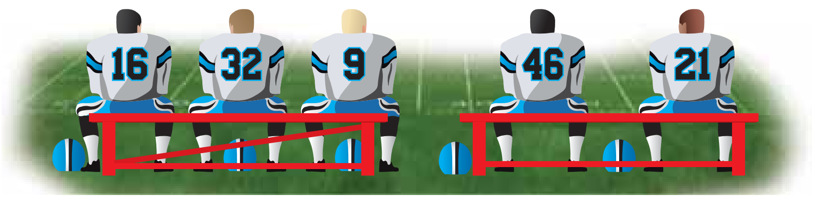


3. $\triangle QPT \cong \triangle RST$



EXAMPLE 2 Solving a Real-Life Problem

Explain why the bench with the diagonal support is stable, while the one without the support can collapse.



SOLUTION

The bench with the diagonal support forms triangles with fixed side lengths. By the SSS Congruence Theorem, these triangles cannot change shape, so the bench is stable. The bench without the diagonal support is not stable because there are many possible quadrilaterals with the given side lengths.

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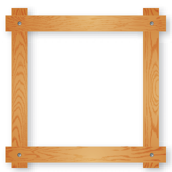
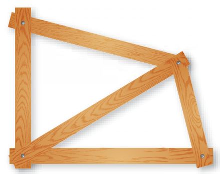
Determine whether the figure is stable. Explain your reasoning.

4.



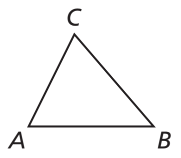
5.



6.

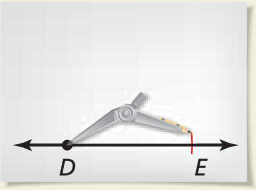
CONSTRUCTION Copying a Triangle Using SSS

Construct a triangle that is congruent to $\triangle ABC$ using the SSS Congruence Theorem. Use a compass and straightedge.



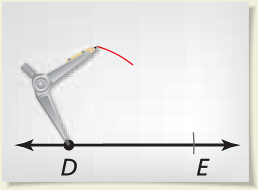
SOLUTION

Step 1



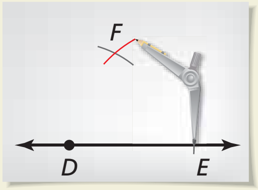
Construct a side
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

Step 2



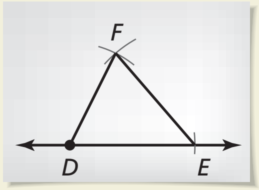
Draw an arc
Open your compass to the length AC. Use this length to draw an arc with center D.

Step 3



Draw an arc
Draw an arc with radius BC and center E that intersects the arc from Step 2. Label the intersection point F.

Step 4

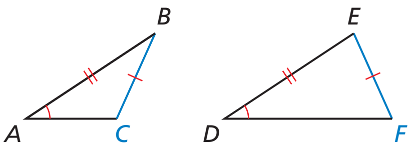


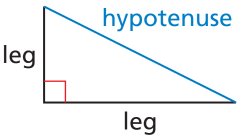
Draw a triangle
Draw $\triangle DEF$. By the SSS Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

Using the Hypotenuse-Leg Congruence Theorem

You know that SAS and SSS are valid methods for proving that triangles are congruent. What about SSA?

In general, SSA is *not* a valid method for proving that triangles are congruent. In the triangles below, two pairs of sides and a pair of angles not included between them are congruent, but the triangles are not congruent.





While SSA is not valid in general, there is a special case for right triangles.

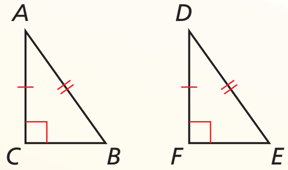
In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.

Theorem

Theorem 5.9 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $m\angle C = m\angle F = 90°$, then $\triangle ABC \cong \triangle DEF$.



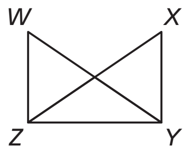
Proof Ex. 38, p. 470; *BigIdeasMath.com*

EXAMPLE 3 Using the Hypotenuse-Leg Congruence Theorem

Write a proof.

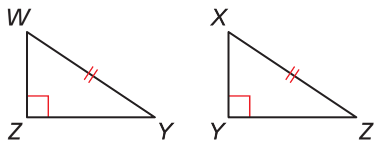
Given $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

Prove $\triangle WYZ \cong \triangle XZY$

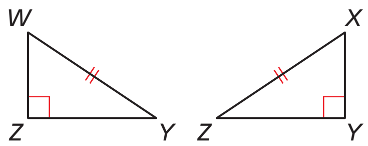


SOLUTION

Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.



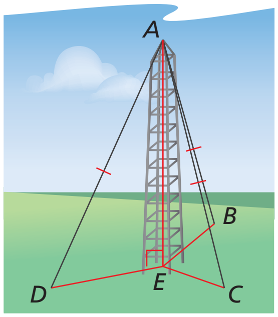
> **STUDY TIP**
>
> If you have trouble matching vertices to letters when you separate the overlapping triangles, leave the triangles in their original orientations.



STATEMENTS	REASONS
H **1.** $\overline{WY} \cong \overline{XZ}$	**1.** Given
2. $\overline{WZ} \perp \overline{ZY}, \overline{XY} \perp \overline{ZY}$	**2.** Given
3. $\angle Z$ and $\angle Y$ are right angles.	**3.** Definition of $\perp$ lines
4. $\triangle WYZ$ and $\triangle XZY$ are right triangles.	**4.** Definition of a right triangle
L **5.** $\overline{ZY} \cong \overline{YZ}$	**5.** Reflexive Property of Congruence (Thm. 2.1)
6. $\triangle WYZ \cong \triangle XZY$	**6.** HL Congruence Theorem

EXAMPLE 4 Using the Hypotenuse-Leg Congruence Theorem

The television antenna is perpendicular to the plane containing points B, C, D, and E. Each of the cables running from the top of the antenna to B, C, and D has the same length. Prove that $\triangle AEB$, $\triangle AEC$, and $\triangle AED$ are congruent.



Given $\overline{AE} \perp \overline{EB}$, $\overline{AE} \perp \overline{EC}$, $\overline{AE} \perp \overline{ED}$, $\overline{AB} \cong \overline{AC} \cong \overline{AD}$

Prove $\triangle AEB \cong \triangle AEC \cong \triangle AED$

SOLUTION

You are given that $\overline{AE} \perp \overline{EB}$ and $\overline{AE} \perp \overline{EC}$. So, $\angle AEB$ and $\angle AEC$ are right angles by the definition of perpendicular lines. By definition, $\triangle AEB$ and $\triangle AEC$ are right triangles. You are given that the hypotenuses of these two triangles, $\overline{AB}$ and $\overline{AC}$, are congruent. Also, $\overline{AE}$ is a leg for both triangles, and $\overline{AE} \cong \overline{AE}$ by the Reflexive Property of Congruence (Thm. 2.1). So, by the Hypotenuse-Leg Congruence Theorem, $\triangle AEB \cong \triangle AEC$. You can use similar reasoning to prove that $\triangle AEC \cong \triangle AED$.

► So, by the Transitive Property of Triangle Congruence (Thm. 5.3), $\triangle AEB \cong \triangle AEC \cong \triangle AED$.

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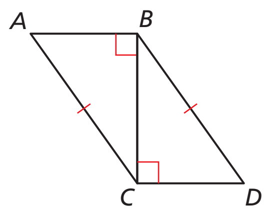
Use the diagram.



7. Redraw $\triangle ABC$ and $\triangle DCB$ side by side with corresponding parts in the same position.

8. Use the information in the diagram to prove that $\triangle ABC \cong \triangle DCB$.

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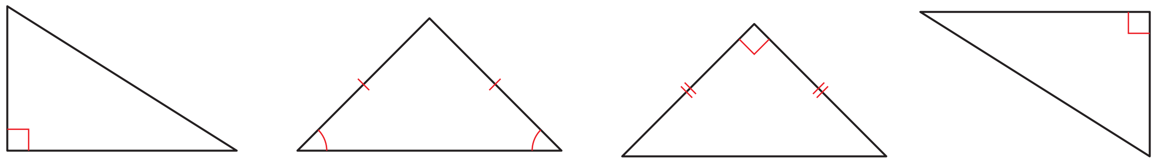
Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The side opposite the right angle is called the ___________ of the right triangle.

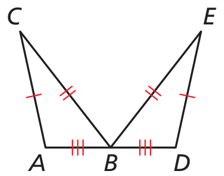
2. **WHICH ONE DOESN'T BELONG?** Which triangle's legs do *not* belong with the other three? Explain your reasoning.



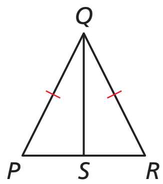
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, decide whether enough information is given to prove that the triangles are congruent using the SSS Congruence Theorem (Theorem 5.8). Explain.

3. $\triangle ABC$, $\triangle DBE$

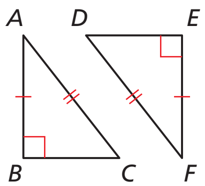


4. $\triangle PQS$, $\triangle RQS$

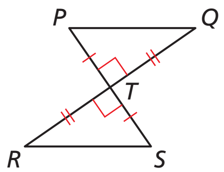


In Exercises 5 and 6, decide whether enough information is given to prove that the triangles are congruent using the HL Congruence Theorem (Theorem 5.9). Explain.

5. $\triangle ABC$, $\triangle FED$

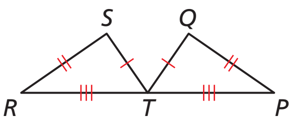


6. $\triangle PQT$, $\triangle SRT$

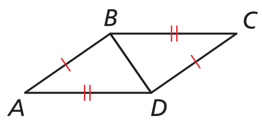


In Exercises 7–10, decide whether the congruence statement is true. Explain your reasoning. *(See Example 1.)*

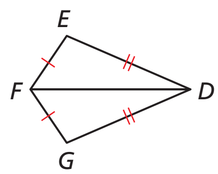
7. $\triangle RST \cong \triangle TQP$



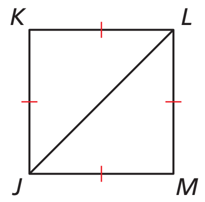
8. $\triangle ABD \cong \triangle CDB$



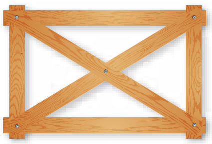
9. $\triangle DEF \cong \triangle DGF$

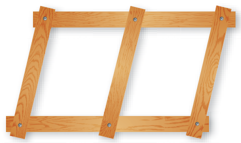


10. $\triangle JKL \cong \triangle LJM$



In Exercises 11 and 12, determine whether the figure is stable. Explain your reasoning. *(See Example 2.)*

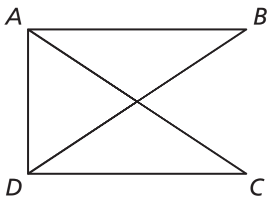
11. 

12. 

In Exercises 13 and 14, redraw the triangles so they are side by side with corresponding parts in the same position. Then write a proof. *(See Example 3.)*

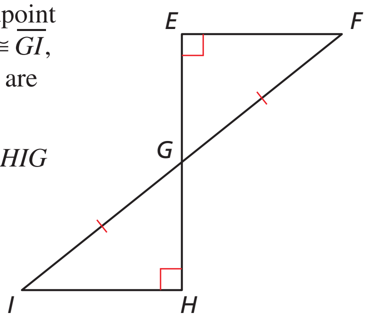
13. **Given** $\overline{AC} \cong \overline{BD}$, $\overline{AB} \perp \overline{AD}$, $\overline{CD} \perp \overline{AD}$

Prove $\triangle BAD \cong \triangle CDA$



14. **Given** G is the midpoint of $\overline{EH}$, $\overline{FG} \cong \overline{GI}$, $\angle E$ and $\angle H$ are right angles.

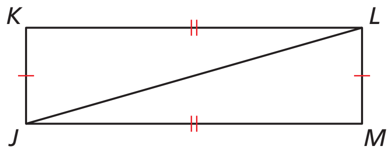
Prove $\triangle EFG \cong \triangle HIG$



In Exercises 15 and 16, write a proof.

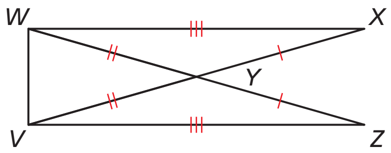
15. **Given** $\overline{LM} \cong \overline{JK}, \overline{MJ} \cong \overline{KL}$

Prove $\triangle LMJ \cong \triangle JKL$



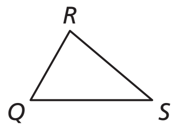
16. **Given** $\overline{WX} \cong \overline{VZ}, \overline{WY} \cong \overline{VY}, \overline{YZ} \cong \overline{YX}$

Prove $\triangle VWX \cong \triangle WVZ$

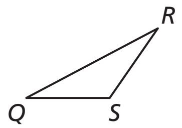


CONSTRUCTION **In Exercises 17 and 18, construct a triangle that is congruent to $\triangle QRS$ using the SSS Congruence Theorem (Theorem 5.8).**

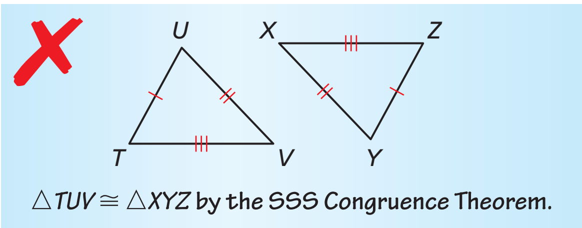
17.



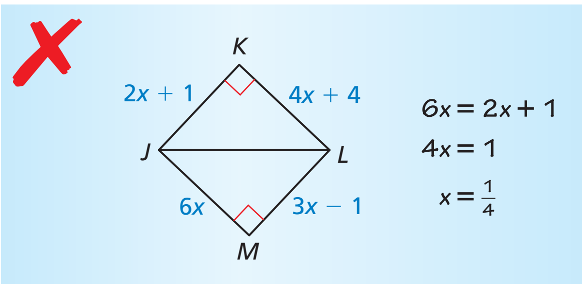
18.



19. **ERROR ANALYSIS** Describe and correct the error in identifying congruent triangles.

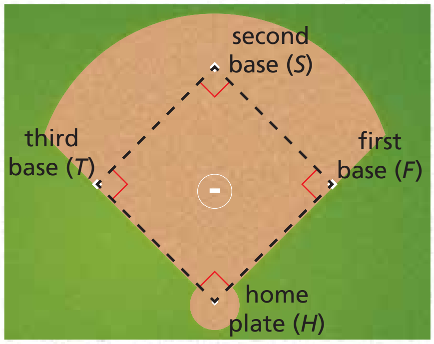


20. **ERROR ANALYSIS** Describe and correct the error in determining the value of x that makes the triangles congruent.

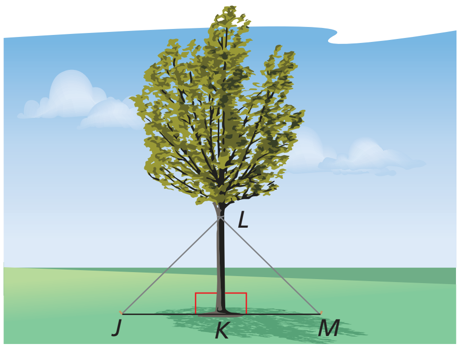


21. **MAKING AN ARGUMENT** Your friend claims that in order to use the SSS Congruence Theorem (Theorem 5.8) to prove that two triangles are congruent, both triangles must be equilateral triangles. Is your friend correct? Explain your reasoning.

22. **MODELING WITH MATHEMATICS** The distances between consecutive bases on a softball field are the same. The distance from home plate to second base is the same as the distance from first base to third base. The angles created at each base are 90°. Prove $\triangle HFS \cong \triangle FST \cong \triangle STH$. *(See Example 4.)*



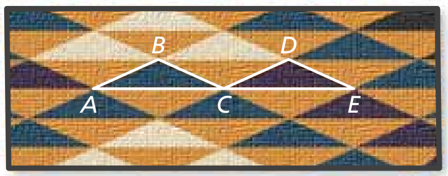
23. **REASONING** To support a tree, you attach wires from the trunk of the tree to stakes in the ground, as shown in the diagram.



a. What additional information do you need to use the HL Congruence Theorem (Theorem 5.9) to prove that $\triangle JKL \cong \triangle MKL$?

b. Suppose K is the midpoint of JM. Name a theorem you could use to prove that $\triangle JKL \cong \triangle MKL$. Explain your reasoning.

24. **REASONING** Use the photo of the Navajo rug, where $\overline{BC} \cong \overline{DE}$ and $\overline{AC} \cong \overline{CE}$.



a. What additional information do you need to use the SSS Congruence Theorem (Theorem 5.8) to prove that $\triangle ABC \cong \triangle CDE$?

b. What additional information do you need to use the HL Congruence Theorem (Theorem 5.9) to prove that $\triangle ABC \cong \triangle CDE$?

In Exercises 25–28, use the given coordinates to determine whether $\triangle ABC \cong \triangle DEF$.

25. $A(-2, -2)$, $B(4, -2)$, $C(4, 6)$, $D(5, 7)$, $E(5, 1)$, $F(13, 1)$

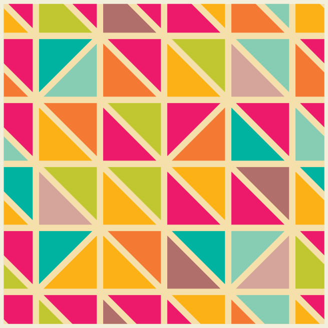
26. $A(-2, 1)$, $B(3, -3)$, $C(7, 5)$, $D(3, 6)$, $E(8, 2)$, $F(10, 11)$

27. $A(0, 0)$, $B(6, 5)$, $C(9, 0)$, $D(0, -1)$, $E(6, -6)$, $F(9, -1)$

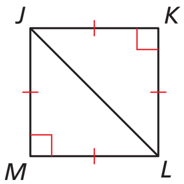
28. $A(-5, 7)$, $B(-5, 2)$, $C(0, 2)$, $D(0, 6)$, $E(0, 1)$, $F(4, 1)$

29. **CRITICAL THINKING** You notice two triangles in the tile floor of a hotel lobby. You want to determine whether the triangles are congruent, but you only have a piece of string. Can you determine whether the triangles are congruent? Explain.

30. **HOW DO YOU SEE IT?** There are several theorems you can use to show that the triangles in the "square" pattern are congruent. Name two of them.



31. **MAKING AN ARGUMENT** Your cousin says that $\triangle JKL$ is congruent to $\triangle LMJ$ by the SSS Congruence Theorem (Thm. 5.8). Your friend says that $\triangle JKL$ is congruent to $\triangle LMJ$ by the HL Congruence Theorem (Thm. 5.9). Who is correct? Explain your reasoning.



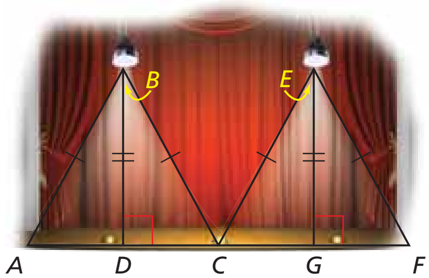
32. **THOUGHT PROVOKING** The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, do you think that two triangles are congruent if their corresponding sides are congruent? Justify your answer.

USING TOOLS **In Exercises 33 and 34, use the given information to sketch $\triangle LMN$ and $\triangle STU$. Mark the triangles with the given information.**

33. $\overline{LM} \perp \overline{MN}$, $\overline{ST} \perp \overline{TU}$, $\overline{LM} \cong \overline{NM} \cong \overline{UT} \cong \overline{ST}$

34. $\overline{LM} \perp \overline{MN}$, $\overline{ST} \perp \overline{TU}$, $\overline{LM} \cong \overline{ST}$, $\overline{LN} \cong \overline{SU}$

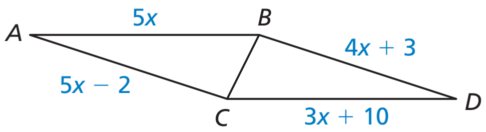
35. **CRITICAL THINKING** The diagram shows the light created by two spotlights. Both spotlights are the same distance from the stage.



a. Show that $\triangle ABD \cong \triangle CBD$. State which theorem or postulate you used and explain your reasoning.

b. Are all four right triangles shown in the diagram congruent? Explain your reasoning.

36. **MATHEMATICAL CONNECTIONS** Find all values of x that make the triangles congruent. Explain.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the congruent triangles. *(Section 5.2)*

37. Name the segment in $\triangle DEF$ that is congruent to $\overline{AC}$.

38. Name the segment in $\triangle ABC$ that is congruent to $\overline{EF}$.

39. Name the angle in $\triangle DEF$ that is congruent to $\angle B$.

40. Name the angle in $\triangle ABC$ that is congruent to $\angle F$.