5.1 Angles of Triangles

Essential Question How are the angle measures of a triangle related?

EXPLORATION 1 Writing a Conjecture

Work with a partner.

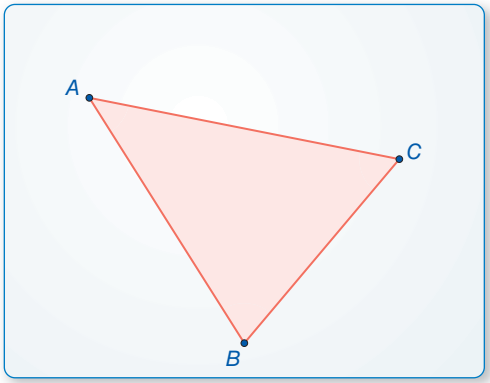
a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

b. Find the measures of the interior angles of the triangle.

c. Find the sum of the interior angle measures.

d. Repeat parts (a)–(c) with several other triangles. Then write a conjecture about the sum of the measures of the interior angles of a triangle.

CONSTRUCTING VIABLE ARGUMENTS
To be proficient in math, you need to reason inductively about data and write conjectures.



Sample

Angles

$m\angle A = 43.67°$

$m\angle B = 81.87°$

$m\angle C = 54.46°$

EXPLORATION 2 Writing a Conjecture

Work with a partner.

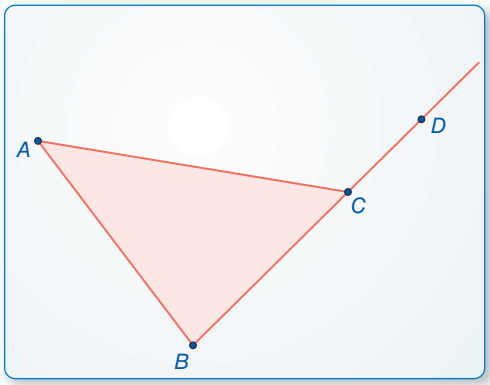
a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

b. Draw an exterior angle at any vertex and find its measure.

c. Find the measures of the two nonadjacent interior angles of the triangle.

d. Find the sum of the measures of the two nonadjacent interior angles. Compare this sum to the measure of the exterior angle.

e. Repeat parts (a)–(d) with several other triangles. Then write a conjecture that compares the measure of an exterior angle with the sum of the measures of the two nonadjacent interior angles.



Sample

Angles

$m\angle A = 43.67°$

$m\angle B = 81.87°$

$m\angle ACD = 125.54°$

Communicate Your Answer

3. How are the angle measures of a triangle related?

4. An exterior angle of a triangle measures 32°. What do you know about the measures of the interior angles? Explain your reasoning.

5.1 Lesson

Core Vocabulary

interior angles, *p. 233*
exterior angles, *p. 233*
corollary to a theorem, *p. 235*

Previous
triangle

What You Will Learn

- Classify triangles by sides and angles.
- Find interior and exterior angle measures of triangles.

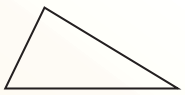
Classifying Triangles by Sides and by Angles

Recall that a *triangle* is a polygon with three sides. You can classify triangles by sides and by angles, as shown below.

Core Concept

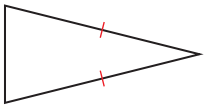
Classifying Triangles by Sides

Scalene Triangle



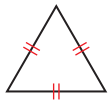
no congruent sides

Isosceles Triangle



at least 2 congruent sides

Equilateral Triangle



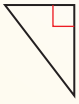
3 congruent sides

Classifying Triangles by Angles

Acute Triangle

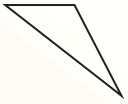
3 acute angles

Right Triangle



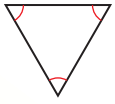
1 right angle

Obtuse Triangle



1 obtuse angle

Equiangular Triangle



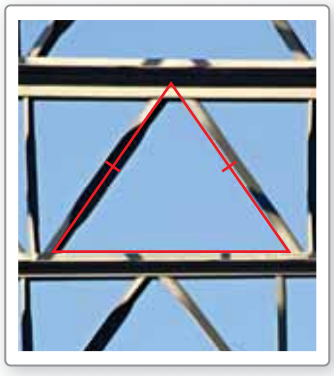
3 congruent angles

READING

Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

EXAMPLE 1 Classifying Triangles by Sides and by Angles

Classify the triangular shape of the support beams in the diagram by its sides and by measuring its angles.



SOLUTION

The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are 55°, 55°, and 70°.

- So, it is an acute isosceles triangle.

Monitoring Progress

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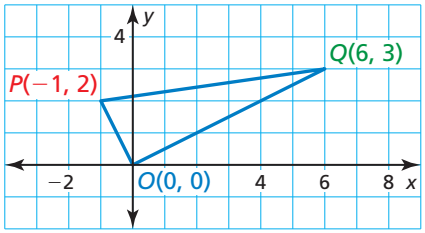
1. Draw an obtuse isosceles triangle and an acute scalene triangle.

EXAMPLE 2 Classifying a Triangle in the Coordinate Plane

Classify $\triangle OPQ$ by its sides. Then determine whether it is a right triangle.



SOLUTION

Step 1 Use the Distance Formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[6 - (-1)]^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

Because no sides are congruent, $\triangle OPQ$ is a scalene triangle.

Step 2 Check for right angles. The slope of $\overline{OP}$ is $\frac{2 - 0}{-1 - 0} = -2$. The slope of $\overline{OQ}$ is $\frac{3 - 0}{6 - 0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$. So, $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

▶ So, $\triangle OPQ$ is a right scalene triangle.

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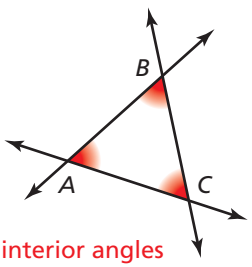
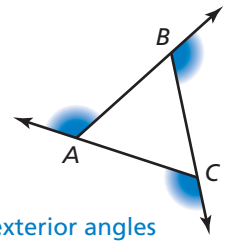
2. $\triangle ABC$ has vertices $A(0, 0)$, $B(3, 3)$, and $C(-3, 3)$. Classify the triangle by its sides. Then determine whether it is a right triangle.

Finding Angle Measures of Triangles

When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.



interior angles



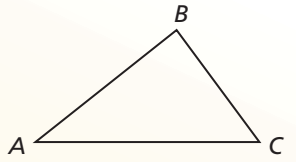
exterior angles

Theorem

Theorem 5.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

Proof p. 234; Ex. 53, p. 238



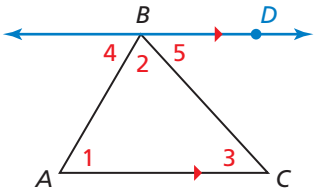
$m\angle A + m\angle B + m\angle C = 180°$

To prove certain theorems, you may need to add a line, a segment, or a ray to a given diagram. An *auxiliary* line is used in the proof of the Triangle Sum Theorem.

PROOF Triangle Sum Theorem

Given $\triangle ABC$

Prove $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$



Plan for Proof

a. Draw an auxiliary line through B that is parallel to $\overline{AC}$.

b. Show that $m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$, $\angle 1 \cong \angle 4$, and $\angle 3 \cong \angle 5$.

c. By substitution, $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$.

Plan in Action

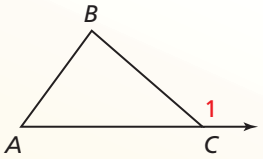
	STATEMENTS	REASONS
a.	**1.** Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	**1.** Parallel Postulate (Post. 3.1)
b.	**2.** $m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$	**2.** Angle Addition Postulate (Post. 1.4) and definition of straight angle
	3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	**3.** Alternate Interior Angles Theorem (Thm. 3.2)
	4. $m\angle 1 = m\angle 4$, $m\angle 3 = m\angle 5$	**4.** Definition of congruent angles
c.	**5.** $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$	**5.** Substitution Property of Equality

Theorem

Theorem 5.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

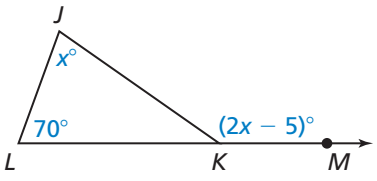
Proof Ex. 42, p. 237



$m\angle 1 = m\angle A + m\angle B$

EXAMPLE 3 Finding an Angle Measure

Find $m\angle JKM$.



SOLUTION

Step 1 Write and solve an equation to find the value of x.

$(2x - 5)^\circ = 70^\circ + x^\circ$ Apply the Exterior Angle Theorem.

$x = 75$ Solve for x.

Step 2 Substitute 75 for x in $2x - 5$ to find $m\angle JKM$.

$2x - 5 = 2 \cdot 75 - 5 = 145$

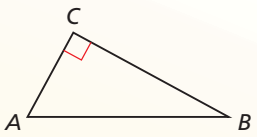
So, the measure of $\angle JKM$ is 145°.

A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

Corollary

Corollary 5.1 Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

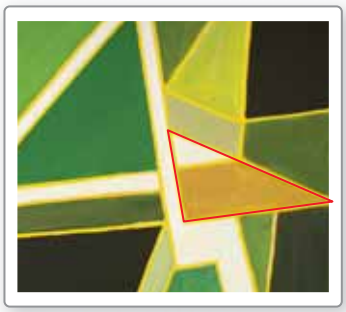


$m\angle A + m\angle B = 90°$

Proof Ex. 41, p. 237

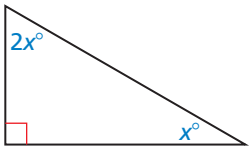
EXAMPLE 4 Modeling with Mathematics

In the painting, the red triangle is a right triangle. The measure of one acute angle in the triangle is twice the measure of the other. Find the measure of each acute angle.



SOLUTION

1. **Understand the Problem** You are given a right triangle and the relationship between the two acute angles in the triangle. You need to find the measure of each acute angle.



2. **Make a Plan** First, sketch a diagram of the situation. You can use the Corollary to the Triangle Sum Theorem and the given relationship between the two acute angles to write and solve an equation to find the measure of each acute angle.
3. **Solve the Problem** Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $2x°$. The Corollary to the Triangle Sum Theorem states that the acute angles of a right triangle are complementary.

   Use the corollary to set up and solve an equation.

   $x° + 2x° = 90°$ Corollary to the Triangle Sum Theorem

   $x = 30$ Solve for x.

   ▶ So, the measures of the acute angles are $30°$ and $2(30°) = 60°$.
4. **Look Back** Add the two angles and check that their sum satisfies the Corollary to the Triangle Sum Theorem.

   $30° + 60° = 90°$ ✓

Monitoring Progress

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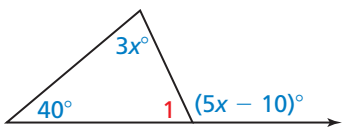
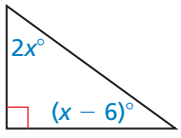
3. Find the measure of $\angle 1$.



4. Find the measure of each acute angle.



5.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

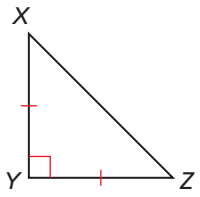
Vocabulary and Core Concept Check

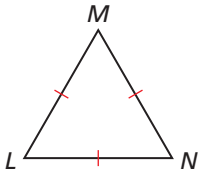
1. **WRITING** Can a right triangle also be obtuse? Explain your reasoning.

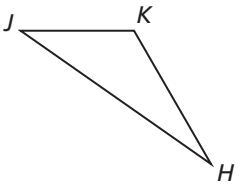
2. **COMPLETE THE SENTENCE** The measure of an exterior angle of a triangle is equal to the sum of the measures of the two _____ interior angles.

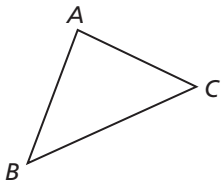
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, classify the triangle by its sides and by measuring its angles. *(See Example 1.)*

3. 

4. 

5. 

6. 

In Exercises 7–10, classify $\triangle ABC$ by its sides. Then determine whether it is a right triangle. *(See Example 2.)*

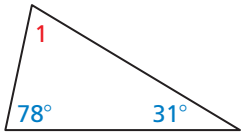
7. $A(2, 3), B(6, 3), C(2, 7)$

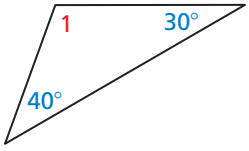
8. $A(3, 3), B(6, 9), C(6, -3)$

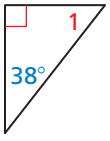
9. $A(1, 9), B(4, 8), C(2, 5)$

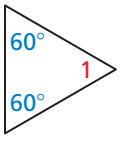
10. $A(-2, 3), B(0, -3), C(3, -2)$

In Exercises 11–14, find $m\angle 1$. Then classify the triangle by its angles.

11. 

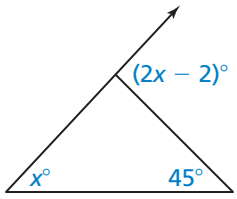
12. 

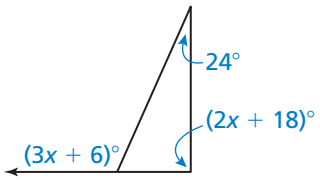
13. 

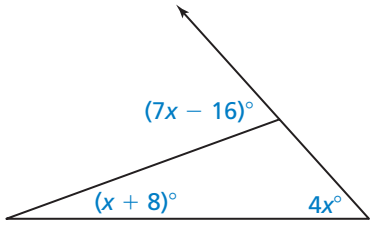
14. 

In Exercises 15–18, find the measure of the exterior angle. *(See Example 3.)*

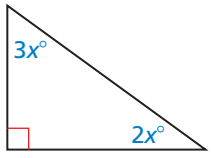
15. 

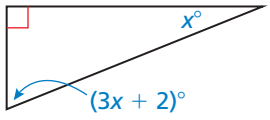
16. 

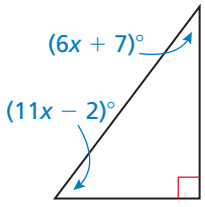
17. 

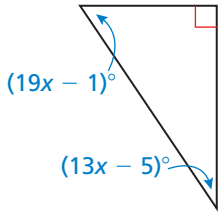
18. 

In Exercises 19–22, find the measure of each acute angle. *(See Example 4.)*

19. 

20. 

21. 

22. 

In Exercises 23–26, find the measure of each acute angle in the right triangle. *(See Example 4.)*

23. The measure of one acute angle is 5 times the measure of the other acute angle.

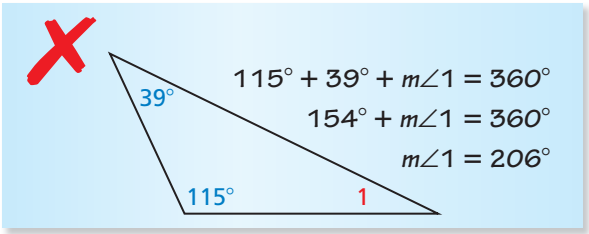
24. The measure of one acute angle is 8 times the measure of the other acute angle.

25. The measure of one acute angle is 3 times the sum of the measure of the other acute angle and 8.

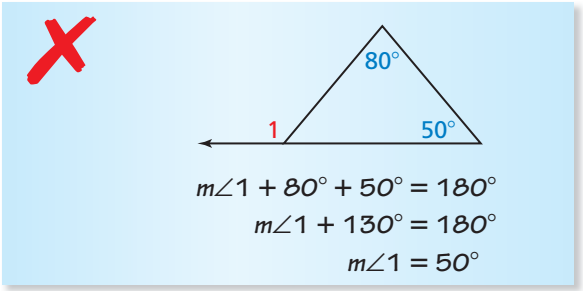
26. The measure of one acute angle is twice the difference of the measure of the other acute angle and 12.

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in finding $m\angle 1$.**

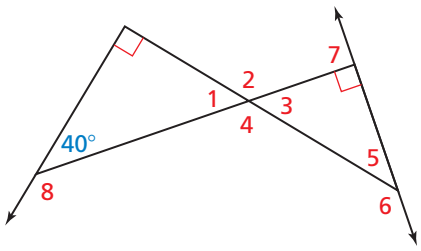
27.



28.



In Exercises 29–36, find the measure of the numbered angle.



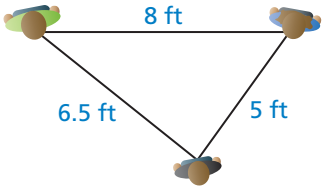
29. $\angle 1$ **30.** $\angle 2$

31. $\angle 3$ **32.** $\angle 4$

33. $\angle 5$ **34.** $\angle 6$

35. $\angle 7$ **36.** $\angle 8$

37. **USING TOOLS** Three people are standing on a stage. The distances between the three people are shown in the diagram. Classify the triangle by its sides and by measuring its angles.



38. **USING STRUCTURE** Which of the following sets of angle measures could form a triangle? Select all that apply.

Ⓐ 100°, 50°, 40° Ⓑ 96°, 74°, 10°

Ⓒ 165°, 113°, 82° Ⓓ 101°, 41°, 38°

Ⓔ 90°, 45°, 45° Ⓕ 84°, 62°, 34°

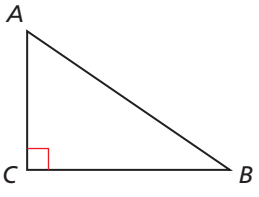
39. **MODELING WITH MATHEMATICS** You are bending a strip of metal into an isosceles triangle for a sculpture. The strip of metal is 20 inches long. The first bend is made 6 inches from one end. Describe two ways you could complete the triangle.

40. **THOUGHT PROVOKING** Find and draw an object (or part of an object) that can be modeled by a triangle and an exterior angle. Describe the relationship between the interior angles of the triangle and the exterior angle in terms of the object.

41. **PROVING A COROLLARY** Prove the Corollary to the Triangle Sum Theorem (Corollary 5.1).

Given $\triangle ABC$ is a right triangle.

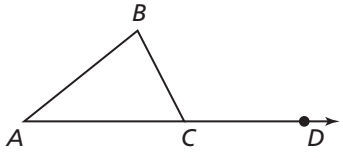
Prove $\angle A$ and $\angle B$ are complementary.



42. **PROVING A THEOREM** Prove the Exterior Angle Theorem (Theorem 5.2).

Given $\triangle ABC$, exterior $\angle BCD$

Prove $m\angle A + m\angle B = m\angle BCD$



43. **CRITICAL THINKING** Is it possible to draw an obtuse isosceles triangle? obtuse equilateral triangle? If so, provide examples. If not, explain why it is not possible.

44. **CRITICAL THINKING** Is it possible to draw a right isosceles triangle? right equilateral triangle? If so, provide an example. If not, explain why it is not possible.

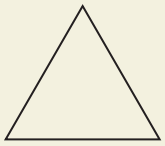
45. **MATHEMATICAL CONNECTIONS** $\triangle ABC$ is isosceles, $AB = x$, and $BC = 2x - 4$.

a. Find two possible values for x when the perimeter of $\triangle ABC$ is 32.

b. How many possible values are there for x when the perimeter of $\triangle ABC$ is 12?

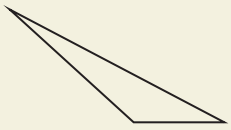
46. **HOW DO YOU SEE IT?** In as many ways as possible, classify each triangle by its appearance.

a.

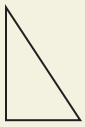


b.

c.



d.



47. **ANALYZING RELATIONSHIPS** Which of the following could represent the measures of an exterior angle and two interior angles of a triangle? Select all that apply.

Ⓐ 100°, 62°, 38° Ⓑ 81°, 57°, 24°

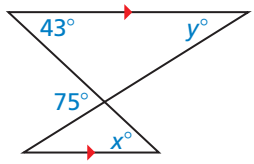
Ⓒ 119°, 68°, 49° Ⓓ 95°, 85°, 28°

Ⓔ 92°, 78°, 68° Ⓕ 149°, 101°, 48°

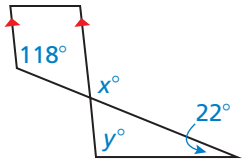
48. **MAKING AN ARGUMENT** Your friend claims the measure of an exterior angle will always be greater than the sum of the nonadjacent interior angle measures. Is your friend correct? Explain your reasoning.

MATHEMATICAL CONNECTIONS **In Exercises 49–52, find the values of x and y.**

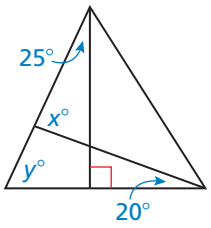
49.



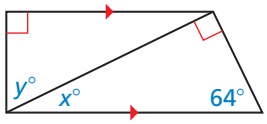
50.



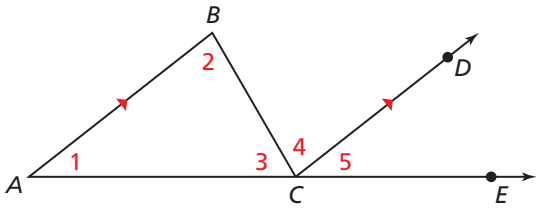
51.



52.



53. **PROVING A THEOREM** Use the diagram to write a proof of the Triangle Sum Theorem (Theorem 5.1). Your proof should be different from the proof of the Triangle Sum Theorem shown in this lesson.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the diagram to find the measure of the segment or angle. *(Section 1.2 and Section 1.5)*

54. $m\angle KHL$

55. $m\angle ABC$

56. GH

57. BC