

2.6 Proving Geometric Relationships

Essential Question How can you use a flowchart to prove a mathematical statement?

EXPLORATION 1 Matching Reasons in a Flowchart Proof

Work with a partner. Match each reason with the correct step in the flowchart.

Given $AC = AB + AB$



Prove $AB = BC$

MODELING WITH MATHEMATICS

To be proficient in math, you need to map relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas.

$$AC = AB + AB$$

$$AB + BC = AC$$

$$AB + AB = AB + BC$$

$$AB = BC$$

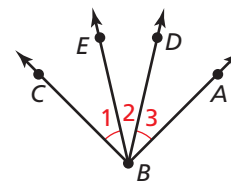
- A. Segment Addition Postulate (Post. 1.2) B. Given
C. Transitive Property of Equality D. Subtraction Property of Equality

EXPLORATION 2 Matching Reasons in a Flowchart Proof

Work with a partner. Match each reason with the correct step in the flowchart.

Given $m\angle 1 = m\angle 3$

Prove $m\angle EBA = m\angle CBD$



$$m\angle 1 = m\angle 3$$

$$m\angle EBA = m\angle 2 + m\angle 3$$

$$m\angle EBA = m\angle 2 + m\angle 1$$

$$m\angle EBA = m\angle 1 + m\angle 2$$

$$m\angle 1 + m\angle 2 = m\angle CBD$$

$$m\angle EBA = m\angle CBD$$

- A. Angle Addition Postulate (Post. 1.4) B. Transitive Property of Equality
C. Substitution Property of Equality D. Angle Addition Postulate (Post. 1.4)
E. Given F. Commutative Property of Addition

Communicate Your Answer

- How can you use a flowchart to prove a mathematical statement?
- Compare the flowchart proofs above with the two-column proofs in the Section 2.5 Explorations. Explain the advantages and disadvantages of each.

2.6 Lesson

Core Vocabulary

flowchart proof, or flow proof, p. 106
 paragraph proof, p. 108

What You Will Learn

- ▶ Write flowchart proofs to prove geometric relationships.
- ▶ Write paragraph proofs to prove geometric relationships.

Writing Flowchart Proofs

Another proof format is a **flowchart proof**, or **flow proof**, which uses boxes and arrows to show the flow of a logical argument. Each reason is below the statement it justifies. A flowchart proof of the *Right Angles Congruence Theorem* is shown in Example 1. This theorem is useful when writing proofs involving right angles.

Theorem

Theorem 2.3 Right Angles Congruence Theorem

All right angles are congruent.

Proof Example 1, p. 106

STUDY TIP

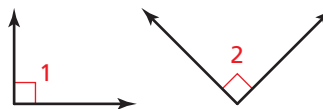
When you prove a theorem, write the hypothesis of the theorem as the **Given** statement. The conclusion is what you must **Prove**.

EXAMPLE 1 Proving the Right Angles Congruence Theorem

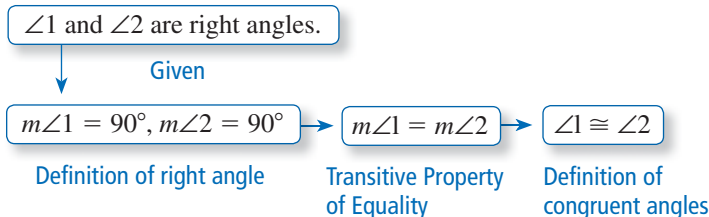
Use the given flowchart proof to write a two-column proof of the Right Angles Congruence Theorem.

Given $\angle 1$ and $\angle 2$ are right angles.

Prove $\angle 1 \cong \angle 2$



Flowchart Proof



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are right angles.	1. Given
2. $m\angle 1 = 90^\circ, m\angle 2 = 90^\circ$	2. Definition of right angle
3. $m\angle 1 = m\angle 2$	3. Transitive Property of Equality
4. $\angle 1 \cong \angle 2$	4. Definition of congruent angles

Monitoring Progress

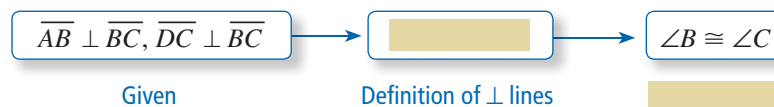
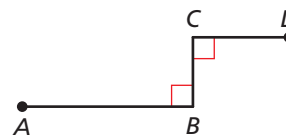


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- Copy and complete the flowchart proof. Then write a two-column proof.

Given $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$

Prove $\angle B \cong \angle C$



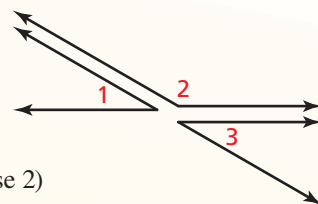
Theorems

Theorem 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

Proof Example 2, p. 107 (case 1); Ex. 20, p. 113 (case 2)

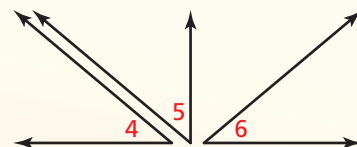


Theorem 2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 4$ and $\angle 5$ are complementary and $\angle 6$ and $\angle 5$ are complementary, then $\angle 4 \cong \angle 6$.

Proof Ex. 19, p. 112 (case 1); Ex. 22, p. 113 (case 2)



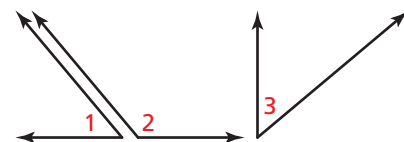
To prove the Congruent Supplements Theorem, you must prove two cases: one with angles supplementary to the same angle and one with angles supplementary to congruent angles. The proof of the Congruent Complements Theorem also requires two cases.

EXAMPLE 2 Proving a Case of Congruent Supplements Theorem

Use the given two-column proof to write a flowchart proof that proves that two angles supplementary to the same angle are congruent.

Given $\angle 1$ and $\angle 2$ are supplementary.
 $\angle 3$ and $\angle 2$ are supplementary.

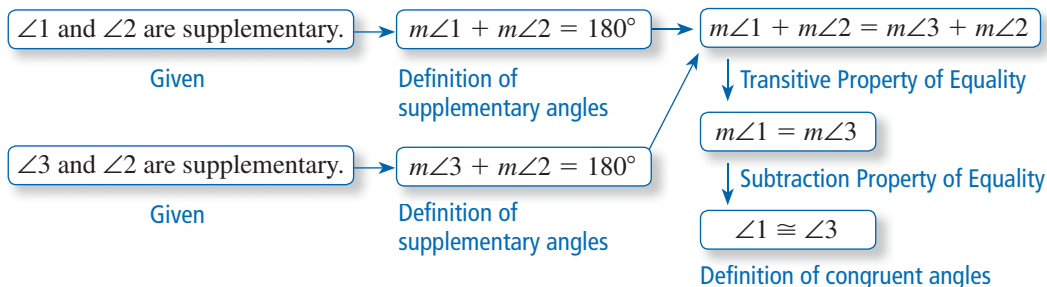
Prove $\angle 1 \cong \angle 3$



Two-Column Proof

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 2$ are supplementary.	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$, $m\angle 3 + m\angle 2 = 180^\circ$	2. Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	3. Transitive Property of Equality
4. $m\angle 1 = m\angle 3$	4. Subtraction Property of Equality
5. $\angle 1 \cong \angle 3$	5. Definition of congruent angles

Flowchart Proof



Writing Paragraph Proofs

Another proof format is a **paragraph proof**, which presents the statements and reasons of a proof as sentences in a paragraph. It uses words to explain the logical flow of the argument.

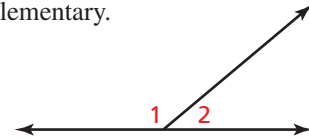
Two intersecting lines form pairs of vertical angles and linear pairs. The *Linear Pair Postulate* formally states the relationship between linear pairs. You can use this postulate to prove the *Vertical Angles Congruence Theorem*.

Postulate and Theorem

Postulate 2.8 Linear Pair Postulate

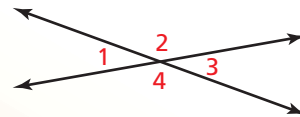
If two angles form a linear pair, then they are supplementary.

$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180^\circ$.



Theorem 2.6 Vertical Angles Congruence Theorem

Vertical angles are congruent.



$$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$$

Proof Example 3, p. 108

EXAMPLE 3

Proving the Vertical Angles Congruence Theorem

Use the given paragraph proof to write a two-column proof of the Vertical Angles Congruence Theorem.

Given $\angle 5$ and $\angle 7$ are vertical angles.

Prove $\angle 5 \cong \angle 7$



STUDY TIP

In paragraph proofs, *transitional words* such as *so*, *then*, and *therefore* help make the logic clear.

Paragraph Proof

$\angle 5$ and $\angle 7$ are vertical angles formed by intersecting lines. As shown in the diagram, $\angle 5$ and $\angle 6$ are a linear pair, and $\angle 6$ and $\angle 7$ are a linear pair. Then, by the Linear Pair Postulate, $\angle 5$ and $\angle 6$ are supplementary and $\angle 6$ and $\angle 7$ are supplementary. So, by the Congruent Supplements Theorem, $\angle 5 \cong \angle 7$.

JUSTIFYING STEPS

You can use information labeled in a diagram in your proof.

Two-Column Proof

STATEMENTS	REASONS
1. $\angle 5$ and $\angle 7$ are vertical angles.	1. Given
2. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	2. Definition of linear pair, as shown in the diagram
3. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	3. Linear Pair Postulate
4. $\angle 5 \cong \angle 7$	4. Congruent Supplements Theorem

2. Copy and complete the two-column proof. Then write a flowchart proof.

Given $AB = DE, BC = CD$

Prove $\overline{AC} \cong \overline{CE}$



STATEMENTS	REASONS
1. $AB = DE, BC = CD$	1. Given
2. $AB + BC = BC + DE$	2. Addition Property of Equality
3. _____	3. Substitution Property of Equality
4. $AB + BC = AC, CD + DE = CE$	4. _____
5. _____	5. Substitution Property of Equality
6. $\overline{AC} \cong \overline{CE}$	6. _____

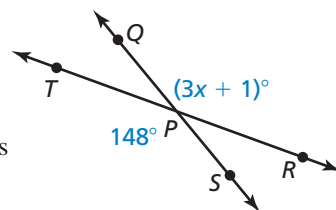
3. Rewrite the two-column proof in Example 3 without using the Congruent Supplements Theorem. How many steps do you save by using the theorem?

EXAMPLE 4 Using Angle Relationships

Find the value of x .

SOLUTION

$\angle TPS$ and $\angle QPR$ are vertical angles. By the Vertical Angles Congruence Theorem, the angles are congruent. Use this fact to write and solve an equation.

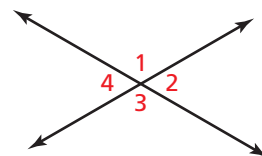


$m\angle TPS = m\angle QPR$	Definition of congruent angles
$148^\circ = (3x + 1)^\circ$	Substitute angle measures.
$147 = 3x$	Subtract 1 from each side.
$49 = x$	Divide each side by 3.

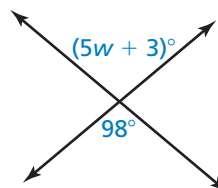
► So, the value of x is 49.

Use the diagram and the given angle measure to find the other three angle measures.

- $m\angle 1 = 117^\circ$
- $m\angle 2 = 59^\circ$
- $m\angle 4 = 88^\circ$



7. Find the value of w .

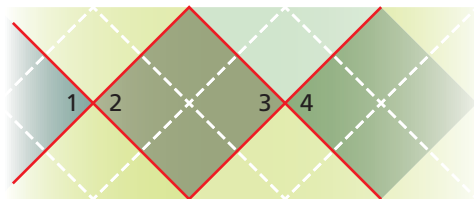


EXAMPLE 5**Using the Vertical Angles Congruence Theorem**

Write a paragraph proof.

Given $\angle 1 \cong \angle 4$

Prove $\angle 2 \cong \angle 3$

**Paragraph Proof**

$\angle 1$ and $\angle 4$ are congruent. By the Vertical Angles Congruence Theorem, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. By the Transitive Property of Angle Congruence (Theorem 2.2), $\angle 2 \cong \angle 4$. Using the Transitive Property of Angle Congruence (Theorem 2.2) once more, $\angle 2 \cong \angle 3$.

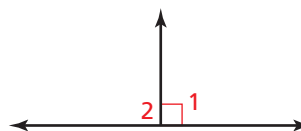
Monitoring Progress

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8. Write a paragraph proof.

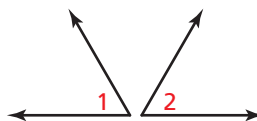
Given $\angle 1$ is a right angle.

Prove $\angle 2$ is a right angle.

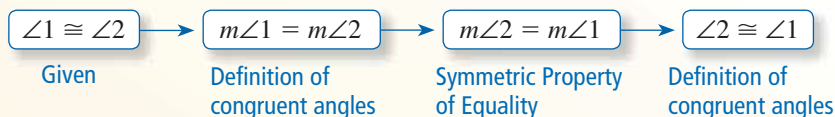
**Concept Summary****Types of Proofs****Symmetric Property of Angle Congruence (Theorem 2.2)**

Given $\angle 1 \cong \angle 2$

Prove $\angle 2 \cong \angle 1$

**Two-Column Proof**

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

Flowchart Proof**Paragraph Proof**

$\angle 1$ is congruent to $\angle 2$. By the definition of congruent angles, the measure of $\angle 1$ is equal to the measure of $\angle 2$. The measure of $\angle 2$ is equal to the measure of $\angle 1$ by the Symmetric Property of Equality. Then by the definition of congruent angles, $\angle 2$ is congruent to $\angle 1$.

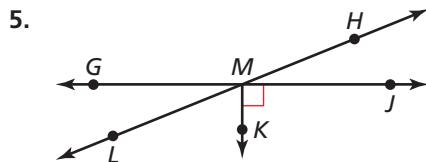
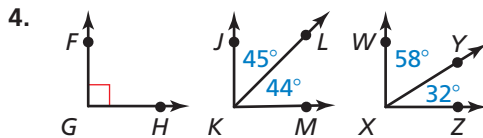
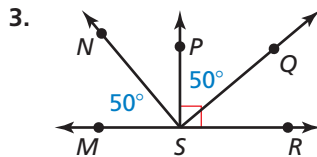
2.6 Exercises

Vocabulary and Core Concept Check

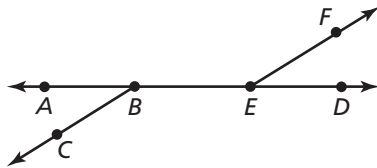
- WRITING** Explain why all right angles are congruent.
- VOCABULARY** What are the two types of angles that are formed by intersecting lines?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, identify the pair(s) of congruent angles in the figures. Explain how you know they are congruent. (See Examples 1, 2, and 3.)

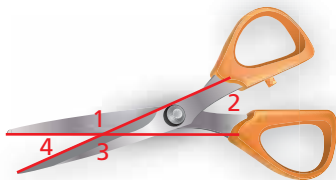


6. $\angle ABC$ is supplementary to $\angle CBD$.
 $\angle CBD$ is supplementary to $\angle DEF$.

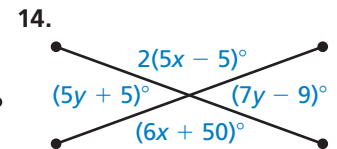
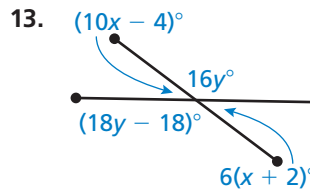
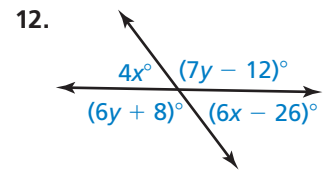
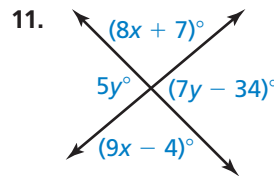


In Exercises 7–10, use the diagram and the given angle measure to find the other three measures. (See Example 3.)

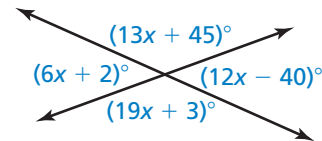
- $m\angle 1 = 143^\circ$
- $m\angle 3 = 159^\circ$
- $m\angle 2 = 34^\circ$
- $m\angle 4 = 29^\circ$



In Exercises 11–14, find the values of x and y . (See Example 4.)



ERROR ANALYSIS In Exercises 15 and 16, describe and correct the error in using the diagram to find the value of x .



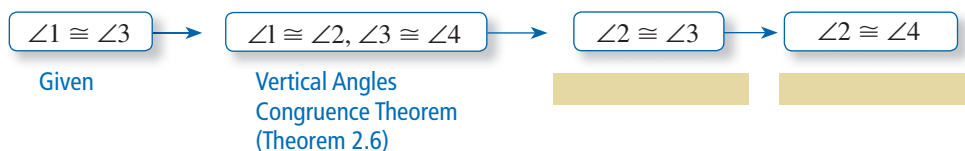
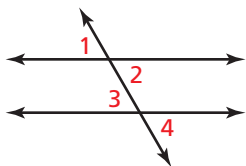
15. $(13x + 45)^\circ + (19x + 3)^\circ = 180^\circ$
 $32x + 48 = 180$
 $32x = 132$
 $x = 4.125$

16. $(13x + 45)^\circ + (12x - 40)^\circ = 90^\circ$
 $25x + 5 = 90$
 $25x = 85$
 $x = 3.4$

17. **PROOF** Copy and complete the flowchart proof. Then write a two-column proof.
(See Example 1.)

Given $\angle 1 \cong \angle 3$

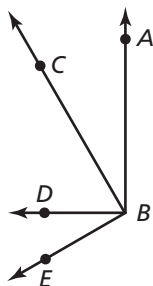
Prove $\angle 2 \cong \angle 4$



18. **PROOF** Copy and complete the two-column proof. Then write a flowchart proof.
(See Example 2.)

Given $\angle ABD$ is a right angle.
 $\angle CBE$ is a right angle.

Prove $\angle ABC \cong \angle DBE$

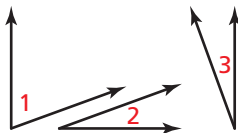


STATEMENTS	REASONS
1. $\angle ABD$ is a right angle. $\angle CBE$ is a right angle.	1. _____
2. $\angle ABC$ and $\angle CBD$ are complementary.	2. Definition of complementary angles
3. $\angle DBE$ and $\angle CBD$ are complementary.	3. _____
4. $\angle ABC \cong \angle DBE$	4. _____

19. **PROVING A THEOREM** Copy and complete the paragraph proof for the Congruent Complements Theorem (Theorem 2.5). Then write a two-column proof. (See Example 3.)

Given $\angle 1$ and $\angle 2$ are complementary.
 $\angle 1$ and $\angle 3$ are complementary.

Prove $\angle 2 \cong \angle 3$



$\angle 1$ and $\angle 2$ are complementary, and $\angle 1$ and $\angle 3$ are complementary. By the definition of _____ angles, $m\angle 1 + m\angle 2 = 90^\circ$ and _____ = 90° . By the _____, $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$. By the Subtraction Property of Equality, _____. So, $\angle 2 \cong \angle 3$ by the definition of _____.

20. **PROVING A THEOREM** Copy and complete the two-column proof for the Congruent Supplement Theorem (Theorem 2.4). Then write a paragraph proof. (See Example 5.)

Given $\angle 1$ and $\angle 2$ are supplementary.
 $\angle 3$ and $\angle 4$ are supplementary.
 $\angle 1 \cong \angle 4$



Prove $\angle 2 \cong \angle 3$

STATEMENTS

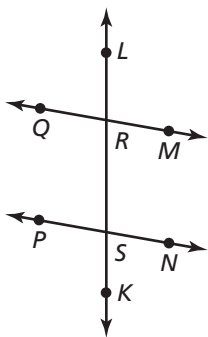
REASONS

1. $\angle 1$ and $\angle 2$ are supplementary.
 $\angle 3$ and $\angle 4$ are supplementary.
 $\angle 1 \cong \angle 4$
2. $m\angle 1 + m\angle 2 = 180^\circ$,
 $m\angle 3 + m\angle 4 = 180^\circ$
3. _____ = $m\angle 3 + m\angle 4$
4. $m\angle 1 = m\angle 4$
5. $m\angle 1 + m\angle 2 =$ _____
6. $m\angle 2 = m\angle 3$
7. _____

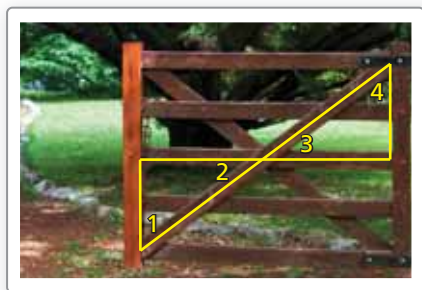
1. Given
2. _____
3. Transitive Property of Equality
4. Definition of congruent angles
5. Substitution Property of Equality
6. _____
7. _____

PROOF In Exercises 21–24, write a proof using any format.

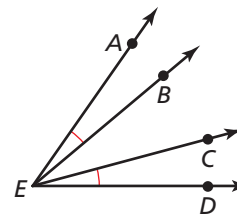
21. **Given** $\angle QRS$ and $\angle PSR$ are supplementary.
Prove $\angle QRL \cong \angle PSR$



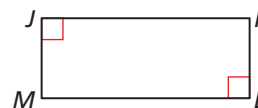
22. **Given** $\angle 1$ and $\angle 3$ are complementary.
 $\angle 2$ and $\angle 4$ are complementary.
Prove $\angle 1 \cong \angle 4$



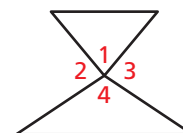
23. **Given** $\angle AEB \cong \angle DEC$
Prove $\angle AEC \cong \angle DEB$



24. **Given** $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$,
 $\angle J \cong \angle M$, $\angle K \cong \angle L$
Prove $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$



25. **MAKING AN ARGUMENT** You overhear your friend discussing the diagram shown with a classmate. Your classmate claims $\angle 1 \cong \angle 4$ because they are vertical angles. Your friend claims they are not congruent because he can tell by looking at the diagram. Who is correct? Support your answer with definitions or theorems.

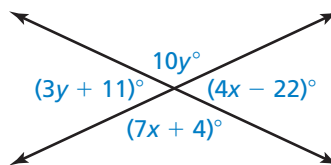


26. **THOUGHT PROVOKING** Draw three lines all intersecting at the same point. Explain how you can give two of the angle measures so that you can find the remaining four angle measures.

27. **CRITICAL THINKING** Is the converse of the Linear Pair Postulate (Postulate 2.8) true? If so, write a biconditional statement. Explain your reasoning.

28. **WRITING** How can you save time writing proofs?

29. **MATHEMATICAL CONNECTIONS** Find the measure of each angle in the diagram.



30. **HOW DO YOU SEE IT?** Use the student's two-column proof.

Given $\angle 1 \cong \angle 2$
 $\angle 1$ and $\angle 2$ are supplementary.

Prove _____

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$ $\angle 1$ and $\angle 2$ are supplementary.	1. Given
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 1 + m\angle 2 = 180^\circ$	3. Definition of supplementary angles
4. $m\angle 1 + m\angle 1 = 180^\circ$	4. Substitution Property of Equality
5. $2m\angle 1 = 180^\circ$	5. Simplify.
6. $m\angle 1 = 90^\circ$	6. Division Property of Equality
7. $m\angle 2 = 90^\circ$	7. Transitive Property of Equality
8. _____	8. _____

- What is the student trying to prove?
- Your friend claims that the last line of the proof should be $\angle 1 \cong \angle 2$, because the measures of the angles are both 90° . Is your friend correct? Explain.

Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Use the cube. (Section 1.1)

- Name three collinear points.
- Name the intersection of plane ABF and plane EHG .
- Name two planes containing \overline{BC} .
- Name three planes containing point D .
- Name three points that are not collinear.
- Name two planes containing point J .

