

# 2.5 Proving Statements about Segments and Angles

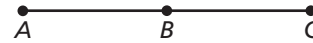
**Essential Question** How can you prove a mathematical statement?

A **proof** is a logical argument that uses deductive reasoning to show that a statement is true.

## EXPLORATION 1 Writing Reasons in a Proof

**Work with a partner.** Four steps of a proof are shown. Write the reasons for each statement.

**Given**  $AC = AB + AB$



**Prove**  $AB = BC$

### REASONING ABSTRACTLY

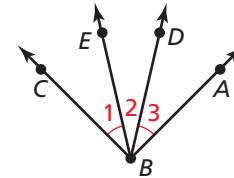
To be proficient in math, you need to know and be able to use algebraic properties.

STATEMENTS	REASONS
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2. _____
3. $AB + AB = AB + BC$	3. _____
4. $AB = BC$	4. _____

## EXPLORATION 2 Writing Steps in a Proof

**Work with a partner.** Six steps of a proof are shown. Complete the statements that correspond to each reason.

**Given**  $m\angle 1 = m\angle 3$



**Prove**  $m\angle EBA = m\angle CBD$

STATEMENTS	REASONS
1. _____	1. Given
2. $m\angle EBA = m\angle 2 + m\angle 3$	2. Angle Addition Postulate (Post.1.4)
3. $m\angle EBA = m\angle 2 + m\angle 1$	3. Substitution Property of Equality
4. $m\angle EBA =$ _____	4. Commutative Property of Addition
5. $m\angle 1 + m\angle 2 =$ _____	5. Angle Addition Postulate (Post.1.4)
6. _____	6. Transitive Property of Equality

## Communicate Your Answer

3. How can you prove a mathematical statement?

4. Use the given information and the figure to write a proof for the statement.

**Given**  $B$  is the midpoint of  $\overline{AC}$ .  
 $C$  is the midpoint of  $\overline{BD}$ .



**Prove**  $AB = CD$

# 2.5 Lesson

## Core Vocabulary

proof, p. 100  
two-column proof, p. 100  
theorem, p. 101

## What You Will Learn

- ▶ Write two-column proofs.
- ▶ Name and prove properties of congruence.

## Writing Two-Column Proofs

A **proof** is a logical argument that uses deductive reasoning to show that a statement is true. There are several formats for proofs. A **two-column proof** has numbered statements and corresponding reasons that show an argument in a logical order.

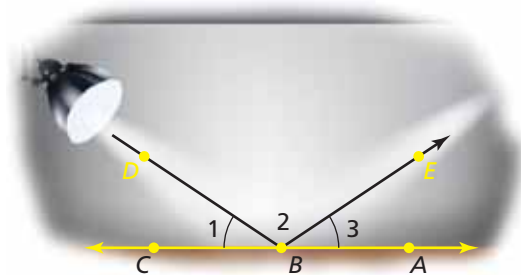
In a two-column proof, each statement in the left-hand column is either given information or the result of applying a known property or fact to statements already made. Each reason in the right-hand column is the explanation for the corresponding statement.

### EXAMPLE 1 Writing a Two-Column Proof

Write a two-column proof for the situation in Example 4 from the Section 2.4 lesson.

**Given**  $m\angle 1 = m\angle 3$

**Prove**  $m\angle DBA = m\angle EBC$



STATEMENTS	REASONS
1. $m\angle 1 = m\angle 3$	1. Given
2. $m\angle DBA = m\angle 3 + m\angle 2$	2. Angle Addition Postulate (Post.1.4)
3. $m\angle DBA = m\angle 1 + m\angle 2$	3. Substitution Property of Equality
4. $m\angle 1 + m\angle 2 = m\angle EBC$	4. Angle Addition Postulate (Post.1.4)
5. $m\angle DBA = m\angle EBC$	5. Transitive Property of Equality

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1. Six steps of a two-column proof are shown. Copy and complete the proof.

**Given**  $T$  is the midpoint of  $\overline{SU}$ .



**Prove**  $x = 5$

STATEMENTS	REASONS
1. $T$ is the midpoint of $\overline{SU}$ .	1. _____
2. $\overline{ST} \cong \overline{TU}$	2. Definition of midpoint
3. $ST = TU$	3. Definition of congruent segments
4. $7x = 3x + 20$	4. _____
5. _____	5. Subtraction Property of Equality
6. $x = 5$	6. _____

## Using Properties of Congruence

The reasons used in a proof can include definitions, properties, postulates, and *theorems*. A **theorem** is a statement that can be proven. Once you have proven a theorem, you can use the theorem as a reason in other proofs.

### Theorems

#### Theorem 2.1 Properties of Segment Congruence

Segment congruence is reflexive, symmetric, and transitive.

**Reflexive** For any segment  $AB$ ,  $\overline{AB} \cong \overline{AB}$ .

**Symmetric** If  $\overline{AB} \cong \overline{CD}$ , then  $\overline{CD} \cong \overline{AB}$ .

**Transitive** If  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{EF}$ , then  $\overline{AB} \cong \overline{EF}$ .

*Proofs* Ex. 11, p. 103; Example 3, p. 101; Chapter Review 2.5 Example, p. 118

#### Theorem 2.2 Properties of Angle Congruence

Angle congruence is reflexive, symmetric, and transitive.

**Reflexive** For any angle  $A$ ,  $\angle A \cong \angle A$ .

**Symmetric** If  $\angle A \cong \angle B$ , then  $\angle B \cong \angle A$ .

**Transitive** If  $\angle A \cong \angle B$  and  $\angle B \cong \angle C$ , then  $\angle A \cong \angle C$ .

*Proofs* Ex. 25, p. 118; 2.5 Concept Summary, p. 102; Ex. 12, p. 103

#### EXAMPLE 2 Naming Properties of Congruence

Name the property that the statement illustrates.

- If  $\angle T \cong \angle V$  and  $\angle V \cong \angle R$ , then  $\angle T \cong \angle R$ .
- If  $\overline{JL} \cong \overline{YZ}$ , then  $\overline{YZ} \cong \overline{JL}$ .

#### SOLUTION

- Transitive Property of Angle Congruence
- Symmetric Property of Segment Congruence

In this lesson, most of the proofs involve showing that congruence and equality are equivalent. You may find that what you are asked to prove seems to be obviously true. It is important to practice writing these proofs to help you prepare for writing more-complicated proofs in later chapters.

#### STUDY TIP

When writing a proof, organize your reasoning by copying or drawing a diagram for the situation described. Then identify the **Given** and **Prove** statements.

#### EXAMPLE 3 Proving a Symmetric Property of Congruence

Write a two-column proof for the Symmetric Property of Segment Congruence.

**Given**  $\overline{LM} \cong \overline{NP}$

**Prove**  $\overline{NP} \cong \overline{LM}$



STATEMENTS	REASONS
1. $\overline{LM} \cong \overline{NP}$	1. Given
2. $LM = NP$	2. Definition of congruent segments
3. $NP = LM$	3. Symmetric Property of Equality
4. $\overline{NP} \cong \overline{LM}$	4. Definition of congruent segments

### EXAMPLE 4 Writing a Two-Column Proof

Prove this property of midpoints: If you know that  $M$  is the midpoint of  $\overline{AB}$ , prove that  $AB$  is two times  $AM$  and  $AM$  is one-half  $AB$ .

**Given**  $M$  is the midpoint of  $\overline{AB}$ .



**Prove**  $AB = 2AM$ ,  $AM = \frac{1}{2}AB$

STATEMENTS	REASONS
1. $M$ is the midpoint of $\overline{AB}$ .	1. Given
2. $\overline{AM} \cong \overline{MB}$	2. Definition of midpoint
3. $AM = MB$	3. Definition of congruent segments
4. $AM + MB = AB$	4. Segment Addition Postulate (Post. 1.2)
5. $AM + AM = AB$	5. Substitution Property of Equality
6. $2AM = AB$	6. Distributive Property
7. $AM = \frac{1}{2}AB$	7. Division Property of Equality

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Name the property that the statement illustrates.

- $\overline{GH} \cong \overline{GH}$
- If  $\angle K \cong \angle P$ , then  $\angle P \cong \angle K$ .
- Look back at Example 4. What would be different if you were proving that  $AB = 2 \cdot MB$  and that  $MB = \frac{1}{2}AB$  instead?

## Concept Summary

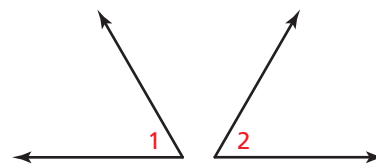
### Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.

### Proof of the Symmetric Property of Angle Congruence

**Given**  $\angle 1 \cong \angle 2$

**Prove**  $\angle 2 \cong \angle 1$



Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

statements based on facts that you know or on conclusions from deductive reasoning

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 2$	1. <b>Given</b>
2. $m\angle 1 = m\angle 2$	2. Definition of congruent angles
3. $m\angle 2 = m\angle 1$	3. Symmetric Property of Equality
4. $\angle 2 \cong \angle 1$	4. Definition of congruent angles

definitions, postulates, or proven theorems that allow you to state the corresponding statement

The number of statements will vary.

Remember to give a reason for the last statement.

# 2.5 Exercises

## Vocabulary and Core Concept Check

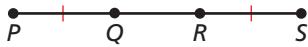
- WRITING** How is a theorem different from a postulate?
- COMPLETE THE SENTENCE** In a two-column proof, each \_\_\_\_\_ is on the left and each \_\_\_\_\_ is on the right.

## Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, copy and complete the proof. (See Example 1.)

3. **Given**  $PQ = RS$

**Prove**  $PR = QS$



### STATEMENTS

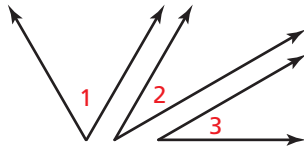
- $PQ = RS$
- $PQ + QR = RS + QR$
- \_\_\_\_\_
- $RS + QR = QS$
- $PR = QS$

### REASONS

- \_\_\_\_\_
- \_\_\_\_\_
- Segment Addition Postulate (Post. 1.2)
- Segment Addition Postulate (Post. 1.2)
- \_\_\_\_\_

4. **Given**  $\angle 1$  is a complement of  $\angle 2$ .  
 $\angle 2 \cong \angle 3$

**Prove**  $\angle 1$  is a complement of  $\angle 3$ .



### STATEMENTS

- $\angle 1$  is a complement of  $\angle 2$ .
- $\angle 2 \cong \angle 3$
- $m\angle 1 + m\angle 2 = 90^\circ$
- $m\angle 2 = m\angle 3$
- \_\_\_\_\_
- $\angle 1$  is a complement of  $\angle 3$ .

### REASONS

- Given
- \_\_\_\_\_
- \_\_\_\_\_
- Definition of congruent angles
- Substitution Property of Equality
- \_\_\_\_\_

In Exercises 5–10, name the property that the statement illustrates. (See Example 2.)

- If  $\overline{PQ} \cong \overline{ST}$  and  $\overline{ST} \cong \overline{UV}$ , then  $\overline{PQ} \cong \overline{UV}$ .
- $\angle F \cong \angle F$
- If  $\angle G \cong \angle H$ , then  $\angle H \cong \angle G$ .
- $\overline{DE} \cong \overline{DE}$
- If  $\overline{XY} \cong \overline{UV}$ , then  $\overline{UV} \cong \overline{XY}$ .
- If  $\angle L \cong \angle M$  and  $\angle M \cong \angle N$ , then  $\angle L \cong \angle N$ .

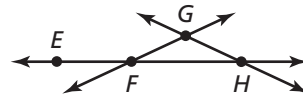
**PROOF** In Exercises 11 and 12, write a two-column proof for the property. (See Example 3.)

- Reflexive Property of Segment Congruence (Thm. 2.1)
- Transitive Property of Angle Congruence (Thm. 2.2)

**PROOF** In Exercises 13 and 14, write a two-column proof. (See Example 4.)

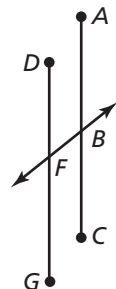
13. **Given**  $\angle GFH \cong \angle GHF$

**Prove**  $\angle EFG$  and  $\angle GHF$  are supplementary.



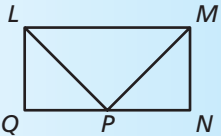
14. **Given**  $\overline{AB} \cong \overline{FG}$ ,  $\overline{BF}$  bisects  $\overline{AC}$  and  $\overline{DG}$ .

**Prove**  $\overline{BC} \cong \overline{DF}$

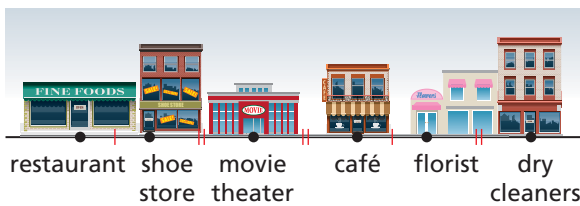


15. **ERROR ANALYSIS** In the diagram,  $\overline{MN} \cong \overline{LQ}$  and  $\overline{LQ} \cong \overline{PN}$ . Describe and correct the error in the reasoning.

**X** Because  $\overline{MN} \cong \overline{LQ}$  and  $\overline{LQ} \cong \overline{PN}$ , then  $\overline{MN} \cong \overline{PN}$  by the Reflexive Property of Segment Congruence (Thm. 2.1).



16. **MODELING WITH MATHEMATICS** The distance from the restaurant to the shoe store is the same as the distance from the café to the florist. The distance from the shoe store to the movie theater is the same as the distance from the movie theater to the café, and from the florist to the dry cleaners.

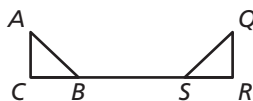


Use the steps below to prove that the distance from the restaurant to the movie theater is the same as the distance from the café to the dry cleaners.

- State what is given and what is to be proven for the situation.
  - Write a two-column proof.
17. **REASONING** In the sculpture shown,  $\angle 1 \cong \angle 2$  and  $\angle 2 \cong \angle 3$ . Classify the triangle and justify your answer.

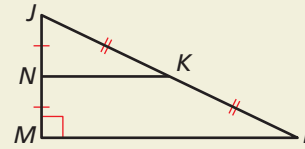


18. **MAKING AN ARGUMENT** In the figure,  $\overline{SR} \cong \overline{CB}$  and  $\overline{AC} \cong \overline{QR}$ . Your friend claims that, because of this,  $\overline{CB} \cong \overline{AC}$  by the Transitive Property of Segment Congruence (Thm. 2.1). Is your friend correct? Explain your reasoning.



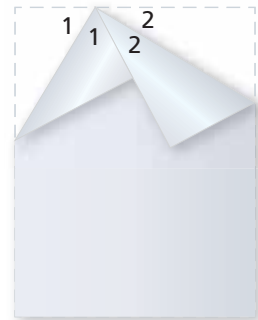
19. **WRITING** Explain why you do not use inductive reasoning when writing a proof.

20. **HOW DO YOU SEE IT?** Use the figure to write Given and Prove statements for each conclusion.



- The acute angles of a right triangle are complementary.
  - A segment connecting the midpoints of two sides of a triangle is half as long as the third side.
21. **REASONING** Fold two corners of a piece of paper so their edges match, as shown.

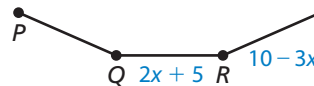
- What do you notice about the angle formed at the top of the page by the folds?
- Write a two-column proof to show that the angle measure is always the same no matter how you make the folds.



22. **THOUGHT PROVOKING** The distance from Springfield to Lakewood City is equal to the distance from Springfield to Bettsville. Janisburg is 50 miles farther from Springfield than Bettsville. Moon Valley is 50 miles farther from Springfield than Lakewood City is. Use line segments to draw a diagram that represents this situation.

23. **MATHEMATICAL CONNECTIONS** Solve for  $x$  using the given information. Justify each step.

Given  $\overline{QR} \cong \overline{PQ}$ ,  $\overline{RS} \cong \overline{PQ}$



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the figure. (Section 1.6)

- $\angle 1$  is a complement of  $\angle 4$ , and  $m\angle 1 = 33^\circ$ . Find  $m\angle 4$ .
- $\angle 3$  is a supplement of  $\angle 2$ , and  $m\angle 2 = 147^\circ$ . Find  $m\angle 3$ .
- Name a pair of vertical angles.

