## 2.4 Algebraic Reasoning

**Essential Question** How can algebraic properties help you solve an equation?

### Justifying Steps in a Solution

**Work with a partner.** In previous courses, you studied different properties, such as the properties of equality and the Distributive, Commutative, and Associative Properties. Write the property that justifies each of the following solution steps.

<table>
<thead>
<tr>
<th>Algebraic Step</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2(x + 3) - 5 = 5x + 4$</td>
<td>Write given equation.</td>
</tr>
<tr>
<td>$2x + 6 - 5 = 5x + 4$</td>
<td></td>
</tr>
<tr>
<td>$2x + 1 = 5x + 4$</td>
<td></td>
</tr>
<tr>
<td>$2x - 2x + 1 = 5x - 2x + 4$</td>
<td></td>
</tr>
<tr>
<td>$1 = 3x + 4$</td>
<td></td>
</tr>
<tr>
<td>$1 - 4 = 3x + 4 - 4$</td>
<td></td>
</tr>
<tr>
<td>$-3 = 3x$</td>
<td></td>
</tr>
<tr>
<td>$-\frac{3}{3} = \frac{3x}{3}$</td>
<td></td>
</tr>
<tr>
<td>$-1 = x$</td>
<td></td>
</tr>
<tr>
<td>$x = -1$</td>
<td></td>
</tr>
</tbody>
</table>

### Stating Algebraic Properties

**Work with a partner.** The symbols ♦ and • represent addition and multiplication (not necessarily in that order). Determine which symbol represents which operation. Justify your answer. Then state each algebraic property being illustrated.

<table>
<thead>
<tr>
<th>Example of Property</th>
<th>Name of Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \diamond 6 = 6 \diamond 5$</td>
<td></td>
</tr>
<tr>
<td>$5 \cdot 6 = 6 \cdot 5$</td>
<td></td>
</tr>
<tr>
<td>$4 \diamond (5 \diamond 6) = (4 \diamond 5) \diamond 6$</td>
<td></td>
</tr>
<tr>
<td>$4 \cdot (5 \cdot 6) = (4 \cdot 5) \cdot 6$</td>
<td></td>
</tr>
<tr>
<td>$0 \diamond 5 = 0$</td>
<td></td>
</tr>
<tr>
<td>$0 \cdot 5 = 5$</td>
<td></td>
</tr>
<tr>
<td>$1 \diamond 5 = 5$</td>
<td></td>
</tr>
<tr>
<td>$4 \diamond (5 \diamond 6) = 4 \diamond (5 \cdot 4) \diamond 6$</td>
<td></td>
</tr>
</tbody>
</table>

### Communicate Your Answer

3. How can algebraic properties help you solve an equation?

What You Will Learn

- Use Algebraic Properties of Equality to justify the steps in solving an equation.
- Use the Distributive Property to justify the steps in solving an equation.
- Use properties of equality involving segment lengths and angle measures.

Using Algebraic Properties of Equality

When you solve an equation, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

Core Concept

Algebraic Properties of Equality

Let \( a, b, \) and \( c \) be real numbers.

- **Addition Property of Equality**
  
  If \( a = b \), then \( a + c = b + c \).

- **Subtraction Property of Equality**
  
  If \( a = b \), then \( a - c = b - c \).

- **Multiplication Property of Equality**
  
  If \( a = b \), then \( a \cdot c = b \cdot c, c \neq 0 \).

- **Division Property of Equality**
  
  If \( a = b \), then \( \frac{a}{c} = \frac{b}{c}, c \neq 0 \).

- **Substitution Property of Equality**
  
  If \( a = b \), then \( a \) can be substituted for \( b \) (or \( b \) for \( a \)) in any equation or expression.

**EXAMPLE 1**

**Justifying Steps**

Solve \( 3x + 2 = 23 - 4x \). Justify each step.

**SOLUTION**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3x + 2 = 23 - 4x )</td>
<td>Write the equation.</td>
<td>Given</td>
</tr>
<tr>
<td>( 3x + 2 + 4x = 23 - )</td>
<td>Add 4x to each side.</td>
<td>Addition Property</td>
</tr>
<tr>
<td></td>
<td></td>
<td>of Equality</td>
</tr>
<tr>
<td>( 7x + 2 = 23 )</td>
<td>Combine like terms.</td>
<td>Simplify</td>
</tr>
<tr>
<td>( 7x + 2 - 2 = 23 - 2 )</td>
<td>Subtract 2 from each side.</td>
<td>Subtraction Property</td>
</tr>
<tr>
<td></td>
<td></td>
<td>of Equality</td>
</tr>
<tr>
<td>( 7x = 21 )</td>
<td>Combine constant terms.</td>
<td>Simplify</td>
</tr>
<tr>
<td>( x = 3 )</td>
<td>Divide each side by 7.</td>
<td>Division Property</td>
</tr>
</tbody>
</table>

- The solution is \( x = 3 \).

**Monitoring Progress**

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Solve the equation. Justify each step.

1. \( 6x - 11 = -35 \)
2. \( -2p - 9 = 10p - 17 \)
3. \( 39 - 5z = -1 + 5z \)
Using the Distributive Property

Core Concept

Distributive Property
Let \( a, b, \) and \( c \) be real numbers.

\[
\text{Sum} \quad a(b + c) = ab + ac \quad \text{Difference} \quad a(b - c) = ab - ac
\]

Example 2 Using the Distributive Property

Solve \(-5(7w + 8) = 30\). Justify each step.

SOLUTION

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5(7w + 8) = 30)</td>
<td>Write the equation.</td>
<td>Given</td>
</tr>
<tr>
<td>(-35w - 40 = 30)</td>
<td>Multiply.</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>(-35w = 70)</td>
<td>Add 40 to each side.</td>
<td>Addition Property of Equality</td>
</tr>
<tr>
<td>(w = -2)</td>
<td>Divide each side by (-35).</td>
<td>Division Property of Equality</td>
</tr>
</tbody>
</table>

The solution is \( w = -2 \).

Example 3 Solving a Real-Life Problem

You get a raise at your part-time job. To write your raise as a percent, use the formula \( p(r + 1) = n \), where \( p \) is your previous wage, \( r \) is the percent increase (as a decimal), and \( n \) is your new wage. Solve the formula for \( r \). What is your raise written as a percent when your hourly wage increases from $7.25 to $7.54 per hour?

SOLUTION

Step 1 Solve for \( r \) in the formula \( p(r + 1) = n \).

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(r + 1) = n )</td>
<td>Write the equation.</td>
<td>Given</td>
</tr>
<tr>
<td>( pr + p = n )</td>
<td>Multiply.</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>( pr = n - p )</td>
<td>Subtract ( p ) from each side.</td>
<td>Subtraction Property of Equality</td>
</tr>
<tr>
<td>( r = \frac{n - p}{p} )</td>
<td>Divide each side by ( p ).</td>
<td>Division Property of Equality</td>
</tr>
</tbody>
</table>

Step 2 Evaluate \( r = \frac{n - p}{p} \) when \( n = 7.54 \) and \( p = 7.25 \).

\[
r = \frac{n - p}{p} = \frac{7.54 - 7.25}{7.25} = \frac{0.29}{7.25} = 0.04
\]

Your raise is 4%.

Monitoring Progress

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Solve the equation. Justify each step.

4. \( 3(3x + 14) = -3 \)  
5. \( 4 = -10b + 6(2 - b) \)

6. Solve the formula \( A = \frac{1}{2}bh \) for \( b \). Justify each step. Then find the base of a triangle whose area is 952 square feet and whose height is 56 feet.

Section 2.4 Algebraic Reasoning 93
Using Other Properties of Equality

The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for all segment lengths and angle measures.

### Reflexive, Symmetric, and Transitive Properties of Equality

<table>
<thead>
<tr>
<th>Property</th>
<th>Real Numbers</th>
<th>Segment Lengths</th>
<th>Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive Property</td>
<td>$a = a$</td>
<td>$AB = AB$</td>
<td>$m\angle A = m\angle A$</td>
</tr>
<tr>
<td>Symmetric Property</td>
<td>If $a = b$, then $b = a$.</td>
<td>If $AB = CD$, then $CD = AB$.</td>
<td>If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.</td>
</tr>
<tr>
<td>Transitive Property</td>
<td>If $a = b$ and $b = c$, then $a = c$.</td>
<td>If $AB = CD$ and $CD = EF$, then $AB = EF$.</td>
<td>If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.</td>
</tr>
</tbody>
</table>

### Example 4

Using Properties of Equality with Angle Measures

You reflect the beam of a spotlight off a mirror lying flat on a stage, as shown. Determine whether $m\angle DBA = m\angle EBC$.

#### Solution

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m\angle 1 = m\angle 3$</td>
<td>Marked in diagram.</td>
<td>Given</td>
</tr>
<tr>
<td>$m\angle DBA = m\angle 3 + m\angle 2$</td>
<td>Add measures of adjacent angles.</td>
<td>Angle Addition Postulate (Post. 1.4)</td>
</tr>
<tr>
<td>$m\angle DBA = m\angle 1 + m\angle 2$</td>
<td>Substitute $m\angle 1$ for $m\angle 3$.</td>
<td>Substitution Property of Equality</td>
</tr>
<tr>
<td>$m\angle 1 + m\angle 2 = m\angle EBC$</td>
<td>Add measures of adjacent angles.</td>
<td>Angle Addition Postulate (Post. 1.4)</td>
</tr>
<tr>
<td>$m\angle DBA = m\angle EBC$</td>
<td>Both measures are equal to the sum $m\angle 1 + m\angle 2$.</td>
<td>Transitive Property of Equality</td>
</tr>
</tbody>
</table>

### Monitoring Progress

Name the property of equality that the statement illustrates.

7. If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.
8. $34^\circ = 34^\circ$
9. $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 5$. So, $m\angle 1 = m\angle 5$. 

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94 Chapter 2 Reasoning and Proofs
A park, a shoe store, a pizza shop, and a movie theater are located in order on a city street. The distance between the park and the shoe store is the same as the distance between the pizza shop and the movie theater. Show that the distance between the park and the pizza shop is the same as the distance between the shoe store and the movie theater.

**SOLUTION**

1. **Understand the Problem** You know that the locations lie in order and that the distance between two of the locations (park and shoe store) is the same as the distance between the other two locations (pizza shop and movie theater). You need to show that two of the other distances are the same.

2. **Make a Plan** Draw and label a diagram to represent the situation.

   ![Diagram](image)

   Modify your diagram by letting the points \( P, S, Z, \) and \( M \) represent the park, the shoe store, the pizza shop, and the movie theater, respectively. Show any mathematical relationships.

   Use the Segment Addition Postulate (Postulate 1.2) to show that \( PZ = SM \).

3. **Solve the Problem**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Explanation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( PS = ZM )</td>
<td>Marked in diagram.</td>
<td>Given</td>
</tr>
<tr>
<td>( PZ = PS + SZ )</td>
<td>Add lengths of adjacent segments.</td>
<td>Segment Addition Postulate (Post. 1.2)</td>
</tr>
<tr>
<td>( SM = SZ + ZM )</td>
<td>Add lengths of adjacent segments.</td>
<td>Segment Addition Postulate (Post. 1.2)</td>
</tr>
<tr>
<td>( PS + SZ = ZM + SZ )</td>
<td>Add SZ to each side of ( PS = ZM ).</td>
<td>Addition Property of Equality</td>
</tr>
<tr>
<td>( PZ = SM )</td>
<td>Substitute ( PZ ) for ( PS + SZ ) and ( SM ) for ( ZM + SZ ).</td>
<td>Substitution Property of Equality</td>
</tr>
</tbody>
</table>

4. **Look Back** Reread the problem. Make sure your diagram is drawn precisely using the given information. Check the steps in your solution.

**Monitoring Progress**

Name the property of equality that the statement illustrates.

10. If \( JK = KL \) and \( KL = 16 \), then \( JK = 16 \).
11. \( PQ = ST \), so \( ST = PQ \).
12. \( ZY = ZY \)
13. In Example 5, a hot dog stand is located halfway between the shoe store and the pizza shop, at point \( H \). Show that \( PH = HM \).
2.4 Exercises

Vocabulary and Core Concept Check

1. **Vocabulary**  The statement “The measure of an angle is equal to itself” is true because of what property?

2. **Different Words, Same Question**  Which is different? Find both answers.

   What property justifies the following statement?

   - If $c = d$, then $d = c$.
   - If $JK = LM$, then $LM = JK$.
   - If $e = f$ and $f = g$, then $e = g$.
   - If $m \angle R = m \angle S$, then $m \angle S = m \angle R$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, write the property that justifies each step.

3. $3x - 12 = 7x + 8$  Given

   - $-4x - 12 = 8$
   - $-4x = 20$
   - $x = -5$

4. $5(x - 1) = 4x + 13$  Given

   - $5x - 5 = 4x + 13$
   - $x = 13$
   - $x = 18$

In Exercises 5–14, solve the equation. Justify each step. (See Examples 1 and 2.)

5. $5x - 10 = -40$
6. $6x + 17 = -7$

7. $2x - 8 = 6x - 20$
8. $4x + 9 = 16 - 3x$

9. $5x - 20 = -10$
10. $3(2x + 11) = 9$
11. $2(-x - 5) = 12$

12. $44 - 2(3x + 4) = -18x$
13. $4(5x - 9) = -2(x + 7)$
14. $3(4x + 7) = 5(3x + 3)$

In Exercises 15–20, solve the equation for $y$. Justify each step. (See Example 3.)

15. $5x + y = 18$
16. $-4x + 2y = 8$

17. $2y + 0.5x = 16$
18. $\frac{1}{2}x - \frac{3}{4}y = -2$

19. $12 - 3y = 30x + 6$
20. $3x + 7 = -7 + 9y$

In Exercises 21–24, solve the equation for the given variable. Justify each step. (See Example 3.)

21. $C = 2 \pi r; r$
22. $I = Prt; P$

23. $S = 180(n - 2); n$
24. $S = 2\pi r^2 + 2\pi rh; h$

In Exercises 25–32, name the property of equality that the statement illustrates.

25. If $x = y$, then $3x = 3y$.
26. If $AM = MB$, then $AM + 5 = MB + 5$.

27. $x = x$
28. If $x = y$, then $y = x$.

29. $m \angle Z = m \angle Z$
30. If $m \angle A = 29^\circ$ and $m \angle B = 29^\circ$, then $m \angle A = m \angle B$.

31. If $AB = LM$, then $LM = AB$.
32. If $BC = XY$ and $XY = 8$, then $BC = 8$. 

Chapter 2  Reasoning and Proofs
In Exercises 33–40, use the property to copy and complete the statement.

33. Substitution Property of Equality: If \( AB = 20 \), then \( AB + CD = \) ____.

34. Symmetric Property of Equality: If \( m \angle 1 = m \angle 2 \), then ____.

35. Addition Property of Equality: If \( AB = CD \), then \( AB + EF = \) ____.

36. Multiplication Property of Equality: If \( AB = CD \), then \( 5 \cdot AB = \) ____.

37. Subtraction Property of Equality: If \( LM = XY \), then \( LM - GH = \) ____.

38. Distributive Property: If \( 5(x + 8) = 2 \), then ____ + ____ = 2.

39. Transitive Property of Equality: If \( m \angle 1 = m \angle 2 \) and \( m \angle 2 = m \angle 3 \), then ____.

40. Reflexive Property of Equality: \( m \angle ABC = \) ____.

ERROR ANALYSIS In Exercises 41 and 42, describe and correct the error in solving the equation.

41. \[
\begin{align*}
7x &= x + 24 & \text{Given} \\
8x &= 24 & \text{Addition Property of Equality} \\
x &= 3 & \text{Division Property of Equality}
\end{align*}
\]

42. \[
\begin{align*}
6x + 14 &= 32 & \text{Given} \\
6x &= 18 & \text{Division Property of Equality} \\
x &= 3 & \text{Simplify.}
\end{align*}
\]

43. REWRITING A FORMULA The formula for the perimeter \( P \) of a rectangle is \( P = 2l + 2w \), where \( l \) is the length and \( w \) is the width. Solve the formula for \( l \). Justify each step. Then find the length of a rectangular lawn with a perimeter of 32 meters and a width of 5 meters.

44. REWRITING A FORMULA The formula for the area \( A \) of a trapezoid is \( A = \frac{1}{2}h(b_1 + b_2) \), where \( h \) is the height and \( b_1 \) and \( b_2 \) are the lengths of the two bases. Solve the formula for \( b_1 \). Justify each step. Then find the length of one of the bases of the trapezoid when the area of the trapezoid is 91 square meters, the height is 7 meters, and the length of the other base is 20 meters.

45. ANALYZING RELATIONSHIPS In the diagram, \( m \angle ABD = m \angle CBE \). Show that \( m \angle 1 = m \angle 3 \). (See Example 4.)

46. ANALYZING RELATIONSHIPS In the diagram, \( AC = BD \). Show that \( AB = CD \). (See Example 5.)

47. ANALYZING RELATIONSHIPS Copy and complete the table to show that \( m \angle 2 = m \angle 3 \).

<table>
<thead>
<tr>
<th>Equation</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m \angle 1 = m \angle 4 ), ( m \angle EHF = 90^\circ ), ( m \angle GHF = 90^\circ )</td>
<td>Given</td>
</tr>
<tr>
<td>( m \angle EHF = m \angle GHF )</td>
<td></td>
</tr>
<tr>
<td>( m \angle EHF = m \angle 1 + m \angle 2 )</td>
<td></td>
</tr>
<tr>
<td>( m \angle GHF = m \angle 3 + m \angle 4 )</td>
<td></td>
</tr>
<tr>
<td>( m \angle 1 + m \angle 2 = m \angle 3 + m \angle 4 )</td>
<td></td>
</tr>
<tr>
<td>( m \angle 2 = m \angle 3 )</td>
<td>Substitution Property of Equality</td>
</tr>
</tbody>
</table>

48. WRITING Compare the Reflexive Property of Equality with the Symmetric Property of Equality. How are the properties similar? How are they different?
Chapter 2  Reasoning and Proofs

REASONING  In Exercises 49 and 50, show that the perimeter of $\triangle ABC$ is equal to the perimeter of $\triangle ADC$.

49.

50.

MATHMATICAL CONNECTIONS  In the figure, $ZY \cong XW$, $ZX = 5x + 17$, $YW = 10 - 2x$, and $YX = 3$. Find $ZY$ and $XW$.

MATHEMATICAL CONNECTIONS  In the fi gure, $\overline{ZY} \cong \overline{XW}$, $\overline{ZX} = 5x + 17$, $\overline{YW} = 10 - 2x$, and $\overline{YX} = 3$. Find $\overline{ZY}$ and $\overline{XW}$.

52. HOW DO YOU SEE IT? The bar graph shows the number of hours each employee works at a grocery store. Give an example of the Reflexive, Symmetric, and Transitive Properties of Equality.

53. ATTENDING TO PRECISION  Which of the following statements illustrate the Symmetric Property of Equality? Select all that apply.

A. If $AC = RS$, then $RS = AC$.
B. If $x = 9$, then $9 = x$.
C. If $AD = BC$, then $DA = CB$.
D. $AB = BA$
E. If $AB = LM$ and $LM = RT$, then $AB = RT$.
F. If $XY = EF$, then $FE = XY$.

54. THOUGHT PROVOKING  Write examples from your everyday life to help you remember the Reflexive, Symmetric, and Transitive Properties of Equality. Justify your answers.

55. MULTIPLE REPRESENTATIONS  The formula to convert a temperature in degrees Fahrenheit ($^\circ F$) to degrees Celsius ($^\circ C$) is $C = \frac{5}{9}(F - 32)$.

a. Solve the formula for $F$. Justify each step.
b. Make a table that shows the conversion to Fahrenheit for each temperature: $0^\circ C$, $20^\circ C$, $32^\circ C$, and $41^\circ C$.
c. Use your table to graph the temperature in degrees Fahrenheit as a function of the temperature in degrees Celsius. Is this a linear function?

56. REASONING  Select all the properties that would also apply to inequalities. Explain your reasoning.

A. Addition Property
B. Subtraction Property
C. Substitution Property
D. Reflexive Property
E. Symmetric Property
F. Transitive Property

Maintaining Mathematical Proficiency  Reviewing what you learned in previous grades and lessons

Name the definition, property, or postulate that is represented by each diagram. (Section 1.2, Section 1.3, and Section 1.5)

57. $XY + YZ = XZ$
58. $m\angle ABD + m\angle DBC = m\angle ABC$
59. $A$
60. $B$