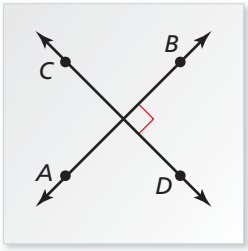
2.3 Postulates and Diagrams

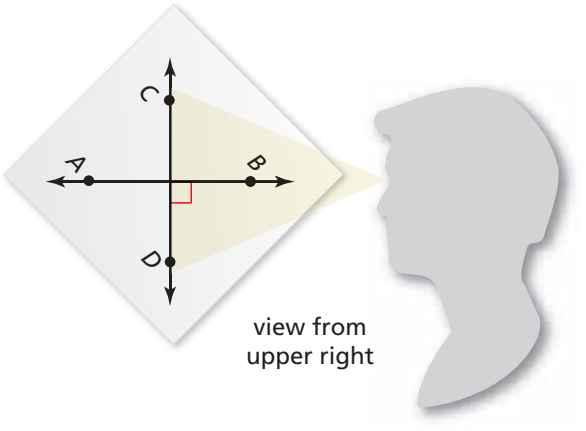
Essential Question In a diagram, what can be assumed and what needs to be labeled?

EXPLORATION 1 Looking at a Diagram

Work with a partner. On a piece of paper, draw two perpendicular lines. Label them $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. Look at the diagram from different angles. Do the lines appear perpendicular regardless of the angle at which you look at them? Describe *all* the angles at which you can look at the lines and have them appear perpendicular.



view from above



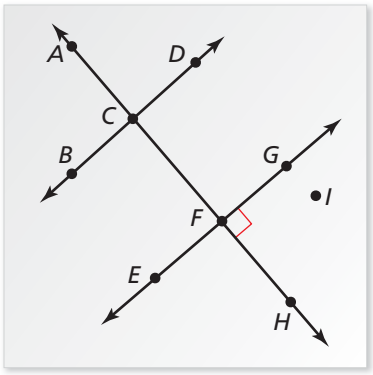
view from upper right

ATTENDING TO PRECISION

To be proficient in math, you need to state the meanings of the symbols you choose.

EXPLORATION 2 Interpreting a Diagram

Work with a partner. When you draw a diagram, you are communicating with others. It is important that you include sufficient information in the diagram. Use the diagram to determine which of the following statements you can assume to be true. Explain your reasoning.



a. All the points shown are coplanar.

b. Points D, G, and I are collinear.

c. Points A, C, and H are collinear.

d. $\overleftrightarrow{EG}$ and $\overleftrightarrow{AH}$ are perpendicular.

e. $\angle BCA$ and $\angle ACD$ are a linear pair.

f. $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are perpendicular.

g. $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ are parallel.

h. $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are coplanar.

i. $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ do not intersect.

j. $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ intersect.

k. $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ are perpendicular.

l. $\angle ACD$ and $\angle BCF$ are vertical angles.

m. $\overleftrightarrow{AC}$ and $\overleftrightarrow{FH}$ are the same line.

Communicate Your Answer

3. In a diagram, what can be assumed and what needs to be labeled?

4. Use the diagram in Exploration 2 to write two statements you can assume to be true and two statements you cannot assume to be true. Your statements should be different from those given in Exploration 2. Explain your reasoning.

2.3 Lesson

Core Vocabulary

line perpendicular to a plane, *p. 86*

Previous
postulate
point
line
plane

What You Will Learn

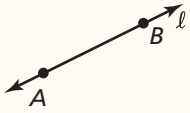
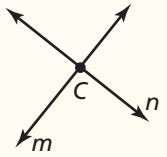
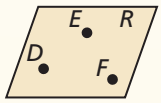
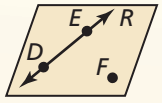
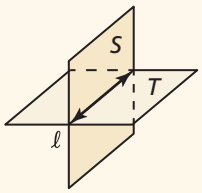
- Identify postulates using diagrams.
- Sketch and interpret diagrams.

Identifying Postulates

Here are seven more postulates involving points, lines, and planes.

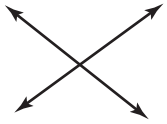
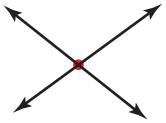
Postulates

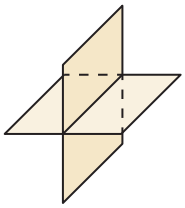
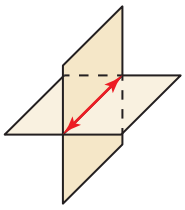
Point, Line, and Plane Postulates

Postulate	Example	
2.1 Two Point Postulate Through any two points, there exists exactly one line.		Through points A and B, there is exactly one line ℓ. Line ℓ contains at least two points.
2.2 Line-Point Postulate A line contains at least two points.		
2.3 Line Intersection Postulate If two lines intersect, then their intersection is exactly one point.		The intersection of line m and line n is point C.
2.4 Three Point Postulate Through any three noncollinear points, there exists exactly one plane.		Through points D, E, and F, there is exactly one plane, plane R. Plane R contains at least three noncollinear points.
2.5 Plane-Point Postulate A plane contains at least three noncollinear points.		
2.6 Plane-Line Postulate If two points lie in a plane, then the line containing them lies in the plane.		Points D and E lie in plane R, so $\overleftrightarrow{DE}$ lies in plane R.
2.7 Plane Intersection Postulate If two planes intersect, then their intersection is a line.		The intersection of plane S and plane T is line ℓ.

EXAMPLE 1 Identifying a Postulate Using a Diagram

State the postulate illustrated by the diagram.

a. If  then 

b. If  then 

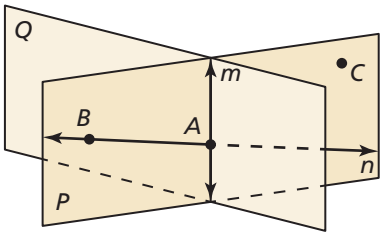
SOLUTION

a. Line Intersection Postulate If two lines intersect, then their intersection is exactly one point.

b. Plane Intersection Postulate If two planes intersect, then their intersection is a line.

EXAMPLE 2 Identifying Postulates from a Diagram

Use the diagram to write examples of the Plane-Point Postulate and the Plane-Line Postulate.



SOLUTION

Plane-Point Postulate Plane P contains at least three noncollinear points, A, B, and C.

Plane-Line Postulate Point A and point B lie in plane P. So, line n containing points A and B also lies in plane P.

Monitoring Progress

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1. Use the diagram in Example 2. Which postulate allows you to say that the intersection of plane P and plane Q is a line?
2. Use the diagram in Example 2 to write an example of the postulate.
   - **a.** Two Point Postulate
   - **b.** Line-Point Postulate
   - **c.** Line Intersection Postulate

Sketching and Interpreting Diagrams

EXAMPLE 3 Sketching a Diagram

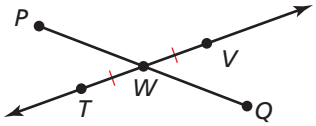
Sketch a diagram showing $\overleftrightarrow{TV}$ intersecting $\overline{PQ}$ at point W, so that $\overline{TW} \cong \overline{WV}$.

SOLUTION

Step 1 Draw $\overleftrightarrow{TV}$ and label points T and V.

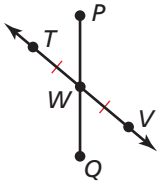
Step 2 Draw point W at the midpoint of $\overline{TV}$. Mark the congruent segments.

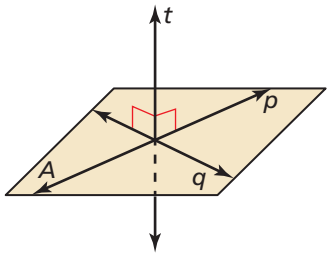
Step 3 Draw $\overline{PQ}$ through W.



ANOTHER WAY

In Example 3, there are many ways you can sketch the diagram. Another way is shown below.





A line is a **line perpendicular to a plane** if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

In a diagram, a line perpendicular to a plane must be marked with a right angle symbol, as shown.

EXAMPLE 4 Interpreting a Diagram

Which of the following statements *cannot* be assumed from the diagram?

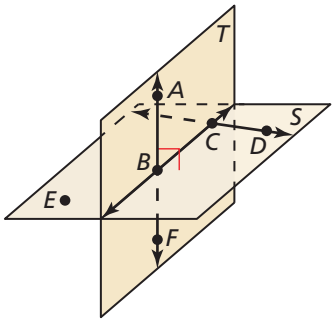
Points A, B, and F are collinear.

Points E, B, and D are collinear.

$\overleftrightarrow{AB} \perp$ plane S

$\overleftrightarrow{CD} \perp$ plane T

$\overleftrightarrow{AF}$ intersects $\overleftrightarrow{BC}$ at point B.



SOLUTION

No drawn line connects points E, B, and D. So, you cannot assume they are collinear. With no right angle marked, you cannot assume $\overleftrightarrow{CD} \perp$ plane T.

Monitoring Progress

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Refer back to Example 3.

3. If the given information states that $\overline{PW}$ and $\overline{QW}$ are congruent, how can you indicate that in the diagram?

4. Name a pair of supplementary angles in the diagram. Explain.

Use the diagram in Example 4.

5. Can you assume that plane S intersects plane T at $\overleftrightarrow{BC}$?

6. Explain how you know that $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$.

2.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** Through any _____ noncollinear points, there exists exactly one plane.

2. **WRITING** Explain why you need at least three noncollinear points to determine a plane.

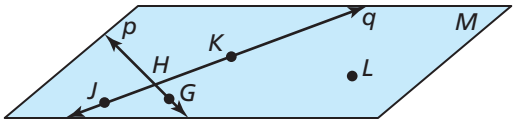
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, state the postulate illustrated by the diagram. *(See Example 1.)*

3. If A, B — then A, B

4. If — then A, B, C

In Exercises 5–8, use the diagram to write an example of the postulate. *(See Example 2.)*



5. Line-Point Postulate (Postulate 2.2)

6. Line Intersection Postulate (Postulate 2.3)

7. Three Point Postulate (Postulate 2.4)

8. Plane-Line Postulate (Postulate 2.6)

In Exercises 9–12, sketch a diagram of the description. *(See Example 3.)*

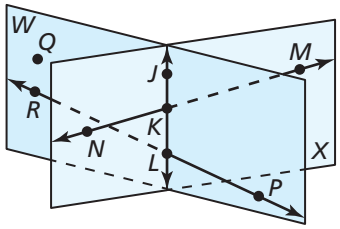
9. plane P and line m intersecting plane P at a 90° angle

10. $\overline{XY}$ in plane P, $\overline{XY}$ bisected by point A, and point C not on $\overline{XY}$

11. $\overline{XY}$ intersecting $\overline{WV}$ at point A, so that $XA = VA$

12. $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are all in plane P, and point X is the midpoint of all three segments.

In Exercises 13–20, use the diagram to determine whether you can assume the statement. *(See Example 4.)*



13. Planes W and X intersect at $\overleftrightarrow{KL}$.

14. Points K, L, M, and N are coplanar.

15. Points Q, J, and M are collinear.

16. $\overleftrightarrow{MN}$ and $\overleftrightarrow{RP}$ intersect.

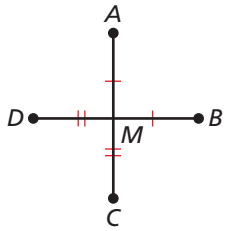
17. $\overleftrightarrow{JK}$ lies in plane X.

18. $\angle PLK$ is a right angle.

19. $\angle NKL$ and $\angle JKM$ are vertical angles.

20. $\angle NKJ$ and $\angle JKM$ are supplementary angles.

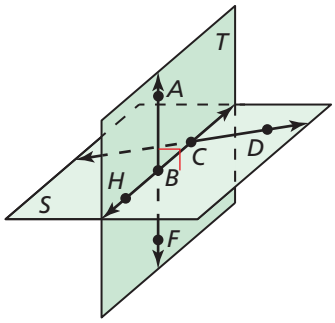
ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in the statement made about the diagram.**



21. ✗ M is the midpoint of $\overline{AC}$ and $\overline{BD}$.

22. ✗ $\overline{AC}$ intersects $\overline{BD}$ at a 90° angle, so $\overline{AC} \perp \overline{BD}$.

23. ATTENDING TO PRECISION Select all the statements about the diagram that you *cannot* conclude.



Ⓐ A, B, and C are coplanar.

Ⓑ Plane T intersects plane S in $\overleftrightarrow{BC}$.

Ⓒ $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$.

Ⓓ $\overleftrightarrow{HC} \perp \overleftrightarrow{CD}$.

Ⓔ Plane $T \perp$ plane S.

Ⓕ Point B bisects $\overline{HC}$.

Ⓖ $\angle ABH$ and $\angle HBF$ are a linear pair.

Ⓗ $\overleftrightarrow{AF} \perp \overleftrightarrow{CD}$.

24. HOW DO YOU SEE IT? Use the diagram of line m and point C. Make a conjecture about how many planes can be drawn so that line m and point C lie in the same plane. Use postulates to justify your conjecture.

25. MATHEMATICAL CONNECTIONS One way to graph a linear equation is to plot two points whose coordinates satisfy the equation and then connect them with a line. Which postulate guarantees this process works for any linear equation?

26. MATHEMATICAL CONNECTIONS A way to solve a system of two linear equations that intersect is to graph the lines and find the coordinates of their intersection. Which postulate guarantees this process works for any two linear equations?

In Exercises 27 and 28, (a) rewrite the postulate in if-then form. Then (b) write the converse, inverse, and contrapositive and state which ones are true.

27. Two Point Postulate (Postulate 2.1)

28. Plane-Point Postulate (Postulate 2.5)

29. REASONING Choose the correct symbol to go between the statements.

number of points to determine a line ☐ number of points to determine a plane

$<$ $\quad \leq \quad$ $=$ $\quad \geq \quad$ $>$

30. CRITICAL THINKING If two lines intersect, then they intersect in exactly one point by the Line Intersection Postulate (Postulate 2.3). Do the two lines have to be in the same plane? Draw a picture to support your answer. Then explain your reasoning.

31. MAKING AN ARGUMENT Your friend claims that even though two planes intersect in a line, it is possible for three planes to intersect in a point. Is your friend correct? Explain your reasoning.

32. MAKING AN ARGUMENT Your friend claims that by the Plane Intersection Postulate (Post. 2.7), any two planes intersect in a line. Is your friend's interpretation of the Plane Intersection Postulate (Post. 2.7) correct? Explain your reasoning.

33. ABSTRACT REASONING Points E, F, and G all lie in plane P and in plane Q. What must be true about points E, F, and G so that planes P and Q are different planes? What must be true about points E, F, and G to force planes P and Q to be the same plane? Make sketches to support your answers.

34. THOUGHT PROVOKING The postulates in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. A plane is the surface of the sphere. Find a postulate on page 84 that is not true in spherical geometry. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Tell which algebraic property of equality you used. *(Skills Review Handbook)*

35. $t - 6 = -4$ **36.** $3x = 21$ **37.** $9 + x = 13$ **38.** $\frac{x}{7} = 5$