*Essential Question* How can you use reasoning to solve problems?

A **conjecture** is an unproven statement based on observations.

**EXPLORATION 1** **Writing a Conjecture**

**Work with a partner.** Write a conjecture about the pattern. Then use your conjecture to draw the 10th object in the pattern.



**a.**

**b.**

**c.**
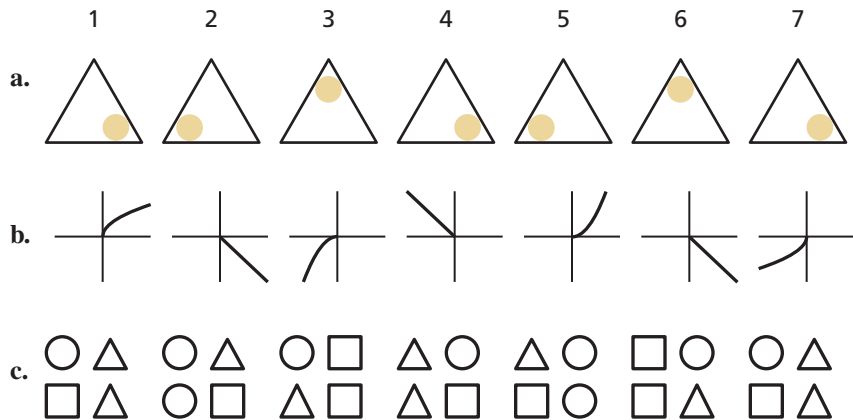
**CONSTRUCTING VIABLE ARGUMENTS**

To be proficient in math, you need to justify your conclusions and communicate them to others.

**EXPLORATION 2** **Using a Venn Diagram**

**Work with a partner.** Use the Venn diagram to determine whether the statement is true or false. Justify your answer. Assume that no region of the Venn diagram is empty.

**a.** If an item has Property B, then it has Property A.

**b.** If an item has Property A, then it has Property B.

**c.** If an item has Property A, then it has Property C.

**d.** Some items that have Property A do not have Property B.

**e.** If an item has Property C, then it does not have Property B.

**f.** Some items have both Properties A and C.

**g.** Some items have both Properties B and C.
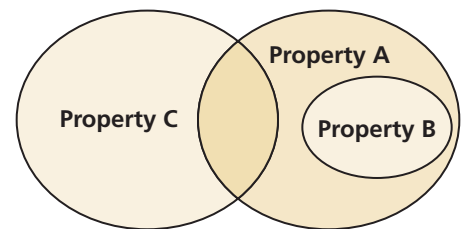


**EXPLORATION 3** **Reasoning and Venn Diagrams**

**Work with a partner.** Draw a Venn diagram that shows the relationship between different types of quadrilaterals: squares, rectangles, parallelograms, trapezoids, rhombuses, and kites. Then write several conditional statements that are shown in your diagram, such as "If a quadrilateral is a square, then it is a rectangle."

## Communicate Your Answer

**4.** How can you use reasoning to solve problems?

**5.** Give an example of how you used reasoning to solve a real-life problem.

## Core Vocabulary

conjecture, *p. 76*
inductive reasoning, *p. 76*
counterexample, *p. 77*
deductive reasoning, *p. 78*

# What You Will Learn

▶ Use inductive reasoning.

▶ Use deductive reasoning.

## Using Inductive Reasoning

### 🔄 Core Concept

**Inductive Reasoning**

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.
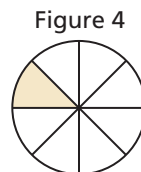
**EXAMPLE 1** **Describing a Visual Pattern**

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

Figure 1          Figure 2          Figure 3
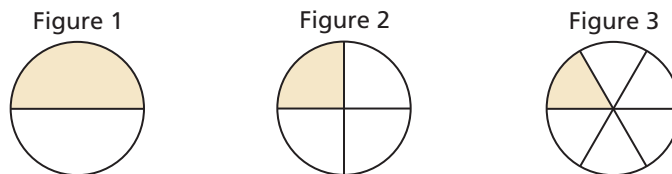
**SOLUTION**

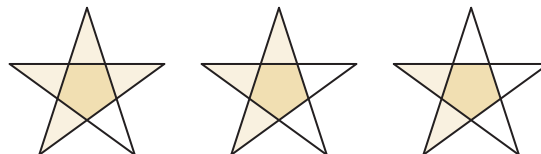Each circle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing a circle into eighths. Shade the section just above the horizontal segment at the left.

Figure 4

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1. Sketch the fifth figure in the pattern in Example 1.

**Sketch the next figure in the pattern.**

2.

3. ○

**EXAMPLE 2**  **Making and Testing a Conjecture**

Numbers such as 3, 4, and 5 are called *consecutive integers*. Make and test a conjecture about the sum of any three consecutive integers.

**SOLUTION**

**Step 1** Find a pattern using a few groups of small numbers.

$$3 + 4 + 5 = 12 = 4 \cdot 3 \qquad\qquad 7 + 8 + 9 = 24 = 8 \cdot 3$$

$$10 + 11 + 12 = 33 = 11 \cdot 3 \qquad 16 + 17 + 18 = 51 = 17 \cdot 3$$

**Step 2** Make a conjecture.

> **Conjecture** The sum of any three consecutive integers is three times the second number.

**Step 3** Test your conjecture using other numbers. For example, test that it works with the groups $-1, 0, 1$ and $100, 101, 102$.

$$-1 + 0 + 1 = 0 = 0 \cdot 3 \quad ✔$$

$$100 + 101 + 102 = 303 = 101 \cdot 3 \quad ✔$$

## Core Concept

**Counterexample**

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

**EXAMPLE 3**  **Finding a Counterexample**

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

**Conjecture** The sum of two numbers is always more than the greater number.

**SOLUTION**

To find a counterexample, you need to find a sum that is less than the greater number.

$$-2 + (-3) = -5$$

$$-5 \not> -2$$

▶ Because a counterexample exists, the conjecture is false.

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4. Make and test a conjecture about the sign of the product of any three negative integers.

5. Make and test a conjecture about the sum of any five consecutive integers.

**Find a counterexample to show that the conjecture is false.**

6. The value of $x^2$ is always greater than the value of $x$.

7. The sum of two numbers is always greater than their difference.

## Using Deductive Reasoning

### 🌀 Core Concept

**Deductive Reasoning**

**Deductive reasoning** uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

**Laws of Logic**

**Law of Detachment**

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

**Law of Syllogism**

If hypothesis $p$, then conclusion $q$.

If hypothesis $q$, then conclusion $r$. ⟩ If these statements are true,

If hypothesis $p$, then conclusion $r$. ← then this statement is true.

---

**EXAMPLE 4**  **Using the Law of Detachment**

If two segments have the same length, then they are congruent. You know that $BC = XY$. Using the Law of Detachment, what statement can you make?

**SOLUTION**

Because $BC = XY$ satisfies the hypothesis of a true conditional statement, the conclusion is also true.

▶ So, $\overline{BC} \cong \overline{XY}$.

---

**EXAMPLE 5**  **Using the Law of Syllogism**

If possible, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

**a.** If $x^2 > 25$, then $x^2 > 20$.

If $x > 5$, then $x^2 > 25$.

**b.** If a polygon is regular, then all angles in the interior of the polygon are congruent.

If a polygon is regular, then all its sides are congruent.

**SOLUTION**

**a.** Notice that the conclusion of the second statement is the hypothesis of the first statement. The order in which the statements are given does not affect whether you can use the Law of Syllogism. So, you can write the following new statement.

▶ If $x > 5$, then $x^2 > 20$.

**b.** Neither statement's conclusion is the same as the other statement's hypothesis.

▶ You cannot use the Law of Syllogism to write a new conditional statement.

**Using Inductive and Deductive Reasoning**

What conclusion can you make about the product of an even integer and any other integer?

**SOLUTION**

**Step 1** Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

$$(-2)(2) = -4 \qquad (-1)(2) = -2 \qquad 2(2) = 4 \qquad 3(2) = 6$$
$$(-2)(-4) = 8 \qquad (-1)(-4) = 4 \qquad 2(-4) = -8 \qquad 3(-4) = -12$$

**Conjecture** Even integer • Any integer = Even integer

**Step 2** Let $n$ and $m$ each be any integer. Use deductive reasoning to show that the conjecture is true.

$2n$ is an even integer because any integer multiplied by 2 is even.

$2nm$ represents the product of an even integer $2n$ and any integer $m$.

$2nm$ is the product of 2 and an integer $nm$. So, $2nm$ is an even integer.

▶ The product of an even integer and any integer is an even integer.

**Comparing Inductive and Deductive Reasoning**

Decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.

**a.** Each time Monica kicks a ball up in the air, it returns to the ground. So, the next time Monica kicks a ball up in the air, it will return to the ground.

**b.** All reptiles are cold-blooded. Parrots are not cold-blooded. Sue's pet parrot is not a reptile.

**SOLUTION**

**a.** Inductive reasoning, because a pattern is used to reach the conclusion.

**b.** Deductive reasoning, because facts about animals and the laws of logic are used to reach the conclusion.

**MAKING SENSE OF PROBLEMS**

In geometry, you will frequently use inductive reasoning to make conjectures. You will also use deductive reasoning to show that conjectures are true or false. You will need to know which type of reasoning to use.

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**8.** If $90° < m\angle R < 180°$, then $\angle R$ is obtuse. The measure of $\angle R$ is 155°. Using the Law of Detachment, what statement can you make?

**9.** Use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements.

If you get an A on your math test, then you can go to the movies.
If you go to the movies, then you can watch your favorite actor.

**10.** Use inductive reasoning to make a conjecture about the sum of a number and itself. Then use deductive reasoning to show that the conjecture is true.

**11.** Decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.

All multiples of 8 are divisible by 4.
64 is a multiple of 8.
So, 64 is divisible by 4.

## Vocabulary and Core Concept Check

1. **VOCABULARY** How does the prefix "counter-" help you understand the term counterexample?

2. **WRITING** Explain the difference between inductive reasoning and deductive reasoning.

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–8, describe the pattern. Then write or draw the next two numbers, letters, or figures.** *(See Example 1.)*

3. $1, -2, 3, -4, 5, \ldots$

4. $0, 2, 6, 12, 20, \ldots$

5. Z, Y, X, W, V, . . .

6. J, F, M, A, M, . . .

7.

8.
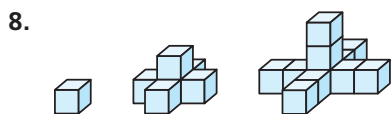
**In Exercises 9–12, make and test a conjecture about the given quantity.** *(See Example 2.)*

9. the product of any two even integers

10. the sum of an even integer and an odd integer

11. the quotient of a number and its reciprocal

12. the quotient of two negative integers

**In Exercises 13–16, find a counterexample to show that the conjecture is false.** *(See Example 3.)*

13. The product of two positive numbers is always greater than either number.

14. If $n$ is a nonzero integer, then $\dfrac{n+1}{n}$ is always greater than 1.

15. If two angles are supplements of each other, then one of the angles must be acute.

16. A line $s$ divides $\overline{MN}$ into two line segments. So, the line $s$ is a segment bisector of $\overline{MN}$.

**In Exercises 17–20, use the Law of Detachment to determine what you can conclude from the given information, if possible.** *(See Example 4.)*

17. If you pass the final, then you pass the class. You passed the final.

18. If your parents let you borrow the car, then you will go to the movies with your friend. You will go to the movies with your friend.

19. If a quadrilateral is a square, then it has four right angles. Quadrilateral $QRST$ has four right angles.

20. If a point divides a line segment into two congruent line segments, then the point is a midpoint. Point $P$ divides $\overline{LH}$ into two congruent line segments.

**In Exercises 21–24, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements, if possible.** *(See Example 5.)*

21. If $x < -2$, then $|x| > 2$. If $x > 2$, then $|x| > 2$.

22. If $a = 3$, then $5a = 15$. If $\frac{1}{2}a = 1\frac{1}{2}$, then $a = 3$.

23. If a figure is a rhombus, then the figure is a parallelogram. If a figure is a parallelogram, then the figure has two pairs of opposite sides that are parallel.

24. If a figure is a square, then the figure has four congruent sides. If a figure is a square, then the figure has four right angles.

**In Exercises 25–28, state the law of logic that is illustrated.**

25. If you do your homework, then you can watch TV. If you watch TV, then you can watch your favorite show.

   If you do your homework, then you can watch your favorite show.

**26.** If you miss practice the day before a game, then you will not be a starting player in the game.

You miss practice on Tuesday. You will not start the game Wednesday.

**27.** If $x > 12$, then $x + 9 > 20$. The value of $x$ is 14.

So, $x + 9 > 20$.

**28.** If $\angle 1$ and $\angle 2$ are vertical angles, then $\angle 1 \cong \angle 2$.
If $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.

If $\angle 1$ and $\angle 2$ are vertical angles, then $m\angle 1 = m\angle 2$.

**In Exercises 29 and 30, use inductive reasoning to make a conjecture about the given quantity. Then use deductive reasoning to show that the conjecture is true.** *(See Example 6.)*

**29.** the sum of two odd integers

**30.** the product of two odd integers

**In Exercises 31–34, decide whether inductive reasoning or deductive reasoning is used to reach the conclusion. Explain your reasoning.** *(See Example 7.)*

**31.** Each time your mom goes to the store, she buys milk. So, the next time your mom goes to the store, she will buy milk.

**32.** Rational numbers can be written as fractions. Irrational numbers cannot be written as fractions. So, $\frac{1}{2}$ is a rational number.

**33.** All men are mortal. Mozart is a man, so Mozart is mortal.

**34.** Each time you clean your room, you are allowed to go out with your friends. So, the next time you clean your room, you will be allowed to go out with your friends.

**ERROR ANALYSIS** **In Exercises 35 and 36, describe and correct the error in interpreting the statement.**

**35.** If a figure is a rectangle, then the figure has four sides. A trapezoid has four sides.

> ✗ Using the Law of Detachment, you can conclude that a trapezoid is a rectangle.

**36.** Each day, you get to school before your friend.

> ✗ Using deductive reasoning, you can conclude that you will arrive at school before your friend tomorrow.

**37.** **REASONING** The table shows the average weights of several subspecies of tigers. What conjecture can you make about the relation between the weights of female tigers and the weights of male tigers? Explain your reasoning.

| | Weight of female (pounds) | Weight of male (pounds) |
|---|---|---|
| **Amur** | 370 | 660 |
| **Bengal** | 300 | 480 |
| **South China** | 240 | 330 |
| **Sumatran** | 200 | 270 |
| **Indo-Chinese** | 250 | 400 |

**38.** **HOW DO YOU SEE IT?** Determine whether you can make each conjecture from the graph. Explain your reasoning.



U.S. High School Girls' Lacrosse
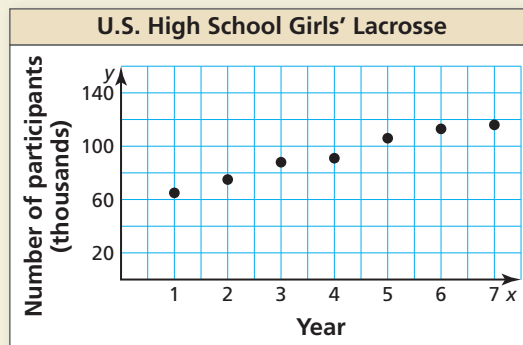
**a.** More girls will participate in high school lacrosse in Year 8 than those who participated in Year 7.

**b.** The number of girls participating in high school lacrosse will exceed the number of boys participating in high school lacrosse in Year 9.

**39.** **MATHEMATICAL CONNECTIONS** Use inductive reasoning to write a formula for the sum of the first $n$ positive even integers.

**40.** **FINDING A PATTERN** The following are the first nine *Fibonacci numbers.*

$$1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

**a.** Make a conjecture about each of the Fibonacci numbers after the first two.

**b.** Write the next three numbers in the pattern.

**c.** Research to find a real-world example of this pattern.

**41. MAKING AN ARGUMENT** Which argument is correct? Explain your reasoning.

**Argument 1:** If two angles measure 30° and 60°, then the angles are complementary. $\angle 1$ and $\angle 2$ are complementary. So, $m\angle 1 = 30°$ and $m\angle 2 = 60°$.

**Argument 2:** If two angles measure 30° and 60°, then the angles are complementary. The measure of $\angle 1$ is 30° and the measure of $\angle 2$ is 60°. So, $\angle 1$ and $\angle 2$ are complementary.

**42. THOUGHT PROVOKING** The first two terms of a sequence are $\frac{1}{4}$ and $\frac{1}{2}$. Describe three different possible patterns for the sequence. List the first five terms for each sequence.

**43. MATHEMATICAL CONNECTIONS** Use the table to make a conjecture about the relationship between $x$ and $y$. Then write an equation for $y$ in terms of $x$. Use the equation to test your conjecture for other values of $x$.

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 2 | 5 | 8 | 11 | 14 |

**44. REASONING** Use the pattern below. Each figure is made of squares that are 1 unit by 1 unit.
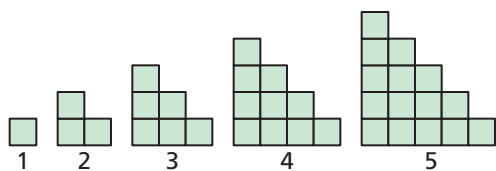


**a.** Find the perimeter of each figure. Describe the pattern of the perimeters.

**b.** Predict the perimeter of the 20th figure.

**45. DRAWING CONCLUSIONS** Decide whether each conclusion is valid. Explain your reasoning.

- Yellowstone is a national park in Wyoming.
- You and your friend went camping at Yellowstone National Park.
- When you go camping, you go canoeing.
- If you go on a hike, your friend goes with you.
- You go on a hike.
- There is a 3-mile-long trail near your campsite.

**a.** You went camping in Wyoming.

**b.** Your friend went canoeing.

**c.** Your friend went on a hike.

**d.** You and your friend went on a hike on a 3-mile-long trail.

**46. CRITICAL THINKING** Geologists use the Mohs' scale to determine a mineral's hardness. Using the scale, a mineral with a higher rating will leave a scratch on a mineral with a lower rating. Testing a mineral's hardness can help identify the mineral.

| Mineral | Talc | Gypsum | Calcite | Fluorite |
|---|---|---|---|---|
| Mohs' rating | 1 | 2 | 3 | 4 |

**a.** The four minerals are randomly labeled $A$, $B$, $C$, and $D$. Mineral $A$ is scratched by Mineral $B$. Mineral $C$ is scratched by all three of the other minerals. What can you conclude? Explain your reasoning.

**b.** What additional test(s) can you use to identify *all* the minerals in part (a)?

## Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

**Determine which postulate is illustrated by the statement.** *(Section 1.2 and Section 1.5)*
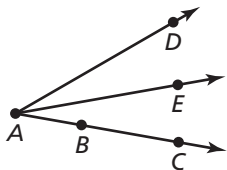


**47.** $AB + BC = AC$

**48.** $m\angle DAC = m\angle DAE + m\angle EAB$

**49.** $AD$ is the absolute value of the difference of the coordinates of $A$ and $D$.

**50.** $m\angle DAC$ is equal to the absolute value of the difference between the real numbers matched with $\overrightarrow{AD}$ and $\overrightarrow{AC}$ on a protractor.