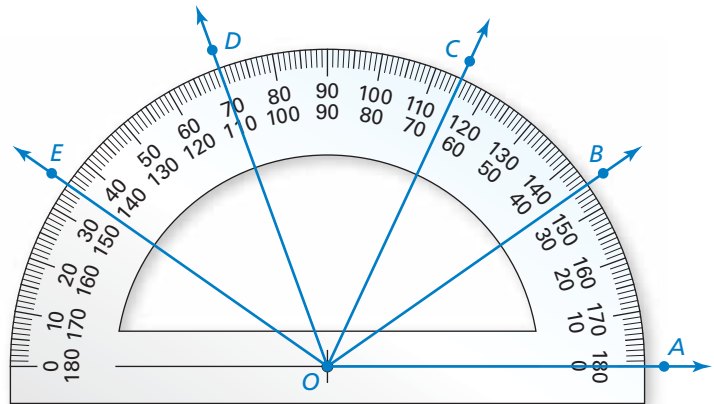


# 1.5 Measuring and Constructing Angles

**Essential Question** How can you measure and classify an angle?

## EXPLORATION 1 Measuring and Classifying Angles

**Work with a partner.** Find the degree measure of each of the following angles. Classify each angle as acute, right, or obtuse.

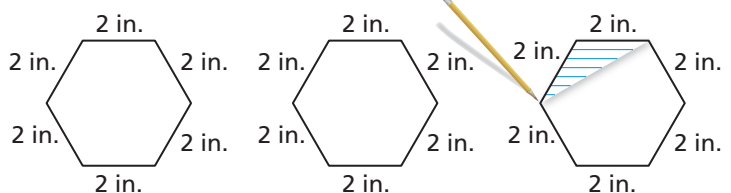
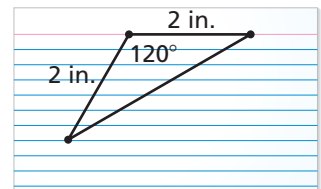


- |                 |                 |                 |                 |
|-----------------|-----------------|-----------------|-----------------|
| a. $\angle AOB$ | b. $\angle AOC$ | c. $\angle BOC$ | d. $\angle BOE$ |
| e. $\angle COE$ | f. $\angle COD$ | g. $\angle BOD$ | h. $\angle AOE$ |

## EXPLORATION 2 Drawing a Regular Polygon

**Work with a partner.**

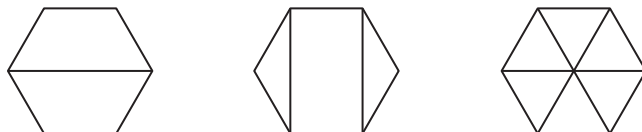
- Use a ruler and protractor to draw the triangular pattern shown at the right.
- Cut out the pattern and use it to draw three regular hexagons, as shown below.



### ATTENDING TO PRECISION

To be proficient in math, you need to calculate and measure accurately and efficiently.

- The sum of the angle measures of a polygon with  $n$  sides is equal to  $180(n - 2)^\circ$ . Do the angle measures of your hexagons agree with this rule? Explain.
- Partition your hexagons into smaller polygons, as shown below. For each hexagon, find the sum of the angle measures of the smaller polygons. Does each sum equal the sum of the angle measures of a hexagon? Explain.



## Communicate Your Answer

- How can you measure and classify an angle?

# 1.5 Lesson

## Core Vocabulary

angle, p. 38  
 vertex, p. 38  
 sides of an angle, p. 38  
 interior of an angle, p. 38  
 exterior of an angle, p. 38  
 measure of an angle, p. 39  
 acute angle, p. 39  
 right angle, p. 39  
 obtuse angle, p. 39  
 straight angle, p. 39  
 congruent angles, p. 40  
 angle bisector, p. 42

### Previous

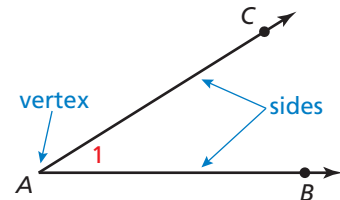
protractor  
 degrees

## What You Will Learn

- ▶ Name angles.
- ▶ Measure and classify angles.
- ▶ Identify congruent angles.
- ▶ Use the Angle Addition Postulate to find angle measures.
- ▶ Bisect angles.

## Naming Angles

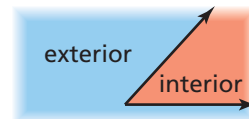
An **angle** is a set of points consisting of two different rays that have the same endpoint, called the **vertex**. The rays are the **sides** of the angle.



You can name an angle in several different ways.

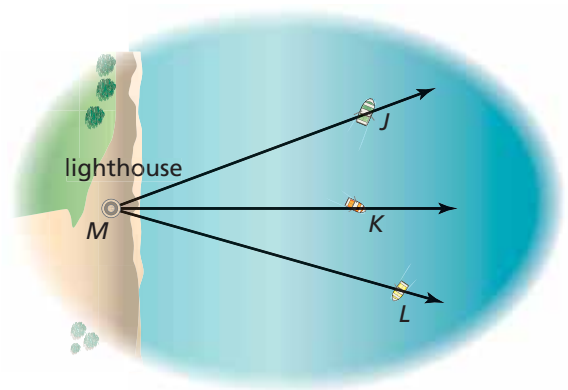
- Use its vertex, such as  $\angle A$ .
- Use a point on each ray and the vertex, such as  $\angle BAC$  or  $\angle CAB$ .
- Use a number, such as  $\angle 1$ .

The region that contains all the points between the sides of the angle is the **interior of the angle**. The region that contains all the points outside the angle is the **exterior of the angle**.



### EXAMPLE 1 Naming Angles

A lighthouse keeper measures the angles formed by the lighthouse at point  $M$  and three boats. Name three angles shown in the diagram.



### SOLUTION

- $\angle JMK$  or  $\angle KMJ$
- $\angle KML$  or  $\angle LMK$
- $\angle JML$  or  $\angle LMJ$

## COMMON ERROR

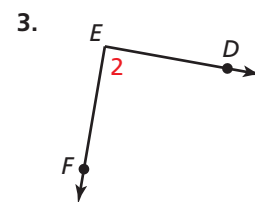
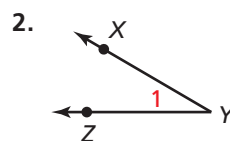
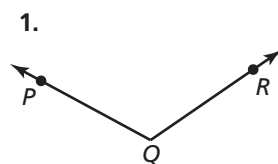
When a point is the vertex of more than one angle, you cannot use the vertex alone to name the angle.

## Monitoring Progress



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Write three names for the angle.



## Measuring and Classifying Angles

A protractor helps you approximate the *measure* of an angle. The measure is usually given in *degrees*.

### COMMON ERROR

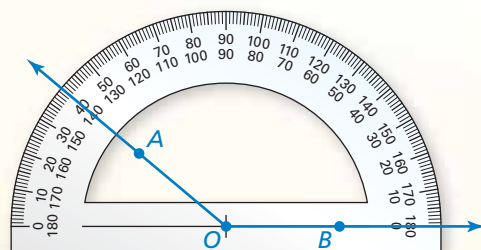
Most protractors have an inner and an outer scale. When measuring, make sure you are using the correct scale.

## Postulate

### Postulate 1.3 Protractor Postulate

Consider  $\overrightarrow{OB}$  and a point  $A$  on one side of  $\overrightarrow{OB}$ . The rays of the form  $\overrightarrow{OA}$  can be matched one to one with the real numbers from 0 to 180.

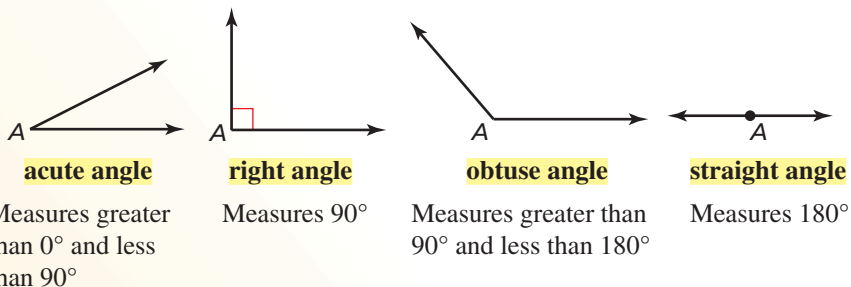
The **measure** of  $\angle AOB$ , which can be written as  $m\angle AOB$ , is equal to the absolute value of the difference between the real numbers matched with  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$  on a protractor.



You can classify angles according to their measures.

## Core Concept

### Types of Angles



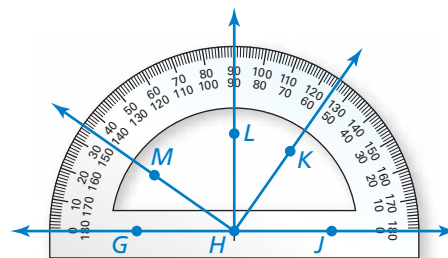
### EXAMPLE 2 Measuring and Classifying Angles

Find the measure of each angle.  
Then classify each angle.

- a.  $\angle GHK$     b.  $\angle JHL$     c.  $\angle LHK$

### SOLUTION

- a.  $\overrightarrow{HG}$  lines up with  $0^\circ$  on the outer scale of the protractor.  $\overrightarrow{HK}$  passes through  $125^\circ$  on the outer scale. So,  $m\angle GHK = 125^\circ$ . It is an *obtuse* angle.
- b.  $\overrightarrow{HJ}$  lines up with  $0^\circ$  on the inner scale of the protractor.  $\overrightarrow{HL}$  passes through  $90^\circ$ . So,  $m\angle JHL = 90^\circ$ . It is a *right* angle.
- c.  $\overrightarrow{HL}$  passes through  $90^\circ$ .  $\overrightarrow{HK}$  passes through  $55^\circ$  on the inner scale. So,  $m\angle LHK = |90 - 55| = 35^\circ$ . It is an *acute* angle.



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Use the diagram in Example 2 to find the angle measure. Then classify the angle.

4.  $\angle JHM$                       5.  $\angle MHK$                       6.  $\angle MHL$

## Identifying Congruent Angles

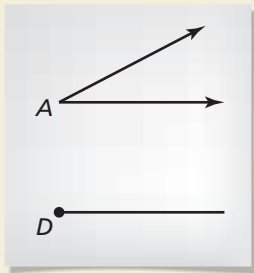
You can use a compass and straightedge to construct an angle that has the same measure as a given angle.

### CONSTRUCTION Copying an Angle

Use a compass and straightedge to construct an angle that has the same measure as  $\angle A$ . In this construction, the *center* of an arc is the point where the compass point rests. The *radius* of an arc is the distance from the center of the arc to a point on the arc drawn by the compass.

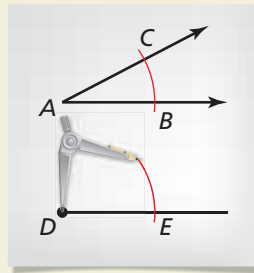
### SOLUTION

Step 1



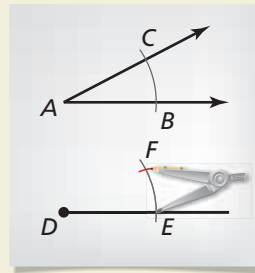
**Draw a segment** Draw an angle such as  $\angle A$ , as shown. Then draw a segment. Label a point  $D$  on the segment.

Step 2



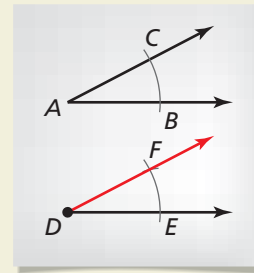
**Draw arcs** Draw an arc with center  $A$ . Using the same radius, draw an arc with center  $D$ .

Step 3



**Draw an arc** Label  $B$ ,  $C$ , and  $E$ . Draw an arc with radius  $BC$  and center  $E$ . Label the intersection  $F$ .

Step 4



**Draw a ray** Draw  $\overrightarrow{DF}$ .  $\angle EDF \cong \angle BAC$ .

Two angles are **congruent angles** when they have the same measure. In the construction above,  $\angle A$  and  $\angle D$  are congruent angles. So,

$$m\angle A = m\angle D \quad \text{The measure of angle } A \text{ is equal to the measure of angle } D.$$

and

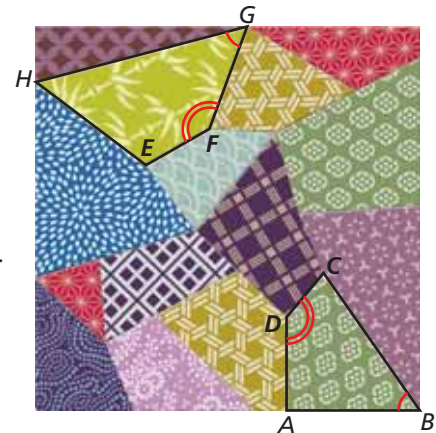
$$\angle A \cong \angle D. \quad \text{Angle } A \text{ is congruent to angle } D.$$

### EXAMPLE 3 Identifying Congruent Angles

- Identify the congruent angles labeled in the quilt design.
- $m\angle ADC = 140^\circ$ . What is  $m\angle EFG$ ?

### SOLUTION

- There are two pairs of congruent angles:  
 $\angle ABC \cong \angle FGH$  and  $\angle ADC \cong \angle EFG$ .
- Because  $\angle ADC \cong \angle EFG$ ,  
 $m\angle ADC = m\angle EFG$ .  
So,  $m\angle EFG = 140^\circ$ .



### READING

In diagrams, matching arcs indicate congruent angles. When there is more than one pair of congruent angles, use multiple arcs.

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- Without measuring, is  $\angle DAB \cong \angle FEH$  in Example 3? Explain your reasoning. Use a protractor to verify your answer.

## Using the Angle Addition Postulate

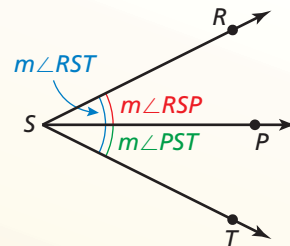
### Postulate

#### Postulate 1.4 Angle Addition Postulate

**Words** If  $P$  is in the interior of  $\angle RST$ , then the measure of  $\angle RST$  is equal to the sum of the measures of  $\angle RSP$  and  $\angle PST$ .

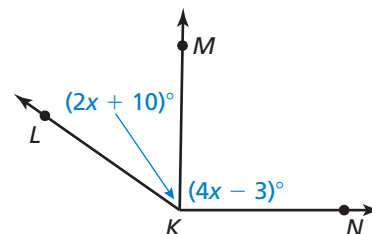
**Symbols** If  $P$  is in the interior of  $\angle RST$ , then

$$m\angle RST = m\angle RSP + m\angle PST.$$



#### EXAMPLE 4 Finding Angle Measures

Given that  $m\angle LKN = 145^\circ$ ,  
find  $m\angle LKM$  and  $m\angle MKN$ .



#### SOLUTION

**Step 1** Write and solve an equation to find the value of  $x$ .

$$m\angle LKN = m\angle LKM + m\angle MKN$$

Angle Addition Postulate

$$145^\circ = (2x + 10)^\circ + (4x - 3)^\circ$$

Substitute angle measures.

$$145 = 6x + 7$$

Combine like terms.

$$138 = 6x$$

Subtract 7 from each side.

$$23 = x$$

Divide each side by 6.

**Step 2** Evaluate the given expressions when  $x = 23$ .

$$m\angle LKM = (2x + 10)^\circ = (2 \cdot 23 + 10)^\circ = 56^\circ$$

$$m\angle MKN = (4x - 3)^\circ = (4 \cdot 23 - 3)^\circ = 89^\circ$$

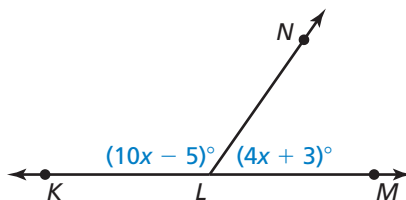
► So,  $m\angle LKM = 56^\circ$ , and  $m\angle MKN = 89^\circ$ .

#### Monitoring Progress

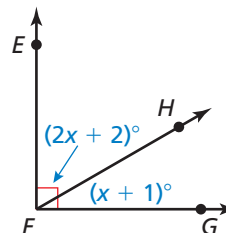
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Find the indicated angle measures.

8. Given that  $\angle KLM$  is a straight angle, find  $m\angle KLN$  and  $m\angle NLM$ .

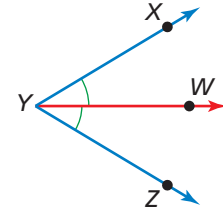


9. Given that  $\angle EFG$  is a right angle, find  $m\angle EFH$  and  $m\angle HFG$ .



## Bisecting Angles

An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the figure,  $\overrightarrow{YW}$  bisects  $\angle XYZ$ , so  $\angle XYW \cong \angle ZYW$ .



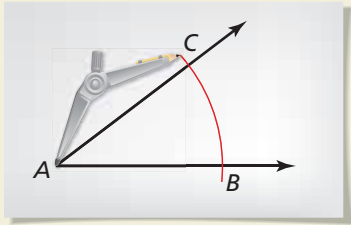
You can use a compass and straightedge to bisect an angle.

### CONSTRUCTION Bisecting an Angle

Construct an angle bisector of  $\angle A$  with a compass and straightedge.

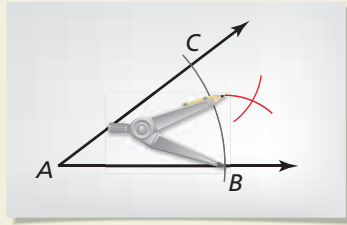
### SOLUTION

#### Step 1



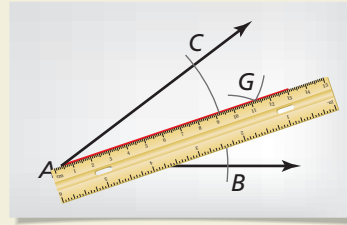
**Draw an arc** Draw an angle such as  $\angle A$ , as shown. Place the compass at  $A$ . Draw an arc that intersects both sides of the angle. Label the intersections  $B$  and  $C$ .

#### Step 2



**Draw arcs** Place the compass at  $C$ . Draw an arc. Then place the compass point at  $B$ . Using the same radius, draw another arc.

#### Step 3



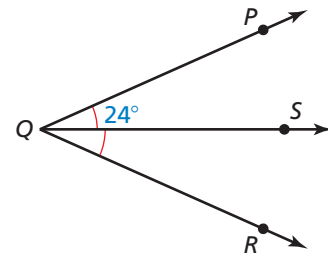
**Draw a ray** Label the intersection  $G$ . Use a straightedge to draw a ray through  $A$  and  $G$ .  $\overrightarrow{AG}$  bisects  $\angle A$ .

### EXAMPLE 5 Using a Bisector to Find Angle Measures

$\overrightarrow{QS}$  bisects  $\angle PQR$ , and  $m\angle PQS = 24^\circ$ . Find  $m\angle PQR$ .

### SOLUTION

**Step 1** Draw a diagram.



**Step 2** Because  $\overrightarrow{QS}$  bisects  $\angle PQR$ ,  $m\angle PQS = m\angle RQS$ . So,  $m\angle RQS = 24^\circ$ . Use the Angle Addition Postulate to find  $m\angle PQR$ .

$$\begin{aligned} m\angle PQR &= m\angle PQS + m\angle RQS \\ &= 24^\circ + 24^\circ \\ &= 48^\circ \end{aligned}$$

Angle Addition Postulate  
Substitute angle measures.  
Add.

► So,  $m\angle PQR = 48^\circ$ .

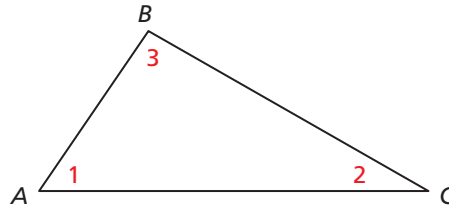
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10. Angle  $MNP$  is a straight angle, and  $\overrightarrow{NQ}$  bisects  $\angle MNP$ . Draw  $\angle MNP$  and  $\overrightarrow{NQ}$ . Use arcs to mark the congruent angles in your diagram. Find the angle measures of these congruent angles.

# 1.5 Exercises

## Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** Two angles are \_\_\_\_\_ angles when they have the same measure.
- WHICH ONE DOESN'T BELONG?** Which angle name does *not* belong with the other three? Explain your reasoning.



$\angle BCA$

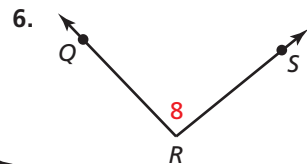
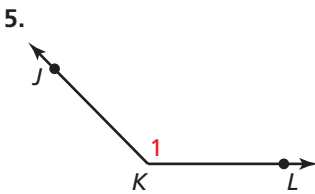
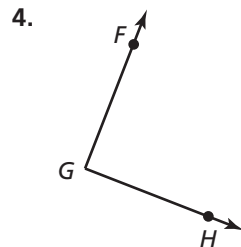
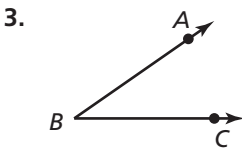
$\angle BAC$

$\angle 1$

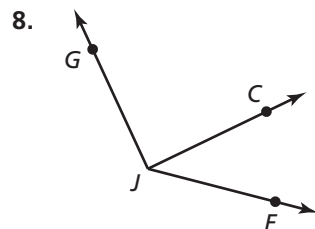
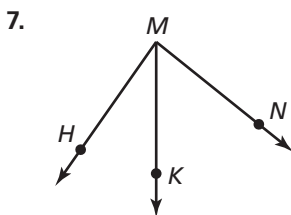
$\angle CAB$

## Monitoring Progress and Modeling with Mathematics

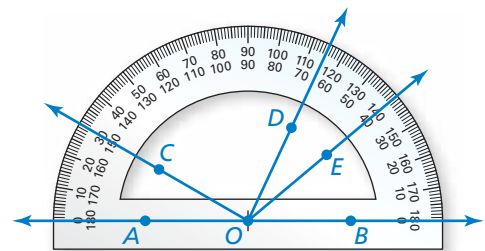
In Exercises 3–6, write three names for the angle.  
(See Example 1.)



In Exercises 7 and 8, name three different angles in the diagram. (See Example 1.)



In Exercises 9–12, find the angle measure. Then classify the angle. (See Example 2.)



9.  $m\angle AOC$

10.  $m\angle BOD$

11.  $m\angle COD$

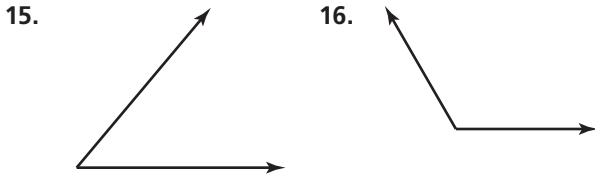
12.  $m\angle EOD$

**ERROR ANALYSIS** In Exercises 13 and 14, describe and correct the error in finding the angle measure. Use the diagram from Exercises 9–12.

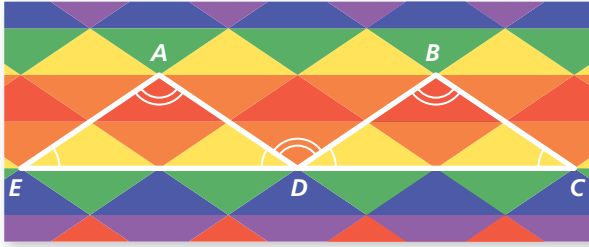
13.  $m\angle BOC = 30^\circ$

14.  $m\angle DOE = 65^\circ$

**CONSTRUCTION** In Exercises 15 and 16, use a compass and straightedge to copy the angle.



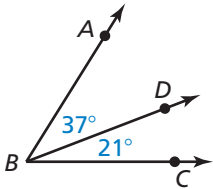
In Exercises 17–20,  $m\angle AED = 34^\circ$  and  $m\angle EAD = 112^\circ$ .  
(See Example 3.)



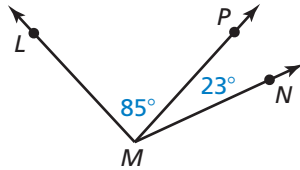
17. Identify the angles congruent to  $\angle AED$ .
18. Identify the angles congruent to  $\angle EAD$ .
19. Find  $m\angle BDC$ .
20. Find  $m\angle ADB$ .

In Exercises 21–24, find the indicated angle measure.

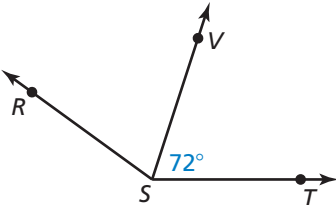
21. Find  $m\angle ABC$ .



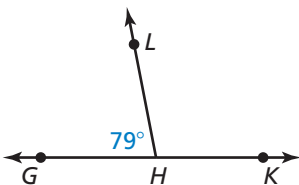
22. Find  $m\angle LMN$ .



23.  $m\angle RST = 114^\circ$ . Find  $m\angle RSV$ .

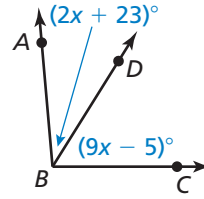


24.  $\angle GHK$  is a straight angle. Find  $m\angle LHK$ .

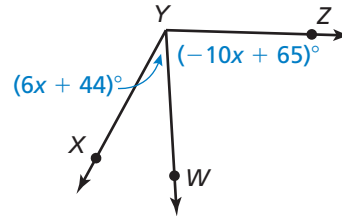


In Exercises 25–30, find the indicated angle measures.  
(See Example 4.)

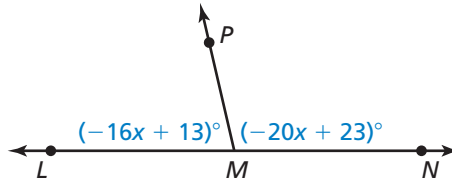
25.  $m\angle ABC = 95^\circ$ . Find  $m\angle ABD$  and  $m\angle DBC$ .



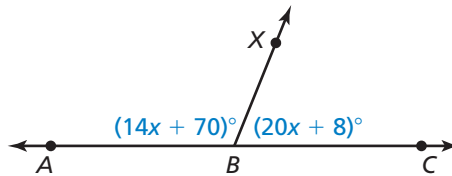
26.  $m\angle XYZ = 117^\circ$ . Find  $m\angle XYW$  and  $m\angle WYZ$ .



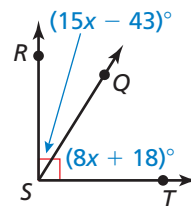
27.  $\angle LMN$  is a straight angle. Find  $m\angle LMP$  and  $m\angle NMP$ .



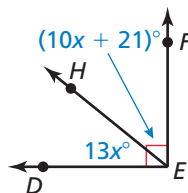
28.  $\angle ABC$  is a straight angle. Find  $m\angle ABX$  and  $m\angle CBX$ .



29. Find  $m\angle RSQ$  and  $m\angle TSQ$ .



30. Find  $m\angle DEH$  and  $m\angle FEH$ .

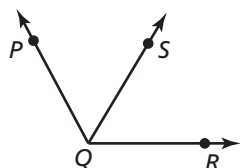




**CONSTRUCTION** In Exercises 31 and 32, copy the angle. Then construct the angle bisector with a compass and straightedge.

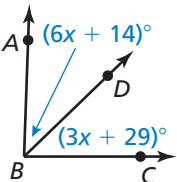
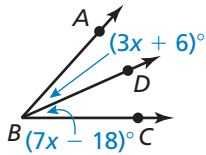
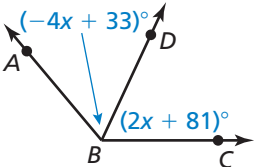
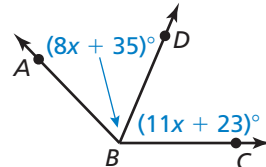


In Exercises 33–36,  $\overrightarrow{QS}$  bisects  $\angle PQR$ . Use the diagram and the given angle measure to find the indicated angle measures. (See Example 5.)

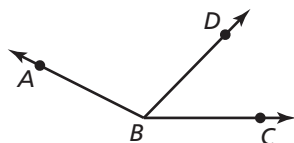


33.  $m\angle PQS = 63^\circ$ . Find  $m\angle RQS$  and  $m\angle PQR$ .
34.  $m\angle RQS = 71^\circ$ . Find  $m\angle PQS$  and  $m\angle PQR$ .
35.  $m\angle PQR = 124^\circ$ . Find  $m\angle PQS$  and  $m\angle RQS$ .
36.  $m\angle PQR = 119^\circ$ . Find  $m\angle PQS$  and  $m\angle RQS$ .

In Exercises 37–40,  $\overrightarrow{BD}$  bisects  $\angle ABC$ . Find  $m\angle ABD$ ,  $m\angle CBD$ , and  $m\angle ABC$ .

37. 
38. 
39. 
40. 

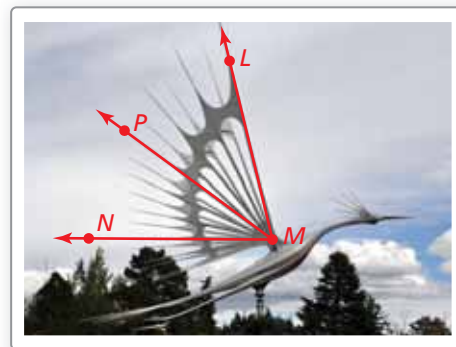
41. **WRITING** Explain how to find  $m\angle ABD$  when you are given  $m\angle ABC$  and  $m\angle CBD$ .



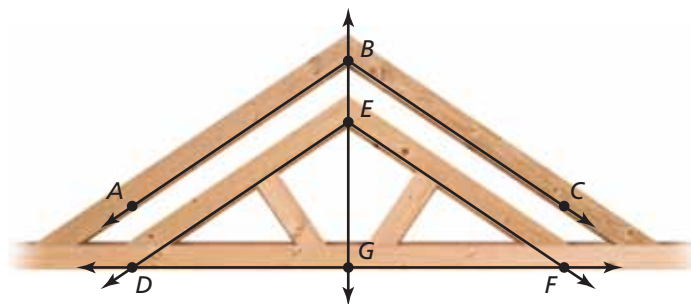
42. **ANALYZING RELATIONSHIPS** The map shows the intersections of three roads. Malcom Way intersects Sydney Street at an angle of  $162^\circ$ . Park Road intersects Sydney Street at an angle of  $87^\circ$ . Find the angle at which Malcom Way intersects Park Road.



43. **ANALYZING RELATIONSHIPS** In the sculpture shown in the photograph, the measure of  $\angle LMN$  is  $76^\circ$  and the measure of  $\angle PMN$  is  $36^\circ$ . What is the measure of  $\angle LMP$ ?



**USING STRUCTURE** In Exercises 44–46, use the diagram of the roof truss.



44. In the roof truss,  $\overrightarrow{BG}$  bisects  $\angle ABC$  and  $\angle DEF$ ,  $m\angle ABC = 112^\circ$ , and  $\angle ABC \cong \angle DEF$ . Find the measure of each angle.
- a.  $m\angle DEF$                       b.  $m\angle ABG$   
c.  $m\angle CBG$                       d.  $m\angle DEG$
45. In the roof truss,  $\angle DGF$  is a straight angle and  $\overrightarrow{GB}$  bisects  $\angle DGF$ . Find  $m\angle DGE$  and  $m\angle FGE$ .
46. Name an example of each of the four types of angles according to their measures in the diagram.

47. **MATHEMATICAL CONNECTIONS** In  $\angle ABC$ ,  $\overrightarrow{BX}$  is in the interior of the angle,  $m\angle ABX$  is 12 more than 4 times  $m\angle CBX$ , and  $m\angle ABC = 92^\circ$ .
- Draw a diagram to represent the situation.
  - Write and solve an equation to find  $m\angle ABX$  and  $m\angle CBX$ .

48. **THOUGHT PROVOKING** The angle between the minute hand and the hour hand of a clock is  $90^\circ$ . What time is it? Justify your answer.

49. **ABSTRACT REASONING** Classify the angles that result from bisecting each type of angle.

- acute angle
- right angle
- obtuse angle
- straight angle

50. **ABSTRACT REASONING** Classify the angles that result from drawing a ray in the interior of each type of angle. Include all possibilities and explain your reasoning.

- acute angle
- right angle
- obtuse angle
- straight angle

51. **CRITICAL THINKING** The ray from the origin through  $(4, 0)$  forms one side of an angle. Use the numbers below as  $x$ - and  $y$ -coordinates to create each type of angle in a coordinate plane.

-2

-1

0

1

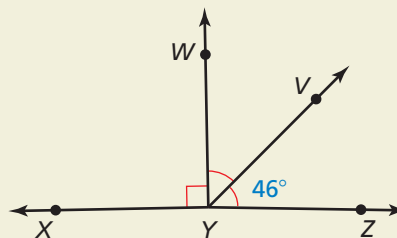
2

- acute angle
- right angle
- obtuse angle
- straight angle

52. **MAKING AN ARGUMENT** Your friend claims it is possible for a straight angle to consist of two obtuse angles. Is your friend correct? Explain your reasoning.

53. **CRITICAL THINKING** Two acute angles are added together. What type(s) of angle(s) do they form? Explain your reasoning.

54. **HOW DO YOU SEE IT?** Use the diagram.



- Is it possible for  $\angle XYZ$  to be a straight angle? Explain your reasoning.
- What can you change in the diagram so that  $\angle XYZ$  is a straight angle?

55. **WRITING** Explain the process of bisecting an angle in your own words. Compare it to bisecting a segment.

56. **ANALYZING RELATIONSHIPS**  $\overrightarrow{SQ}$  bisects  $\angle RST$ ,  $\overrightarrow{SP}$  bisects  $\angle RSQ$ , and  $\overrightarrow{SV}$  bisects  $\angle RSP$ . The measure of  $\angle VSP$  is  $17^\circ$ . Find  $m\angle TSQ$ . Explain.

57. **ABSTRACT REASONING** A bubble level is a tool used to determine whether a surface is horizontal, like the top of a picture frame. If the bubble is not exactly in the middle when the level is placed on the surface, then the surface is not horizontal. What is the most realistic type of angle formed by the level and a horizontal line when the bubble is not in the middle? Explain your reasoning.



## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Solve the equation. (*Skills Review Handbook*)

58.  $x + 67 = 180$

59.  $x + 58 = 90$

60.  $16 + x = 90$

61.  $109 + x = 180$

62.  $(6x + 7) + (13x + 21) = 180$

63.  $(3x + 15) + (4x - 9) = 90$

64.  $(11x - 25) + (24x + 10) = 90$

65.  $(14x - 18) + (5x + 8) = 180$