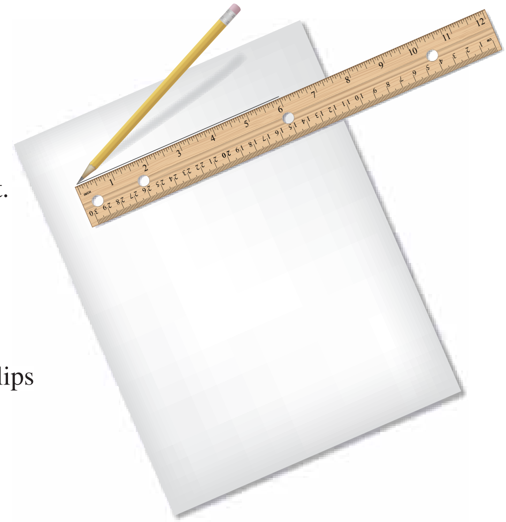
1.2 Measuring and Constructing Segments

Essential Question How can you measure and construct a line segment?

EXPLORATION 1 Measuring Line Segments Using Nonstandard Units

Work with a partner.



a. Draw a line segment that has a length of 6 inches.

b. Use a standard-sized paper clip to measure the length of the line segment. Explain how you measured the line segment in “paper clips.”

MAKING SENSE OF PROBLEMS

To be proficient in math, you need to explain to yourself the meaning of a problem and look for entry points to its solution.

c. Write conversion factors from paper clips to inches and vice versa.

1 paper clip = ____ in.

1 in. = ____ paper clip

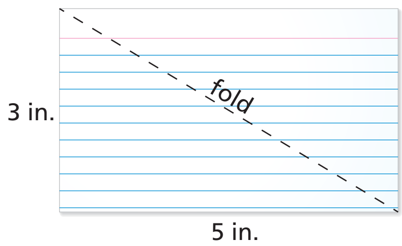
d. A *straightedge* is a tool that you can use to draw a straight line. An example of a straightedge is a ruler. Use only a pencil, straightedge, paper clip, and paper to draw another line segment that is 6 inches long. Explain your process.

EXPLORATION 2 Measuring Line Segments Using Nonstandard Units

Work with a partner.

a. Fold a 3-inch by 5-inch index card on one of its diagonals.

b. Use the Pythagorean Theorem to algebraically determine the length of the diagonal in inches. Use a ruler to check your answer.



c. Measure the length and width of the index card in paper clips.

d. Use the Pythagorean Theorem to algebraically determine the length of the diagonal in paper clips. Then check your answer by measuring the length of the diagonal in paper clips. Does the Pythagorean Theorem work for any unit of measure? Justify your answer.

EXPLORATION 3 Measuring Heights Using Nonstandard Units

Work with a partner. Consider a unit of length that is equal to the length of the diagonal you found in Exploration 2. Call this length “1 diag.” How tall are you in diags? Explain how you obtained your answer.

Communicate Your Answer

4. How can you measure and construct a line segment?

1.2 Lesson

Core Vocabulary

postulate, *p. 12*
axiom, *p. 12*
coordinate, *p. 12*
distance, *p. 12*
construction, *p. 13*
congruent segments, *p. 13*
between, *p. 14*

What You Will Learn

- Use the Ruler Postulate.
- Copy segments and compare segments for congruence.
- Use the Segment Addition Postulate.

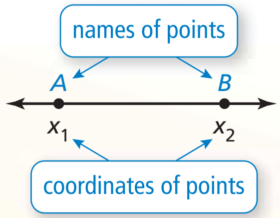
Using the Ruler Postulate

In geometry, a rule that is accepted without proof is called a **postulate** or an **axiom**. A rule that can be proved is called a *theorem*, as you will see later. Postulate 1.1 shows how to find the distance between two points on a line.

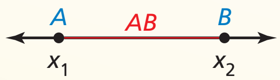
Postulate

Postulate 1.1 Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.



The **distance** between points A and B, written as AB, is the absolute value of the difference of the coordinates of A and B.



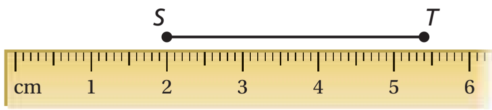
$AB = |x_2 - x_1|$

EXAMPLE 1 **Using the Ruler Postulate**

Measure the length of $\overline{ST}$ to the nearest tenth of a centimeter.

SOLUTION

Align one mark of a metric ruler with S. Then estimate the coordinate of T. For example, when you align S with 2, T appears to align with 5.4.



$ST = |5.4 - 2| = 3.4$ Ruler Postulate

▶ So, the length of $\overline{ST}$ is about 3.4 centimeters.

Monitoring Progress

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Use a ruler to measure the length of the segment to the nearest $\frac{1}{8}$ inch.

1. 

2. 

3. 

4. 

Constructing and Comparing Congruent Segments

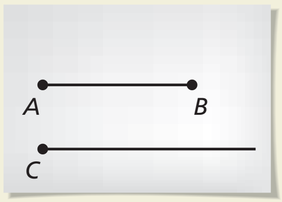
A **construction** is a geometric drawing that uses a limited set of tools, usually a *compass* and *straightedge*.

CONSTRUCTION **Copying a Segment**

Use a compass and straightedge to construct a line segment that has the same length as $\overline{AB}$.

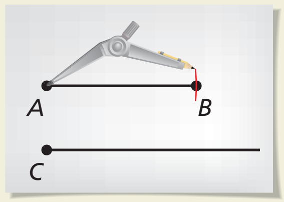
SOLUTION

Step 1



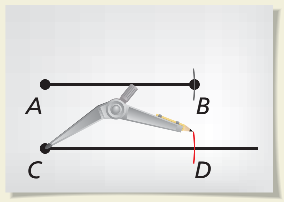
Draw a segment Use a straightedge to draw a segment longer than $\overline{AB}$. Label point C on the new segment.

Step 2



Measure length Set your compass at the length of $\overline{AB}$.

Step 3

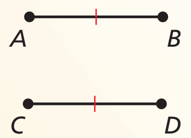


Copy length Place the compass at C. Mark point D on the new segment. So, $\overline{CD}$ has the same length as $\overline{AB}$.

Core Concept

Congruent Segments

Line segments that have the same length are called **congruent segments**. You can say "the length of $\overline{AB}$ is equal to the length of $\overline{CD}$," or you can say "$\overline{AB}$ *is congruent to* $\overline{CD}$." The symbol $\cong$ means "is congruent to."



Lengths are equal.
$AB = CD$
"is equal to"

Segments are congruent.
$\overline{AB} \cong \overline{CD}$
"is congruent to"

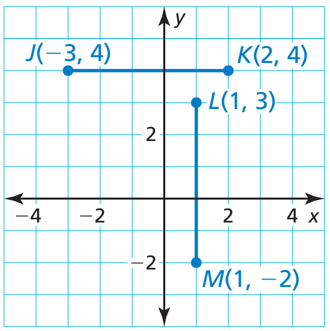
READING

In the diagram, the red tick marks indicate $\overline{AB} \cong \overline{CD}$. When there is more than one pair of congruent segments, use multiple tick marks.

EXAMPLE 2 **Comparing Segments for Congruence**

Plot $J(-3, 4)$, $K(2, 4)$, $L(1, 3)$, and $M(1, -2)$ in a coordinate plane. Then determine whether $\overline{JK}$ and $\overline{LM}$ are congruent.

SOLUTION



Plot the points, as shown. To find the length of a horizontal segment, find the absolute value of the difference of the x-coordinates of the endpoints.

$JK = |2 - (-3)| = 5$ Ruler Postulate

To find the length of a vertical segment, find the absolute value of the difference of the y-coordinates of the endpoints.

$LM = |-2 - 3| = 5$ Ruler Postulate

▶ $\overline{JK}$ and $\overline{LM}$ have the same length. So, $\overline{JK} \cong \overline{LM}$.

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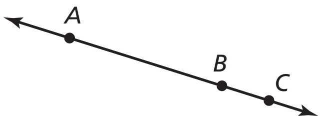
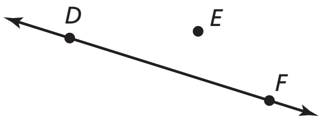
5. Plot $A(-2, 4)$, $B(3, 4)$, $C(0, 2)$, and $D(0, -2)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

Using the Segment Addition Postulate

When three points are collinear, you can say that one point is **between** the other two.



Point B is between points A and C.



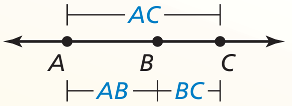
Point E is not between points D and F.

Postulate

Postulate 1.2 Segment Addition Postulate

If B is between A and C, then $AB + BC = AC$.

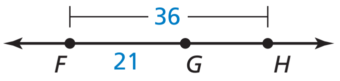
If $AB + BC = AC$, then B is between A and C.



EXAMPLE 3 Using the Segment Addition Postulate

a. Find DF.

b. Find GH.



SOLUTION

a. Use the Segment Addition Postulate to write an equation. Then solve the equation to find DF.

$DF = DE + EF$	Segment Addition Postulate
$DF = 23 + 35$	Substitute 23 for DE and 35 for EF.
$DF = 58$	Add.

b. Use the Segment Addition Postulate to write an equation. Then solve the equation to find GH.

$FH = FG + GH$	Segment Addition Postulate
$36 = 21 + GH$	Substitute 36 for FH and 21 for FG.
$15 = GH$	Subtract 21 from each side.

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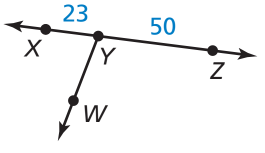
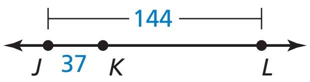
Use the diagram at the right.

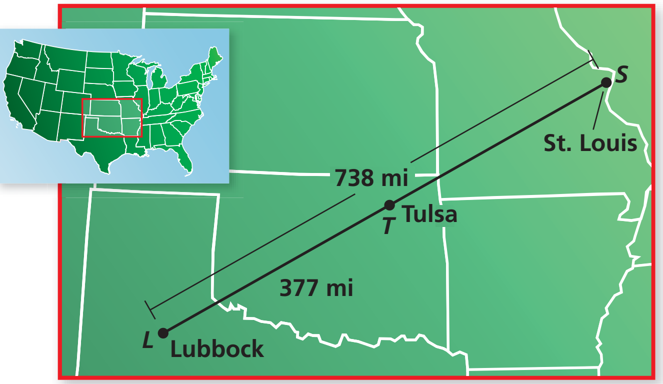


6. Use the Segment Addition Postulate to find XZ.
7. In the diagram, $WY = 30$. Can you use the Segment Addition Postulate to find the distance between points W and Z? Explain your reasoning.
8. Use the diagram at the left to find KL.



EXAMPLE 4 Using the Segment Addition Postulate

The cities shown on the map lie approximately in a straight line. Find the distance from Tulsa, Oklahoma, to St. Louis, Missouri.



SOLUTION

1. **Understand the Problem** You are given the distance from Lubbock to St. Louis and the distance from Lubbock to Tulsa. You need to find the distance from Tulsa to St. Louis.
2. **Make a Plan** Use the Segment Addition Postulate to find the distance from Tulsa to St. Louis.
3. **Solve the Problem** Use the Segment Addition Postulate to write an equation. Then solve the equation to find TS.

$LS = LT + TS$	Segment Addition Postulate
$738 = 377 + TS$	Substitute 738 for LS and 377 for LT.
$361 = TS$	Subtract 377 from each side.

So, the distance from Tulsa to St. Louis is about 361 miles.

4. **Look Back** Does the answer make sense in the context of the problem? The distance from Lubbock to St. Louis is 738 miles. By the Segment Addition Postulate, the distance from Lubbock to Tulsa plus the distance from Tulsa to St. Louis should equal 738 miles.

$377 + 361 = 738$ ✓

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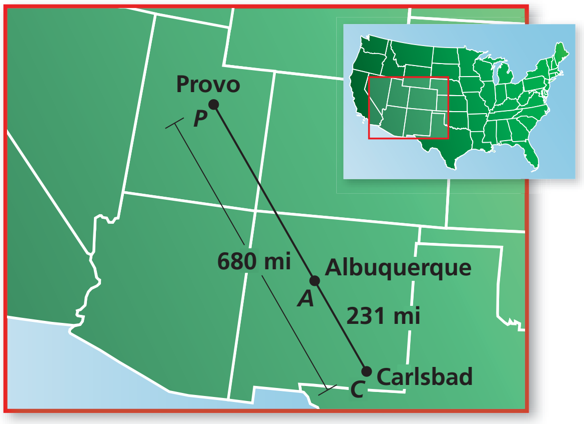
9. The cities shown on the map lie approximately in a straight line. Find the distance from Albuquerque, New Mexico, to Provo, Utah.



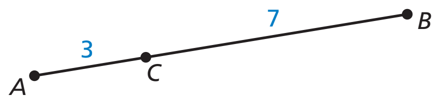
1.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how $\overline{XY}$ and XY are different.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.



Find $AC + CB$.	Find $BC - AC$.
Find AB.	Find $CA + BC$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, use a ruler to measure the length of the segment to the nearest tenth of a centimeter. *(See Example 1.)*

3.

4.

5.

6.

CONSTRUCTION **In Exercises 7 and 8, use a compass and straightedge to construct a copy of the segment.**

7. Copy the segment in Exercise 3.

8. Copy the segment in Exercise 4.

In Exercises 9–14, plot the points in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent. *(See Example 2.)*

9. $A(-4, 5)$, $B(-4, 8)$, $C(2, -3)$, $D(2, 0)$

10. $A(6, -1)$, $B(1, -1)$, $C(2, -3)$, $D(4, -3)$

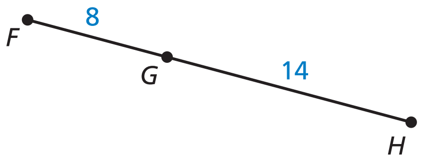
11. $A(8, 3)$, $B(-1, 3)$, $C(5, 10)$, $D(5, 3)$

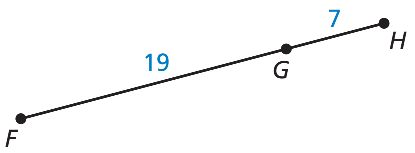
12. $A(6, -8)$, $B(6, 1)$, $C(7, -2)$, $D(-2, -2)$

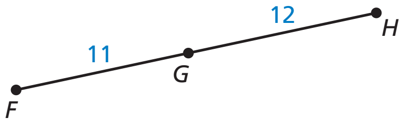
13. $A(-5, 6)$, $B(-5, -1)$, $C(-4, 3)$, $D(3, 3)$

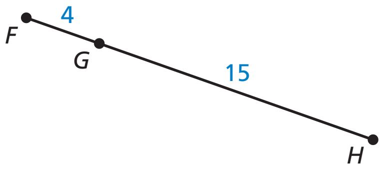
14. $A(10, -4)$, $B(3, -4)$, $C(-1, 2)$, $D(-1, 5)$

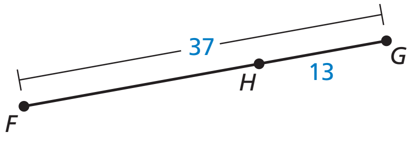
In Exercises 15–22, find FH. *(See Example 3.)*

15. 

16. 

17. 

18. 

19. 

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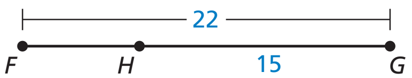

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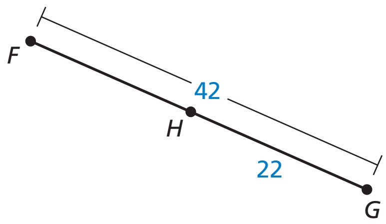

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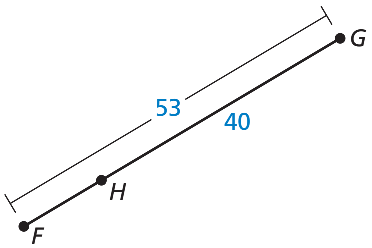
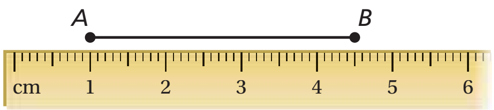


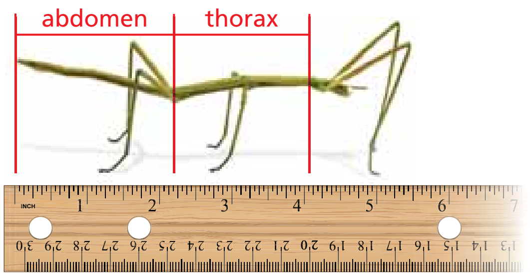
ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in finding the length of $\overline{AB}$.**



23. ✗ $AB = 1 - 4.5 = -3.5$

24. ✗ $AB = |1 + 4.5| = 5.5$

25. **ATTENDING TO PRECISION** The diagram shows an insect called a walking stick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest $\frac{1}{4}$ inch. How much longer is the walking stick's abdomen than its thorax? How many times longer is its abdomen than its thorax?

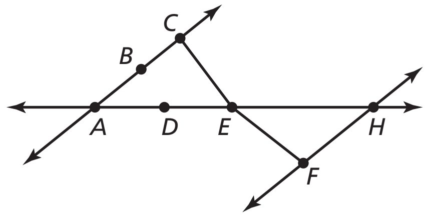


26. **MODELING WITH MATHEMATICS** In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane's position at three different points during its flight. Point A represents Cape Spear, Newfoundland, point B represents the approximate position after 1 day, and point C represents Mannin Bay, Ireland. The airplane left from Cape Spear and landed in Mannin Bay. *(See Example 4.)*

a. Find the total distance the model airplane flew.

b. The model airplane's flight lasted nearly 38 hours. Estimate the airplane's average speed in miles per hour.

27. **USING STRUCTURE** Determine whether the statements are true or false. Explain your reasoning.



a. B is between A and C.

b. C is between B and E.

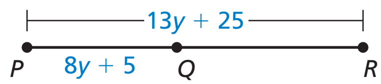
c. D is between A and H.

d. E is between C and F.

28. **MATHEMATICAL CONNECTIONS** Write an expression for the length of the segment.

a. $\overline{AC}$

b. $\overline{QR}$



29. MATHEMATICAL CONNECTIONS Point S is between points R and T on $\overline{RT}$. Use the information to write an equation in terms of x. Then solve the equation and find RS, ST, and RT.

a. $RS = 2x + 10$
$ST = x - 4$
$RT = 21$

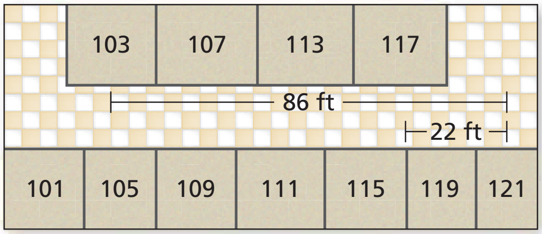
b. $RS = 3x - 16$
$ST = 4x - 8$
$RT = 60$

c. $RS = 2x - 8$
$ST = 11$
$RT = x + 10$

d. $RS = 4x - 9$
$ST = 19$
$RT = 8x - 14$

30. THOUGHT PROVOKING Is it possible to design a table where no two legs have the same length? Assume that the endpoints of the legs must all lie in the same plane. Include a diagram as part of your answer.

31. MODELING WITH MATHEMATICS You have to walk from Room 103 to Room 117.



a. How many feet do you travel from Room 103 to Room 117?

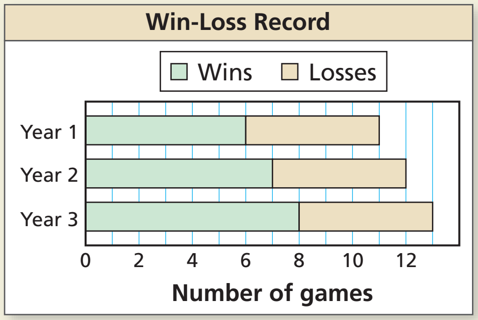
b. You can walk 4.4 feet per second. How many minutes will it take you to get to Room 117?

c. Why might it take you longer than the time in part (b)?

32. MAKING AN ARGUMENT Your friend and your cousin discuss measuring with a ruler. Your friend says that you must always line up objects at the zero on a ruler. Your cousin says it does not matter. Decide who is correct and explain your reasoning.

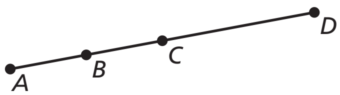
33. REASONING You travel from City X to City Y. You know that the round-trip distance is 647 miles. City Z, a city you pass on the way, is 27 miles from City X. Find the distance from City Z to City Y. Justify your answer.

34. HOW DO YOU SEE IT? The bar graph shows the win-loss record for a lacrosse team over a period of three years. Explain how you can apply the Ruler Postulate (Post. 1.1) and the Segment Addition Postulate (Post. 1.2) when interpreting a stacked bar graph like the one shown.



35. ABSTRACT REASONING The points (a, b) and (c, b) form a segment, and the points (d, e) and (d, f) form a segment. Create an equation assuming the segments are congruent. Are there any letters not used in the equation? Explain.

36. MATHEMATICAL CONNECTIONS In the diagram, $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{CD}$, and $AD = 12$. Find the lengths of all segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measure of the segment is greater than 3? Explain your reasoning.



37. CRITICAL THINKING Is it possible to use the Segment Addition Postulate (Post. 1.2) to show $FB > CB$ or that $AC > DB$? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify. *(Skills Review Handbook)*

38. $\dfrac{-4 + 6}{2}$

39. $\sqrt{20 + 5}$

40. $\sqrt{25 + 9}$

41. $\dfrac{7 + 6}{2}$

Solve the equation. *(Skills Review Handbook)*

42. $5x + 7 = 9x - 17$

43. $\dfrac{3 + y}{2} = 6$

44. $\dfrac{-5 + x}{2} = -9$

45. $-6x - 13 = -x - 23$