9.4 Graphing Sine and Cosine Functions

Essential Question What are the characteristics of the graphs of the sine and cosine functions?

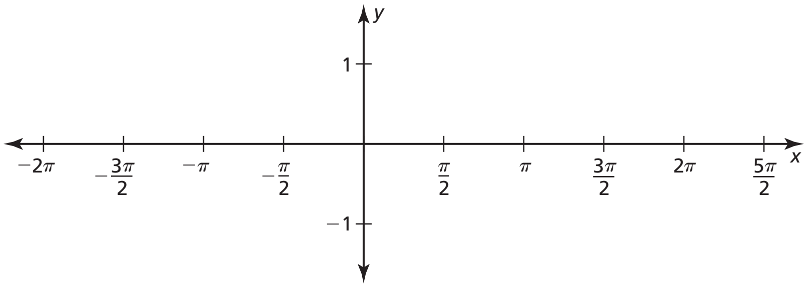
EXPLORATION 1 Graphing the Sine Function

Work with a partner.

a. Complete the table for $y = \sin x$, where x is an angle measure in radians.

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0
$y = \sin x$									
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$
$y = \sin x$									

b. Plot the points (x, y) from part (a). Draw a smooth curve through the points to sketch the graph of $y = \sin x$.



c. Use the graph to identify the x-intercepts, the x-values where the local maximums and minimums occur, and the intervals for which the function is increasing or decreasing over $-2\pi \le x \le 2\pi$. Is the sine function *even*, *odd*, or *neither*?

EXPLORATION 2 Graphing the Cosine Function

Work with a partner.

a. Complete a table for $y = \cos x$ using the same values of x as those used in Exploration 1.

b. Plot the points (x, y) from part (a) and sketch the graph of $y = \cos x$.

c. Use the graph to identify the x-intercepts, the x-values where the local maximums and minimums occur, and the intervals for which the function is increasing or decreasing over $-2\pi \le x \le 2\pi$. Is the cosine function *even*, *odd*, or *neither*?

LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

3. What are the characteristics of the graphs of the sine and cosine functions?

4. Describe the end behavior of the graph of $y = \sin x$.

9.4 Lesson

Core Vocabulary

amplitude, *p. 486*
periodic function, *p. 486*
cycle, *p. 486*
period, *p. 486*
phase shift, *p. 488*
midline, *p. 488*

Previous
transformations
x-intercept

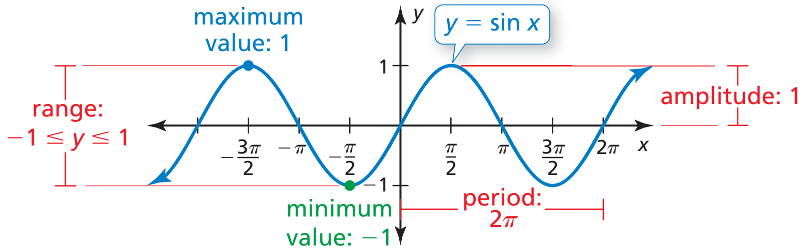
What You Will Learn

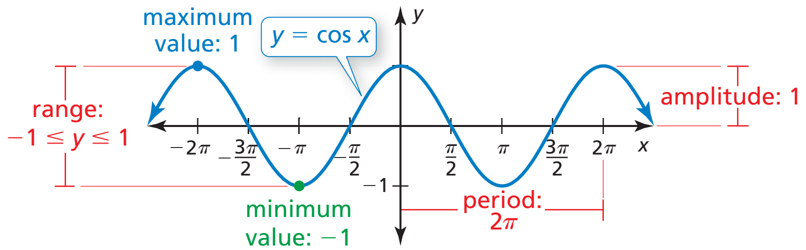
- Explore characteristics of sine and cosine functions.
- Stretch and shrink graphs of sine and cosine functions.
- Translate graphs of sine and cosine functions.
- Reflect graphs of sine and cosine functions.

Exploring Characteristics of Sine and Cosine Functions

In this lesson, you will learn to graph sine and cosine functions. The graphs of sine and cosine functions are related to the graphs of the parent functions $y = \sin x$ and $y = \cos x$, which are shown below.

x	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin x$	0	1	0	−1	0	1	0	−1	0
$y = \cos x$	1	0	−1	0	1	0	−1	0	1





Core Concept

Characteristics of $y = \sin x$ and $y = \cos x$

- The domain of each function is all real numbers.
- The range of each function is $-1 \leq y \leq 1$. So, the minimum value of each function is -1 and the maximum value is 1.
- The **amplitude** of the graph of each function is one-half of the difference of the maximum value and the minimum value, or $\frac{1}{2}[1 - (-1)] = 1$.
- Each function is **periodic**, which means that its graph has a repeating pattern. The shortest repeating portion of the graph is called a **cycle**. The horizontal length of each cycle is called the **period**. Each graph shown above has a period of 2π.
- The x-intercepts for $y = \sin x$ occur when $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$.
- The x-intercepts for $y = \cos x$ occur when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$.

Stretching and Shrinking Sine and Cosine Functions

The graphs of $y = a \sin bx$ and $y = a \cos bx$ represent transformations of their parent functions. The value of a indicates a vertical stretch ($a > 1$) or a vertical shrink ($0 < a < 1$) and changes the amplitude of the graph. The value of b indicates a horizontal stretch ($0 < b < 1$) or a horizontal shrink ($b > 1$) and changes the period of the graph.

REMEMBER

The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink of the graph of $y = f(x)$ by a factor of a.

The graph of $y = f(bx)$ is a horizontal stretch or shrink of the graph of $y = f(x)$ by a factor of $\frac{1}{b}$.

$$y = a \sin bx$$

$$y = a \cos bx$$

vertical stretch or shrink by a factor of a — horizontal stretch or shrink by a factor of $\frac{1}{b}$

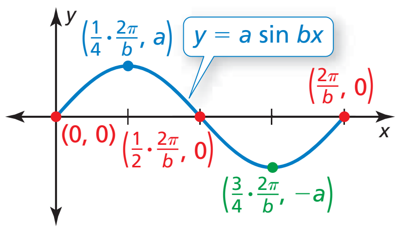
Core Concept

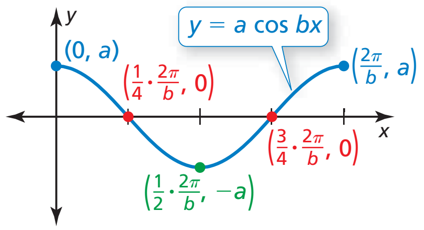
Amplitude and Period

The amplitude and period of the graphs of $y = a \sin bx$ and $y = a \cos bx$, where a and b are nonzero real numbers, are as follows:

$$\text{Amplitude} = |a| \qquad \text{Period} = \frac{2\pi}{|b|}$$

Each graph below shows five key points that partition the interval $0 \le x \le \frac{2\pi}{b}$ into four equal parts. You can use these points to sketch the graphs of $y = a \sin bx$ and $y = a \cos bx$. The x-intercepts, maximum, and minimum occur at these points.





EXAMPLE 1 **Graphing a Sine Function**

Identify the amplitude and period of $g(x) = 4 \sin x$. Then graph the function and describe the graph of g as a transformation of the graph of $f(x) = \sin x$.

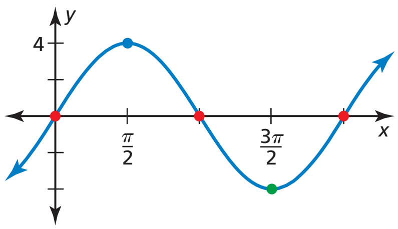
SOLUTION

The function is of the form $g(x) = a \sin bx$ where $a = 4$ and $b = 1$. So, the amplitude is $a = 4$ and the period is $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$.

Intercepts: $(0, 0)$; $\left(\frac{1}{2} \cdot 2\pi, 0\right) = (\pi, 0)$; $(2\pi, 0)$

Maximum: $\left(\frac{1}{4} \cdot 2\pi, 4\right) = \left(\frac{\pi}{2}, 4\right)$

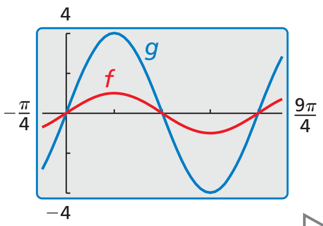
Minimum: $\left(\frac{3}{4} \cdot 2\pi, -4\right) = \left(\frac{3\pi}{2}, -4\right)$



▶ The graph of g is a vertical stretch by a factor of 4 of the graph of f.

REMEMBER

A vertical stretch of a graph does not change its x-intercept(s). So, it makes sense that the x-intercepts of $g(x) = 4 \sin x$ and $f(x) = \sin x$ are the same.



EXAMPLE 2 Graphing a Cosine Function

Identify the amplitude and period of $g(x) = \frac{1}{2} \cos 2\pi x$. Then graph the function and describe the graph of g as a transformation of the graph of $f(x) = \cos x$.

SOLUTION

The function is of the form $g(x) = a \cos bx$ where $a = \frac{1}{2}$ and $b = 2\pi$. So, the amplitude is $a = \frac{1}{2}$ and the period is $\frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$.

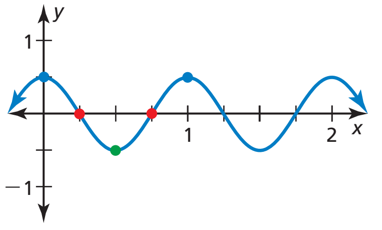
STUDY TIP

After you have drawn one complete cycle of the graph in Example 2 on the interval $0 \leq x \leq 1$, you can extend the graph by repeating the cycle as many times as desired to the left and right of $0 \leq x \leq 1$.

Intercepts: $\left(\frac{1}{4} \cdot 1, 0\right) = \left(\frac{1}{4}, 0\right)$; $\left(\frac{3}{4} \cdot 1, 0\right) = \left(\frac{3}{4}, 0\right)$

Maximums: $\left(0, \frac{1}{2}\right)$; $\left(1, \frac{1}{2}\right)$

Minimum: $\left(\frac{1}{2} \cdot 1, -\frac{1}{2}\right) = \left(\frac{1}{2}, -\frac{1}{2}\right)$



▶ The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ and a horizontal shrink by a factor of $\frac{1}{2\pi}$ of the graph of f.

Monitoring Progress

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Identify the amplitude and period of the function. Then graph the function and describe the graph of g as a transformation of the graph of its parent function.

1. $g(x) = \frac{1}{4} \sin x$ **2.** $g(x) = \cos 2x$ **3.** $g(x) = 2 \sin \pi x$ **4.** $g(x) = \frac{1}{3} \cos \frac{1}{2}x$

Translating Sine and Cosine Functions

REMEMBER

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$.

The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$.

The graphs of $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$ represent translations of $y = a \sin bx$ and $y = a \cos bx$. The value of k indicates a translation up $(k > 0)$ or down $(k < 0)$. The value of h indicates a translation left $(h < 0)$ or right $(h > 0)$. A horizontal translation of a periodic function is called a **phase shift**.

Core Concept

Graphing $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$

To graph $y = a \sin b(x - h) + k$ or $y = a \cos b(x - h) + k$ where $a > 0$ and $b > 0$, follow these steps:

Step 1 Identify the amplitude a, the period $\frac{2\pi}{b}$, the horizontal shift h, and the vertical shift k of the graph.

Step 2 Draw the horizontal line $y = k$, called the **midline** of the graph.

Step 3 Find the five key points by translating the key points of $y = a \sin bx$ or $y = a \cos bx$ horizontally h units and vertically k units.

Step 4 Draw the graph through the five translated key points.

EXAMPLE 3 Graphing a Vertical Translation

Graph $g(x) = 2 \sin 4x + 3$.

LOOKING FOR STRUCTURE

The graph of g is a translation 3 units up of the graph of $f(x) = 2 \sin 4x$. So, add 3 to the y-coordinates of the five key points of f.

SOLUTION

Step 1 Identify the amplitude, period, horizontal shift, and vertical shift.

Amplitude: $a = 2$ Horizontal shift: $h = 0$

Period: $\frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$ Vertical shift: $k = 3$

Step 2 Draw the midline of the graph, $y = 3$.

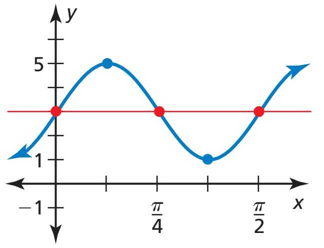
Step 3 Find the five key points.

On $y = k$: $(0, 0 + 3) = (0, 3)$; $\left(\frac{\pi}{4}, 0 + 3\right) = \left(\frac{\pi}{4}, 3\right)$; $\left(\frac{\pi}{2}, 0 + 3\right) = \left(\frac{\pi}{2}, 3\right)$

Maximum: $\left(\frac{\pi}{8}, 2 + 3\right) = \left(\frac{\pi}{8}, 5\right)$

Minimum: $\left(\frac{3\pi}{8}, -2 + 3\right) = \left(\frac{3\pi}{8}, 1\right)$

Step 4 Draw the graph through the key points.



EXAMPLE 4 Graphing a Horizontal Translation

Graph $g(x) = 5 \cos \frac{1}{2}(x - 3\pi)$.

LOOKING FOR STRUCTURE

The graph of g is a translation 3π units right of the graph of $f(x) = 5 \cos \frac{1}{2}x$. So, add 3π to the x-coordinates of the five key points of f.

SOLUTION

Step 1 Identify the amplitude, period, horizontal shift, and vertical shift.

Amplitude: $a = 5$ Horizontal shift: $h = 3\pi$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$ Vertical shift: $k = 0$

Step 2 Draw the midline of the graph. Because $k = 0$, the midline is the x-axis.

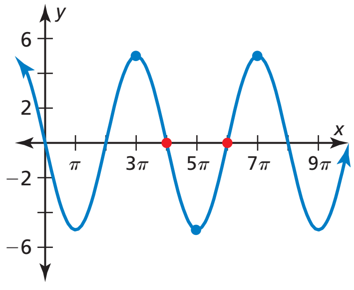
Step 3 Find the five key points.

On $y = k$: $(\pi + 3\pi, 0) = (4\pi, 0)$;
$(3\pi + 3\pi, 0) = (6\pi, 0)$

Maximums: $(0 + 3\pi, 5) = (3\pi, 5)$;
$(4\pi + 3\pi, 5) = (7\pi, 5)$

Minimum: $(2\pi + 3\pi, -5) = (5\pi, -5)$

Step 4 Draw the graph through the key points.



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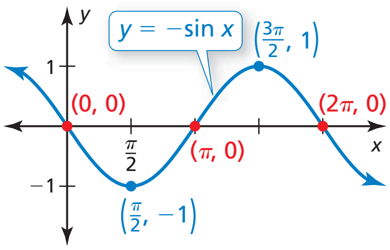
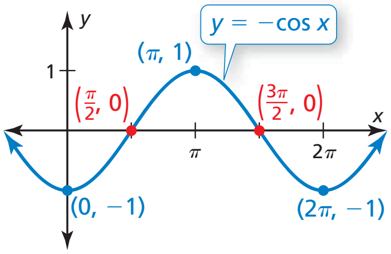
Graph the function.

5. $g(x) = \cos x + 4$ **6.** $g(x) = \frac{1}{2}\sin\left(x - \frac{\pi}{2}\right)$ **7.** $g(x) = \sin(x + \pi) - 1$

Reflecting Sine and Cosine Functions

You have graphed functions of the form $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$, where $a > 0$ and $b > 0$. To see what happens when $a < 0$, consider the graphs of $y = -\sin x$ and $y = -\cos x$.





REMEMBER

This result makes sense because the graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.

The graphs are reflections of the graphs of $y = \sin x$ and $y = \cos x$ in the x-axis. In general, when $a < 0$, the graphs of $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$ are reflections of the graphs of $y = |a| \sin b(x - h) + k$ and $y = |a| \cos b(x - h) + k$, respectively, in the midline $y = k$.

EXAMPLE 5 Graphing a Reflection

Graph $g(x) = -2 \sin \frac{2}{3}\left(x - \frac{\pi}{2}\right)$.

SOLUTION

Step 1 Identify the amplitude, period, horizontal shift, and vertical shift.

Amplitude: $|a| = |-2| = 2$ Horizontal shift: $h = \frac{\pi}{2}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\frac{2}{3}} = 3\pi$ Vertical shift: $k = 0$

Step 2 Draw the midline of the graph. Because $k = 0$, the midline is the x-axis.

Step 3 Find the five key points of $f(x) = |-2| \sin \frac{2}{3}\left(x - \frac{\pi}{2}\right)$.

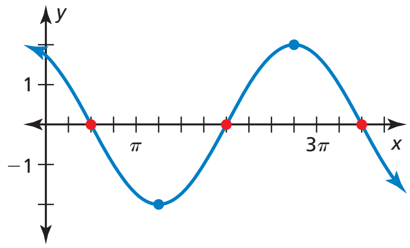
On $y = k$: $\left(0 + \frac{\pi}{2}, 0\right) = \left(\frac{\pi}{2}, 0\right)$; $\left(\frac{3\pi}{2} + \frac{\pi}{2}, 0\right) = (2\pi, 0)$; $\left(3\pi + \frac{\pi}{2}, 0\right) = \left(\frac{7\pi}{2}, 0\right)$

Maximum: $\left(\frac{3\pi}{4} + \frac{\pi}{2}, 2\right) = \left(\frac{5\pi}{4}, 2\right)$ Minimum: $\left(\frac{9\pi}{4} + \frac{\pi}{2}, -2\right) = \left(\frac{11\pi}{4}, -2\right)$

STUDY TIP

In Example 5, the maximum value and minimum value of f are the minimum value and maximum value, respectively, of g.

Step 4 Reflect the graph. Because $a < 0$, the graph is reflected in the midline $y = 0$. So, $\left(\frac{5\pi}{4}, 2\right)$ becomes $\left(\frac{5\pi}{4}, -2\right)$ and $\left(\frac{11\pi}{4}, -2\right)$ becomes $\left(\frac{11\pi}{4}, 2\right)$.



Step 5 Draw the graph through the key points.

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Graph the function.

8. $g(x) = -\cos\left(x + \frac{\pi}{2}\right)$ **9.** $g(x) = -3 \sin \frac{1}{2}x + 2$ **10.** $g(x) = -2 \cos 4x - 1$

9.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The shortest repeating portion of the graph of a periodic function is called a(n) __________.

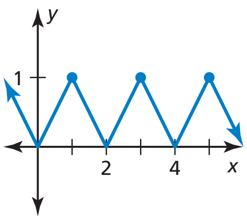
2. **WRITING** Compare the amplitudes and periods of the functions $y = \frac{1}{2}\cos x$ and $y = 3\cos 2x$.

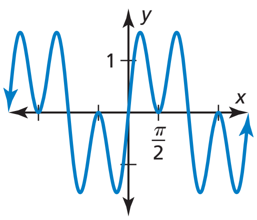
3. **VOCABULARY** What is a phase shift? Give an example of a sine function that has a phase shift.

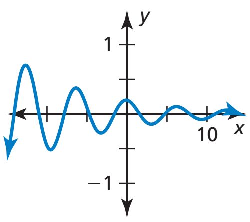
4. **VOCABULARY** What is the midline of the graph of the function $y = 2\sin 3(x + 1) - 2$?

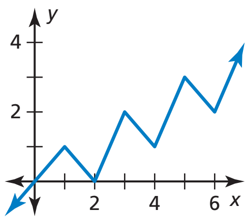
Monitoring Progress and Modeling with Mathematics

USING STRUCTURE In Exercises 5–8, determine whether the graph represents a periodic function. If so, identify the period.

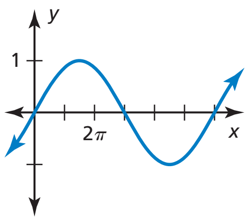
5. 

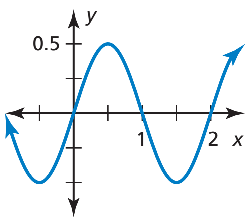
6. 

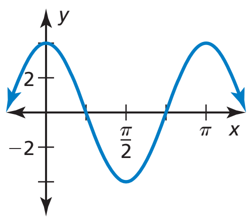
7. 

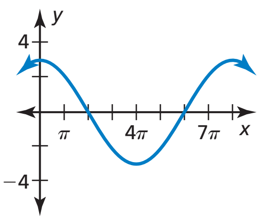
8. 

In Exercises 9–12, identify the amplitude and period of the graph of the function.

9. 

10. 

11. 

12. 

In Exercises 13–20, identify the amplitude and period of the function. Then graph the function and describe the graph of *g* as a transformation of the graph of its parent function. *(See Examples 1 and 2.)*

13. $g(x) = 3\sin x$
14. $g(x) = 2\sin x$
15. $g(x) = \cos 3x$
16. $g(x) = \cos 4x$
17. $g(x) = \sin 2\pi x$
18. $g(x) = 3\sin 2x$
19. $g(x) = \frac{1}{3}\cos 4x$
20. $g(x) = \frac{1}{2}\cos 4\pi x$

21. **ANALYZING EQUATIONS** Which functions have an amplitude of 4 and a period of 2?

    Ⓐ $y = 4\cos 2x$

    Ⓑ $y = -4\sin \pi x$

    Ⓒ $y = 2\sin 4x$

    Ⓓ $y = 4\cos \pi x$

22. **WRITING EQUATIONS** Write an equation of the form $y = a\sin bx$, where $a > 0$ and $b > 0$, so that the graph has the given amplitude and period.

    a. amplitude: 1
    period: 5

    b. amplitude: 10
    period: 4

    c. amplitude: 2
    period: 2π

    d. amplitude: $\frac{1}{2}$
    period: 3π

23. **MODELING WITH MATHEMATICS** The motion of a pendulum can be modeled by the function $d = 4\cos 8\pi t$, where d is the horizontal displacement (in inches) of the pendulum relative to its position at rest and t is the time (in seconds). Find and interpret the period and amplitude in the context of this situation. Then graph the function.

24. MODELING WITH MATHEMATICS A buoy bobs up and down as waves go past. The vertical displacement y (in feet) of the buoy with respect to sea level can be modeled by $y = 1.75 \cos \frac{\pi}{3}t$, where t is the time (in seconds). Find and interpret the period and amplitude in the context of the problem. Then graph the function.

In Exercises 25–34, graph the function. *(See Examples 3 and 4.)*

25. $g(x) = \sin x + 2$ **26.** $g(x) = \cos x - 4$

27. $g(x) = \cos\left(x - \frac{\pi}{2}\right)$ **28.** $g(x) = \sin\left(x + \frac{\pi}{4}\right)$

29. $g(x) = 2 \cos x - 1$ **30.** $g(x) = 3 \sin x + 1$

31. $g(x) = \sin 2(x + \pi)$

32. $g(x) = \cos 2(x - \pi)$

33. $g(x) = \sin \frac{1}{2}(x + 2\pi) + 3$

34. $g(x) = \cos \frac{1}{2}(x - 3\pi) - 5$

35. ERROR ANALYSIS Describe and correct the error in finding the period of the function $y = \sin \frac{2}{3}x$.

✗ Period: $\frac{|b|}{2\pi} = \frac{\left|\frac{2}{3}\right|}{2\pi} = \frac{1}{3\pi}$

36. ERROR ANALYSIS Describe and correct the error in determining the point where the maximum value of the function $y = 2 \sin\left(x - \frac{\pi}{2}\right)$ occurs.

✗ Maximum:

$\left(\left(\frac{1}{4} \cdot 2\pi\right) - \frac{\pi}{2}, 2\right) = \left(\frac{\pi}{2} - \frac{\pi}{2}, 2\right)$

$= (0, 2)$

USING STRUCTURE In Exercises 37–40, describe the transformation of the graph of f represented by the function g.

37. $f(x) = \cos x$, $g(x) = 2 \cos\left(x - \frac{\pi}{2}\right) + 1$

38. $f(x) = \sin x$, $g(x) = 3 \sin\left(x + \frac{\pi}{4}\right) - 2$

39. $f(x) = \sin x$, $g(x) = \sin 3(x + 3\pi) - 5$

40. $f(x) = \cos x$, $g(x) = \cos 6(x - \pi) + 9$

In Exercises 41–48, graph the function. *(See Example 5.)*

41. $g(x) = -\cos x + 3$ **42.** $g(x) = -\sin x - 5$

43. $g(x) = -\sin \frac{1}{2}x - 2$ **44.** $g(x) = -\cos 2x + 1$

45. $g(x) = -\sin(x - \pi) + 4$

46. $g(x) = -\cos(x + \pi) - 2$

47. $g(x) = -4 \cos\left(x + \frac{\pi}{4}\right) - 1$

48. $g(x) = -5 \sin\left(x - \frac{\pi}{2}\right) + 3$

49. USING EQUATIONS Which of the following is a point where the maximum value of the graph of $y = -4 \cos\left(x - \frac{\pi}{2}\right)$ occurs?

Ⓐ $\left(-\frac{\pi}{2}, 4\right)$ Ⓑ $\left(\frac{\pi}{2}, 4\right)$

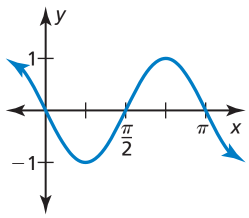
Ⓒ $(0, 4)$ Ⓓ $(\pi, 4)$

50. ANALYZING RELATIONSHIPS Match each function with its graph. Explain your reasoning.

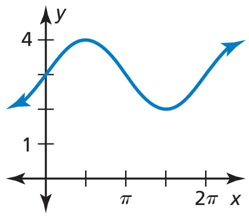
a. $y = 3 + \sin x$ **b.** $y = -3 + \cos x$

c. $y = \sin 2\left(x - \frac{\pi}{2}\right)$ **d.** $y = \cos 2\left(x - \frac{\pi}{2}\right)$

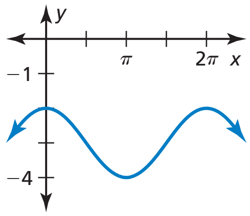
A.



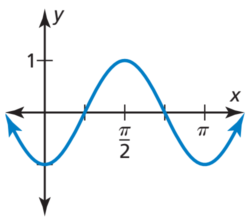
B.



C.



D.



WRITING EQUATIONS **In Exercises 51–54, write a rule for g that represents the indicated transformations of the graph of f.**

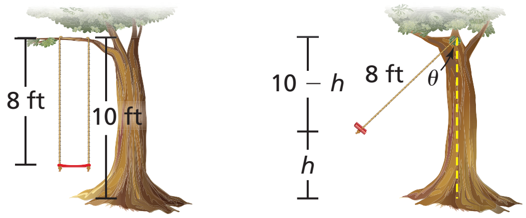
51. $f(x) = 3 \sin x$; translation 2 units up and π units right

52. $f(x) = \cos 2\pi x$; translation 4 units down and 3 units left

53. $f(x) = \frac{1}{3} \cos \pi x$; translation 1 unit down, followed by a reflection in the line $y = -1$

54. $f(x) = \frac{1}{2} \sin 6x$; translation $\frac{3}{2}$ units down and 1 unit right, followed by a reflection in the line $y = -\frac{3}{2}$

55. **MODELING WITH MATHEMATICS** The height h (in feet) of a swing above the ground can be modeled by the function $h = -8 \cos \theta + 10$, where the pivot is 10 feet above the ground, the rope is 8 feet long, and θ is the angle that the rope makes with the vertical. Graph the function. What is the height of the swing when θ is 45°?



Front view Side view

56. **DRAWING A CONCLUSION** In a particular region, the population L (in thousands) of lynx (the predator) and the population H (in thousands) of hares (the prey) can be modeled by the equations

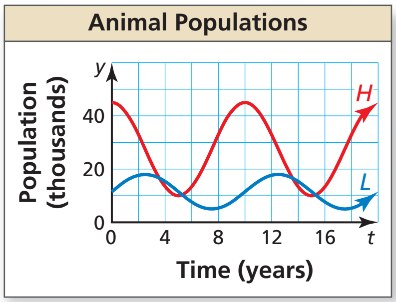
$$L = 11.5 + 6.5 \sin \frac{\pi}{5} t$$

$$H = 27.5 + 17.5 \cos \frac{\pi}{5} t$$

where t is the time in years.

a. Determine the ratio of hares to lynx when $t = 0$, 2.5, 5, and 7.5 years.

b. Use the figure to explain how the changes in the two populations appear to be related.

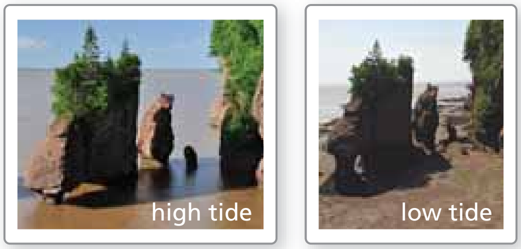


57. **USING TOOLS** The average wind speed s (in miles per hour) in the Boston Harbor can be approximated by

$$s = 3.38 \sin \frac{\pi}{180}(t + 3) + 11.6$$

where t is the time in days and $t = 0$ represents January 1. Use a graphing calculator to graph the function. On which days of the year is the average wind speed 10 miles per hour? Explain your reasoning.

58. **USING TOOLS** The water depth d (in feet) for the Bay of Fundy can be modeled by $d = 35 - 28 \cos \frac{\pi}{6.2} t$, where t is the time in hours and $t = 0$ represents midnight. Use a graphing calculator to graph the function. At what time(s) is the water depth 7 feet? Explain.



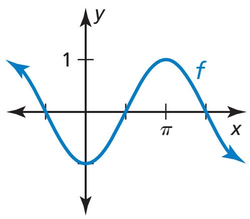
59. **MULTIPLE REPRESENTATIONS** Find the average rate of change of each function over the interval $0 < x < \pi$.

a. $y = 2 \cos x$

b.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$f(x) = -\cos x$	-1	0	1	0	-1

c.



60. **REASONING** Consider the functions $y = \sin(-x)$ and $y = \cos(-x)$.

a. Construct a table of values for each equation using the quadrantal angles in the interval $-2\pi \le x \le 2\pi$.

b. Graph each function.

c. Describe the transformations of the graphs of the parent functions.

61. MODELING WITH MATHEMATICS You are riding a Ferris wheel that turns for 180 seconds. Your height h (in feet) above the ground at any time t (in seconds) can be modeled by the equation

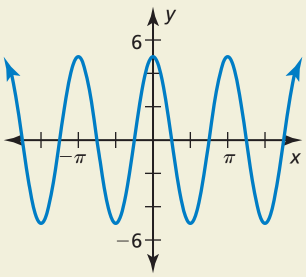
$$h = 85 \sin \frac{\pi}{20}(t - 10) + 90.$$

a. Graph the function.

b. How many cycles does the Ferris wheel make in 180 seconds?

c. What are your maximum and minimum heights?

62. HOW DO YOU SEE IT? Use the graph to answer each question.



a. Does the graph represent a function of the form $f(x) = a \sin bx$ or $f(x) = a \cos bx$? Explain.

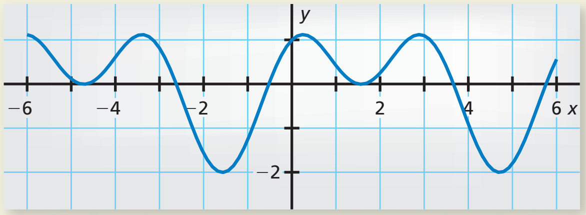
b. Identify the maximum value, minimum value, period, and amplitude of the function.

63. FINDING A PATTERN Write an expression in terms of the integer n that represents all the x-intercepts of the graph of the function $y = \cos 2x$. Justify your answer.

64. MAKING AN ARGUMENT Your friend states that for functions of the form $y = a \sin bx$ and $y = a \cos bx$, the values of a and b affect the x-intercepts of the graph of the function. Is your friend correct? Explain.

65. CRITICAL THINKING Describe a transformation of the graph of $f(x) = \sin x$ that results in the graph of $g(x) = \cos x$.

66. THOUGHT PROVOKING Use a graphing calculator to find a function of the form $y = \sin b_1 x + \cos b_2 x$ whose graph matches that shown below.



67. PROBLEM SOLVING For a person at rest, the blood pressure P (in millimeters of mercury) at time t (in seconds) is given by the function

$$P = 100 - 20 \cos \frac{8\pi}{3}t.$$

Graph the function. One cycle is equivalent to one heartbeat. What is the pulse rate (in heartbeats per minute) of the person?

68. PROBLEM SOLVING The motion of a spring can be modeled by $y = A \cos kt$, where y is the vertical displacement (in feet) of the spring relative to its position at rest, A is the initial displacement (in feet), k is a constant that measures the elasticity of the spring, and t is the time (in seconds).

a. You have a spring whose motion can be modeled by the function $y = 0.2 \cos 6t$. Find the initial displacement and the period of the spring. Then graph the function.

b. When a damping force is applied to the spring, the motion of the spring can be modeled by the function $y = 0.2e^{-4.5t} \cos 4t$. Graph this function. What effect does damping have on the motion?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the rational expression, if possible. *(Section 7.3)*

69. $\dfrac{x^2 + x - 6}{x + 3}$ **70.** $\dfrac{x^3 - 2x^2 - 24x}{x^2 - 2x - 24}$ **71.** $\dfrac{x^2 - 4x - 5}{x^2 + 4x - 5}$ **72.** $\dfrac{x^2 - 16}{x^2 + x - 20}$

Find the least common multiple of the expressions. *(Section 7.4)*

73. $2x, 2(x - 5)$ **74.** $x^2 - 4, x + 2$ **75.** $x^2 + 8x + 12, x + 6$