11.4 Two-Way Tables

Essential Question How can you read and make a two-way table?

EXPLORATION 1 Reading a Two-Way Table

Work with a partner. You are the manager of a sports shop. The two-way tables show the numbers of soccer T-shirts in stock at your shop at the beginning and end of the selling season. (a) Complete the totals for the rows and columns in each table. (b) How would you alter the number of T-shirts you order for next season? Explain your reasoning.

Beginning of season		T-Shirt Size					
		S	M	L	XL	XXL	Total
Color	blue/white	5	6	7	6	5	
	blue/gold	5	6	7	6	5	
	red/white	5	6	7	6	5	
	black/white	5	6	7	6	5	
	black/gold	5	6	7	6	5	
	Total						145

| End of season		T-Shirt Size					
		S	M	L	XL	XXL	Total
Color	blue/white	5	4	1	0	2	
	blue/gold	3	6	5	2	0	
	red/white	4	2	4	1	3	
	black/white	3	4	1	2	1	
	black/gold	5	2	3	0	2	
	Total						

MODELING WITH MATHEMATICS

To be proficient in math, you need to identify important quantities and map their relationships using tools such as graphs and two-way tables.

EXPLORATION 2 Making a Two-Way Table

Work with a partner. The three-dimensional bar graph shows the numbers of hours students work at part-time jobs.

a. Make a two-way table showing the data. Use estimation to find the entries in your table.

b. Write two observations that summarize the data in your table.

Part-Time Jobs of Students at a High School
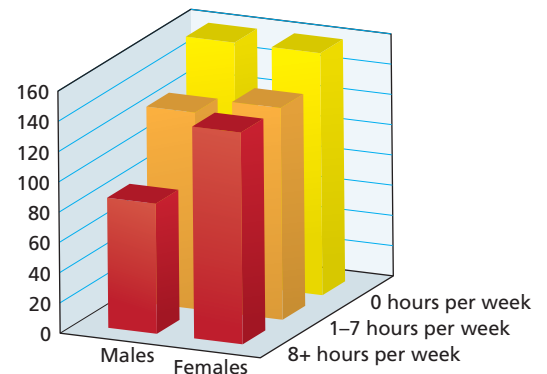
Communicate Your Answer

3. How can you read and make a two-way table?

What You Will Learn

▶ Find and interpret marginal frequencies.
▶ Make two-way tables.
▶ Find relative and conditional relative frequencies.
▶ Use two-way tables to recognize associations in data.

Core Vocabulary

two-way table, *p. 610*
joint frequency, *p. 610*
marginal frequency, *p. 610*
joint relative frequency, *p. 611*
marginal relative frequency,
  p. 611
conditional relative frequency,
  p. 612

Finding and Interpreting Marginal Frequencies

A **two-way table** is a frequency table that displays data collected from one source that belong to two different categories. One category of data is represented by rows, and the other is represented by columns. For instance, the two-way table below shows the results of a survey that asked freshmen and sophomores whether they access the Internet using a mobile device, such as a smartphone.

The two categories of data are *class* and *mobile access*. Class is further divided into *freshman* and *sophomore*, and mobile access is further divided into *yes* and *no*.

Each entry in the table is called a **joint frequency**. The sums of the rows and columns in a two-way table are called **marginal frequencies**.

categories		Mobile Access	
		Yes	No
Class	Freshman	55	22
	Sophomore	63	12

joint frequency

> **REMEMBER**
> The frequency of an event is the number of times the event occurs.

EXAMPLE 1   Finding and Interpreting Marginal Frequencies

Find and interpret the marginal frequencies for the two-way table above.

SOLUTION

Create a new column and a new row for the marginal frequencies. Then add the entries in each row and column.

> **STUDY TIP**
> The sum of the "total" row should be equal to the sum of the "total" column. Place this sum of the marginal frequencies at the bottom right of your two-way table.

		Mobile Access		
		Yes	No	Total
Class	Freshman	55	22	77
	Sophomore	63	12	75
	Total	118	34	152

77 freshmen responded.
75 sophomores responded.
152 students were surveyed.
118 students access the Internet using a mobile device.
34 students do not access the Internet using a mobile device.

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1. You conduct a technology survey to publish on your school's website. You survey students in the school cafeteria about the technological devices they own. The results are shown in the two-way table. Find and interpret the marginal frequencies.

		Tablet Computer	
		Yes	No
Cell Phone	Yes	34	124
	No	18	67

Making Two-Way Tables

EXAMPLE 2   **Making a Two-Way Table**

You conduct a survey that asks 286 students in your freshman class whether they play a sport or a musical instrument. One hundred eighteen of the students play a sport, and 64 of those students play an instrument. Ninety-three of the students do not play a sport or an instrument. Organize the results in a two-way table. Include the marginal frequencies.

SOLUTION

Step 1   Determine the two categories for the table: *sport* and *instrument*.

		Instrument		
		Yes	**No**	**Total**
Sport	**Yes**	64		118
	No		93	
	Total			286

Step 2   Use the given joint and marginal frequencies to fill in parts of the table.

Step 3   Use reasoning to find the missing joint and marginal frequencies. For instance, you can conclude that there are $286 - 118 = 168$ students who do not play a sport, and $118 - 64 = 54$ students who play a sport but do not play an instrument.

		Instrument		
		Yes	**No**	**Total**
Sport	**Yes**	64	54	118
	No	75	93	168
	Total	139	147	286

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2. You survey students about whether they are getting a summer job. Seventy-five males respond, with 18 of them responding "no." Fifty-seven females respond, with 45 of them responding "yes." Organize the results in a two-way table. Include the marginal frequencies.

Finding Relative and Conditional Relative Frequencies

You can display entries of a two-way table as frequency counts (as in Examples 1 and 2) or as *relative frequencies*.

🔵 Core Concept

Relative Frequencies

A **joint relative frequency** is the ratio of a frequency that is not in the "total" row or the "total" column to the total number of values or observations.

A **marginal relative frequency** is the sum of the joint relative frequencies in a row or a column.

When finding relative frequencies in a two-way table, you can use the corresponding decimals or percents.

EXAMPLE 3   Finding Relative Frequencies

Major in Medical Field		
	Yes	No
Junior	124	219
Senior	101	236

Class

The two-way table shows the results of a survey that asked college-bound high school students whether they plan to major in a medical field. Make a two-way table that shows the joint and marginal relative frequencies.

SOLUTION

There are $124 + 219 + 101 + 236 = 680$ students in the survey. To find the joint relative frequencies, divide each frequency by 680. Then find the sum of each row and each column to find the marginal relative frequencies.

STUDY TIP

The sum of the marginal relative frequencies in the "total" row and the "total" column should each be equal to 1.

Major in Medical Field			
	Yes	No	Total
Junior	$\frac{124}{680} \approx 0.18$	$\frac{219}{680} \approx 0.32$	0.50
Senior	$\frac{101}{680} \approx 0.15$	$\frac{236}{680} \approx 0.35$	0.50
Total	0.33	0.67	1

Class

About 50% of the students are juniors.

About 35% of the students are seniors and are not planning to major in a medical field.

Core Concept

Conditional Relative Frequencies

A **conditional relative frequency** is the ratio of a joint relative frequency to the marginal relative frequency. You can find a conditional relative frequency using a row total or a column total of a two-way table.

EXAMPLE 4   Finding Conditional Relative Frequencies

Use the survey results in Example 3 to make a two-way table that shows the conditional relative frequencies based on the column totals.

SOLUTION

Use the marginal relative frequency of each *column* to calculate the conditional relative frequencies.

STUDY TIP

When you use column totals, the sum of the conditional relative frequencies for each column should be equal to 1.

Major in Medical Field		
	Yes	No
Junior	$\frac{0.18}{0.33} \approx 0.55$	$\frac{0.32}{0.67} \approx 0.48$
Senior	$\frac{0.15}{0.33} \approx 0.45$	$\frac{0.35}{0.67} \approx 0.52$

Class

Given that a student is not planning to major in a medical field, the conditional relative frequency that he or she is a junior is about 48%.

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3. Use the survey results in Monitoring Progress Question 2 to make a two-way table that shows the joint and marginal relative frequencies. What percent of students are not getting a summer job?

4. Use the survey results in Example 3 to make a two-way table that shows the conditional relative frequencies based on the row totals. Given that a student is a senior, what is the conditional relative frequency that he or she is planning to major in a medical field?

Recognizing Associations in Data

EXAMPLE 5   Recognizing Associations in Data

You survey students and find that 40% exercise regularly, 35% eat fruits and vegetables each day, and 52% do not exercise and do not eat fruits and vegetables each day. Is there an association between exercising regularly and eating fruits and vegetables each day?

SOLUTION

Use the given information to make a two-way table. Use reasoning to find the missing joint and marginal relative frequencies.

Use conditional relative frequencies based on the column totals to determine whether there is an association. Of the students who exercise regularly, 67.5% eat fruits and vegetables each day. Of the students who do not exercise regularly, only about 13% eat fruits and vegetables each day. It appears that students who exercise regularly are more likely to eat more fruits and vegetables than students who do not exercise regularly.

| | | **Exercise Regularly** | | |
		Yes	**No**	**Total**
Eats Fruit/Vegetables	**Yes**	27%	8%	35%
	No	13%	52%	65%
	Total	40%	60%	100%

| | | **Exercise Regularly** | |
		Yes	**No**
Eats Fruit/Vegetables	**Yes**	$\frac{0.27}{0.4} = 0.675$	$\frac{0.08}{0.6} \approx 0.133$
	No	$\frac{0.13}{0.4} = 0.325$	$\frac{0.52}{0.6} \approx 0.867$

▶ So, there is an association between exercising regularly and eating fruits and vegetables each day.

You can also find the conditional relative frequencies by dividing each joint frequency by its corresponding column total or row total.

EXAMPLE 6   Recognizing Associations in Data

| | | **Age** | | | |
		12–13	**14–15**	**16–17**	**18–19**
Share a Computer	**Yes**	40	47	42	22
	No	10	25	36	34

The two-way table shows the results of a survey that asked students whether they share a computer at home with other family members. Is there an association between age and sharing a computer?

SOLUTION

Use conditional relative frequencies based on column totals to determine whether there is an association. Based on this sample, 80% of students ages 12–13 share a computer and only about 39% of students ages 18–19 share a computer.

▶ The table shows that as age increases, students are less likely to share a computer with other family members. So, there is an association.

| | | **Age** | | | |
		12–13	**14–15**	**16–17**	**18–19**
Share a Computer	**Yes**	$\frac{40}{50} = 0.8$	$\frac{47}{72} \approx 0.65$	$\frac{42}{78} \approx 0.54$	$\frac{22}{56} \approx 0.39$
	No	$\frac{10}{50} \approx 0.2$	$\frac{25}{72} \approx 0.35$	$\frac{36}{78} \approx 0.46$	$\frac{34}{56} \approx 0.61$

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5. Using the results of the survey in Monitoring Progress Question 1, is there an association between owning a tablet computer and owning a cell phone? Explain your reasoning.

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** Each entry in a two-way table is called a(n) _____.

2. **WRITING** When is it appropriate to use a two-way table to organize data?

3. **VOCABULARY** Explain the relationship between joint relative frequencies, marginal relative frequencies, and conditional relative frequencies.

4. **WRITING** Describe two ways you can find conditional relative frequencies.

Monitoring Progress and Modeling with Mathematics

You conduct a survey that asks 346 students whether they buy lunch at school. In Exercises 5–8, use the results of the survey shown in the two-way table.

		Buy Lunch at School	
		Yes	No
Class	Freshman	92	86
	Sophomore	116	52

5. How many freshmen were surveyed?

6. How many sophomores were surveyed?

7. How many students buy lunch at school?

8. How many students do not buy lunch at school?

In Exercises 9 and 10, find and interpret the marginal frequencies. *(See Example 1.)*

9.

		Set Academic Goals	
		Yes	No
Gender	Male	64	168
	Female	54	142

10.

		Cat	
		Yes	No
Dog	Yes	104	208
	No	186	98

11. **USING TWO-WAY TABLES** You conduct a survey that asks students whether they plan to participate in school spirit week. The results are shown in the two-way table. Find and interpret the marginal frequencies.

		Participate in Spirit Week		
		Yes	No	Undecided
Class	Freshman	112	56	54
	Sophomore	92	68	32

12. **USING TWO-WAY TABLES** You conduct a survey that asks college-bound high school seniors about the type of degree they plan to receive. The results are shown in the two-way table. Find and interpret the marginal frequencies.

		Type of Degree		
		Associate's	Bachelor's	Master's
Gender	Male	58	126	42
	Female	62	118	48

USING STRUCTURE In Exercises 13 and 14, complete the two-way table.

13.

		Traveled on an Airplane		
		Yes	No	Total
Class	Freshman		62	
	Sophomore	184		
	Total	274		352

14.

	Plan to Attend School Dance		
Gender	**Yes**	**No**	**Total**
Male	38		
Female		24	112
Total			196

15. MAKING TWO-WAY TABLES You conduct a survey that asks 245 students in your school whether they have taken a Spanish or a French class. One hundred nine of the students have taken a Spanish class, and 45 of those students have taken a French class. Eighty-two of the students have not taken a Spanish or a French class. Organize the results in a two-way table. Include the marginal frequencies. *(See Example 2.)*

16. MAKING TWO-WAY TABLES A car dealership has 98 cars on its lot. Fifty-five of the cars are new. Of the new cars, 36 are domestic cars. There are 15 used foreign cars on the lot. Organize this information in a two-way table. Include the marginal frequencies.

In Exercises 17 and 18, make a two-way table that shows the joint and marginal relative frequencies. *(See Example 3.)*

17.

	Exercise Preference	
Gender	**Aerobic**	**Anaerobic**
Male	88	104
Female	96	62

18.

	Meat	
Bread	**Turkey**	**Ham**
White	452	146
Wheat	328	422

19. USING TWO-WAY TABLES Refer to Exercise 17. What percent of students prefer aerobic exercise? What percent of students are males who prefer anaerobic exercise?

20. USING TWO-WAY TABLES Refer to Exercise 18. What percent of the sandwiches are on wheat bread? What percent of the sandwiches are turkey on white bread?

ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in using the two-way table.

	Participate in Fundraiser	
Class	**Yes**	**No**
Freshman	187	85
Sophomore	123	93

21.

✗ One hundred eighty-seven freshmen responded to the survey.

22.

✗ The two-way table shows the joint relative frequencies.

	Participate in Fundraiser	
Class	**Yes**	**No**
Freshman	$\frac{187}{272} \approx 0.69$	$\frac{85}{272} \approx 0.31$
Sophomore	$\frac{123}{216} \approx 0.57$	$\frac{93}{216} \approx 0.43$

23. USING TWO-WAY TABLES A company is hosting an event for its employees to celebrate the end of the year. It asks the employees whether they prefer a lunch event or a dinner event. It also asks whether they prefer a catered event or a potluck. The results are shown in the two-way table. Make a two-way table that shows the conditional relative frequencies based on the row totals. Given that an employee prefers a lunch event, what is the conditional relative frequency that he or she prefers a catered event? *(See Example 4.)*

	Menu	
Meal	**Potluck**	**Catered**
Lunch	36	48
Dinner	44	72

24. USING TWO-WAY TABLES The two-way table shows the results of a survey that asked students about their preference for a new school mascot. Make a two-way table that shows the conditional relative frequencies based on the column totals. Given that a student prefers a hawk as a mascot, what is the conditional relative frequency that he or she prefers a cartoon mascot?

		Type		
		Tiger	**Hawk**	**Dragon**
Style	**Realistic**	67	74	51
	Cartoon	58	18	24

25. ANALYZING TWO-WAY TABLES You survey college-bound seniors and find that 85% plan to live on campus, 35% plan to have a car while at college, and 5% plan to live off campus and not have a car. Is there an association between living on campus and having a car at college? Explain. *(See Example 5.)*

26. ANALYZING TWO-WAY TABLES You survey students and find that 70% watch sports on TV, 48% participate in a sport, and 16% do not watch sports on TV or participate in a sport. Is there an association between participating in a sport and watching sports on TV? Explain.

27. ANALYZING TWO-WAY TABLES The two-way table shows the results of a survey that asked adults whether they participate in recreational skiing. Is there an association between age and recreational skiing? *(See Example 6.)*

		Age				
		21–30	**31–40**	**41–50**	**51–60**	**61–70**
Ski	**Yes**	87	93	68	37	20
	No	165	195	148	117	125

28. ANALYZING TWO-WAY TABLES Refer to Exercise 12. Is there an association between gender and type of degree? Explain.

29. WRITING Compare Venn diagrams and two-way tables.

30. HOW DO YOU SEE IT? The graph shows the results of a survey that asked students about their favorite movie genre.
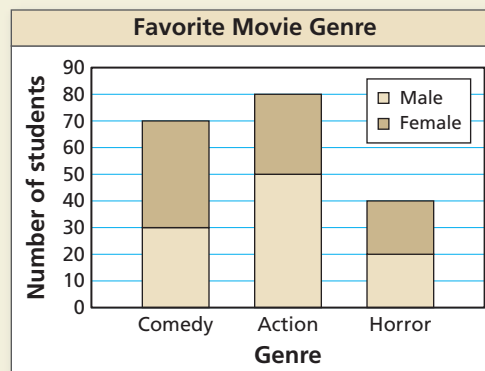
a. Display the given information in a two-way table.

b. Which of the data displays do you prefer? Explain.

31. PROBLEM SOLVING A box office sells 1809 tickets to a play, 800 of which are for the main floor. The tickets consist of $2x + y$ adult tickets on the main floor, $x - 40$ child tickets on the main floor, $x + 2y$ adult tickets in the balcony, and $3x - y - 80$ child tickets in the balcony.

a. Organize this information in a two-way table.

b. Find the values of x and y.

c. What percent of tickets are adult tickets?

d. What percent of child tickets are balcony tickets?

32. THOUGHT PROVOKING Compare "one-way tables" and "two-way tables." Is it possible to have a "three-way table?" If so, give an example of a three-way table.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Tell whether the table of values represents a *linear*, an *exponential*, or a *quadratic* function. *(Section 8.6)*

33.

x	0	1	2	3	4
y	144	24	4	$\frac{2}{3}$	$\frac{1}{9}$

34.

x	−1	0	1	2	3
y	3	0	−1	0	3