10.2 Graphing Cube Root Functions

Essential Question What are some of the characteristics of the graph of a cube root function?

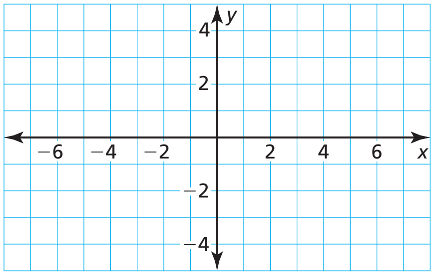
EXPLORATION 1 Graphing Cube Root Functions

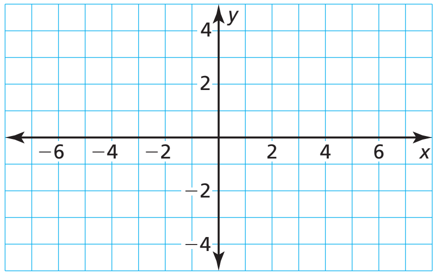
Work with a partner.

- Make a table of values for each function. Use positive and negative values of x.
- Use the table to sketch the graph of each function.
- Describe the domain of each function.
- Describe the range of each function.

a. $y = \sqrt[3]{x}$

b. $y = \sqrt[3]{x + 3}$





LOOKING FOR REGULARITY IN REPEATED REASONING

To be proficient in math, you need to notice whether calculations are repeated and look for both general methods and shortcuts.

EXPLORATION 2 Writing Cube Root Functions

Work with a partner. Write a cube root function, $y = f(x)$, that has the given values. Then use the function to complete the table.

a.

x	$f(x)$
−4	0
−3	
−2	
−1	$\sqrt[3]{3}$
0	

x	$f(x)$
1	
2	
3	
4	2
5	

b.

x	$f(x)$
−4	1
−3	
−2	
−1	$1 + \sqrt[3]{3}$
0	

x	$f(x)$
1	
2	
3	
4	3
5	

Communicate Your Answer

3. What are some of the characteristics of the graph of a cube root function?

4. Graph each function. Then compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

a. $g(x) = \sqrt[3]{x - 1}$

b. $g(x) = \sqrt[3]{x} - 1$

c. $g(x) = 2\sqrt[3]{x}$

d. $g(x) = -2\sqrt[3]{x}$

10.2 Lesson

Core Vocabulary

cube root function, *p. 552*

Previous
radical function
index

What You Will Learn

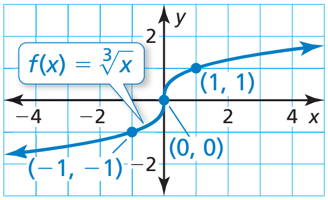
- Graph cube root functions.
- Compare cube root functions using average rates of change.
- Solve real-life problems involving cube root functions.

Graphing Cube Root Functions

Core Concept

Cube Root Functions

A **cube root function** is a radical function with an index of 3. The parent function for the family of cube root functions is $f(x) = \sqrt[3]{x}$. The domain and range of f are all real numbers.



The graph of $f(x) = \sqrt[3]{x}$ increases on the entire domain.

You can transform graphs of cube root functions in the same way you transformed graphs of square root functions.

LOOKING FOR STRUCTURE

Use x-values so that the cube root of the radicand is an integer. This makes it easier to perform the calculations and plot the points.

EXAMPLE 1 Comparing Graphs of Cube Root Functions

Graph $h(x) = \sqrt[3]{x} - 4$. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

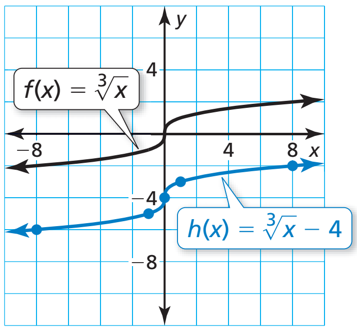
SOLUTION

Step 1 Make a table of values.

x	−8	−1	0	1	8
$h(x)$	−6	−5	−4	−3	−2

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.



The graph of h is a translation 4 units down of the graph of f.

Monitoring Progress

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Graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

1. $h(x) = \sqrt[3]{x} + 3$

2. $m(x) = \sqrt[3]{x - 5}$

3. $g(x) = 4\sqrt[3]{x}$

EXAMPLE 2 Comparing Graphs of Cube Root Functions

Graph $g(x) = -\sqrt[3]{x + 2}$. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

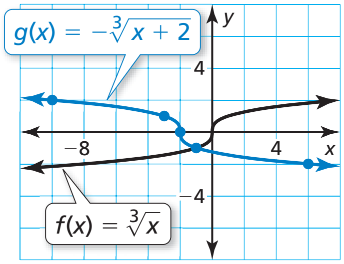
SOLUTION

Step 1 Make a table of values.

x	-10	-3	-2	-1	6
$g(x)$	2	1	0	-1	-2

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.



▶ The graph of g is a translation 2 units left and a reflection in the x-axis of the graph of f.

REMEMBER

The graph of $y = a \cdot f(x - h) + k$ can be obtained from the graph of $y = f(x)$ using the steps you learned in Section 3.6.

EXAMPLE 3 Graphing $y = a\sqrt[3]{x - h} + k$

Let $g(x) = 2\sqrt[3]{x - 3} + 4$. (a) Describe the transformations from the graph of $f(x) = \sqrt[3]{x}$ to the graph of g. (b) Graph g.

SOLUTION

a. Step 1 Translate the graph of f horizontally 3 units right to get the graph of $t(x) = \sqrt[3]{x - 3}$.

Step 2 Stretch the graph of t vertically by a factor of 2 to get the graph of $h(x) = 2\sqrt[3]{x - 3}$.

Step 3 Because $a > 0$, there is no reflection.

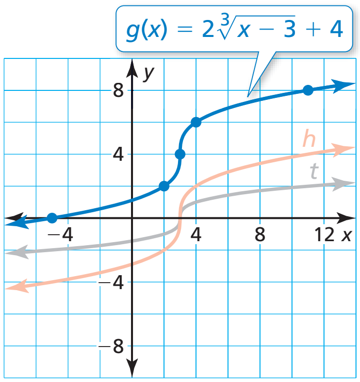
Step 4 Translate the graph of h vertically 4 units up to get the graph of $g(x) = 2\sqrt[3]{x - 3} + 4$.

b. Step 1 Make a table of values.

x	-5	2	3	4	11
$g(x)$	0	2	4	6	8

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.



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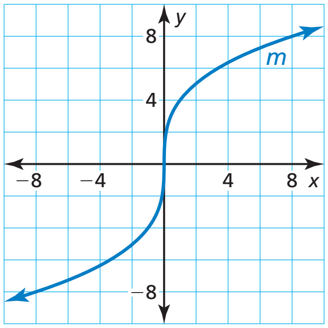
Graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

4. $g(x) = \sqrt[3]{0.5x} + 5$ **5.** $h(x) = 4\sqrt[3]{x} - 1$ **6.** $n(x) = \sqrt[3]{4 - x}$

7. Let $g(x) = -\frac{1}{2}\sqrt[3]{x + 2} - 4$. Describe the transformations from the graph of $f(x) = \sqrt[3]{x}$ to the graph of g. Then graph g.

Comparing Average Rates of Change

EXAMPLE 4 **Comparing Cube Root Functions**



The graph of cube root function m is shown. Compare the average rate of change of m to the average rate of change of $h(x) = \sqrt[3]{\frac{1}{4}x}$ over the interval $x = 0$ to $x = 8$.

SOLUTION

To calculate the average rates of change, use points whose x-coordinates are 0 and 8.

Function m: Use the graph to estimate. Use (0, 0) and (8, 8).

$$\frac{m(8) - m(0)}{8 - 0} \approx \frac{8 - 0}{8} = 1 \qquad \text{Average rate of change of } m$$

Function h: Evaluate h when $x = 0$ and $x = 8$.

$$h(0) = \sqrt[3]{\frac{1}{4}(0)} = 0 \quad \text{and} \quad h(8) = \sqrt[3]{\frac{1}{4}(8)} = \sqrt[3]{2} \approx 1.3$$

Use (0, 0) and $\left(8, \sqrt[3]{2}\right)$.

$$\frac{h(8) - h(0)}{8 - 0} = \frac{\sqrt[3]{2} - 0}{8} \approx 0.16 \qquad \text{Average rate of change of } h$$

▶ The average rate of change of m is $1 \div \frac{\sqrt[3]{2}}{8} \approx 6.3$ times greater than the average rate of change of h over the interval $x = 0$ to $x = 8$.

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8. In Example 4, compare the average rates of change over the interval $x = 2$ to $x = 10$.

Solving Real-Life Problems

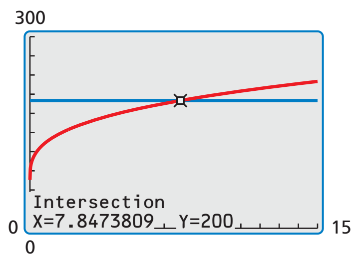
EXAMPLE 5 **Real-Life Application**

The shoulder height h (in centimeters) of a male Asian elephant can be modeled by the function $h = 62.5\sqrt[3]{t} + 75.8$, where t is the age (in years) of the elephant. Use a graphing calculator to graph the function. Estimate the age of an elephant whose shoulder height is 200 centimeters.

SOLUTION

Step 1 Enter $y_1 = 62.5\sqrt[3]{t} + 75.8$ and $y_2 = 200$ into your calculator and graph the equations. Choose a viewing window that shows the point where the graphs intersect.

Step 2 Use the *intersect* feature to find the x-coordinate of the intersection point.



▶ The two graphs intersect at about (8, 200). So, the elephant is about 8 years old.

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9. **WHAT IF?** Estimate the age of an elephant whose shoulder height is 175 centimeters.

10.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The __________ of the radical in a cube root function is 3.

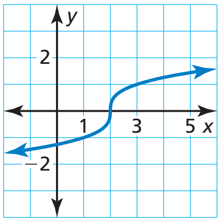
2. **WRITING** Describe the domain and range of the function $f(x) = \sqrt[3]{x-4} + 1$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, match the function with its graph.

3. $y = \sqrt[3]{x+2}$
4. $y = \sqrt[3]{x-2}$
5. $y = \sqrt[3]{x} + 2$
6. $y = \sqrt[3]{x} - 2$

A.



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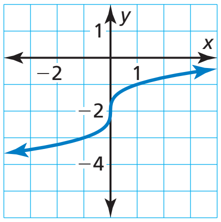

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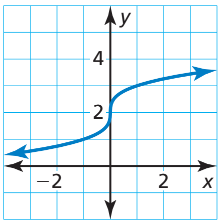

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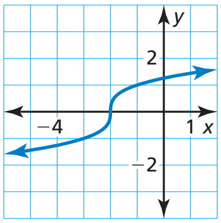
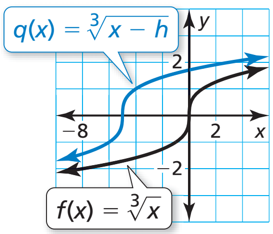


In Exercises 7–12, graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$. *(See Example 1.)*

7. $h(x) = \sqrt[3]{x-4}$
8. $g(x) = \sqrt[3]{x+1}$
9. $m(x) = \sqrt[3]{x} + 5$
10. $q(x) = \sqrt[3]{x} - 3$
11. $p(x) = 6\sqrt[3]{x}$
12. $j(x) = \sqrt[3]{\frac{1}{2}x}$

In Exercises 13–16, compare the graphs. Find the value of h, k, or a.

13.



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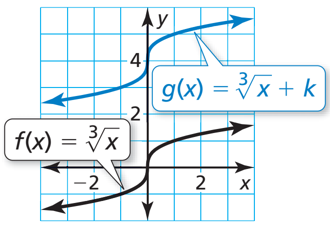

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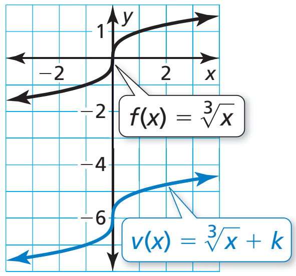

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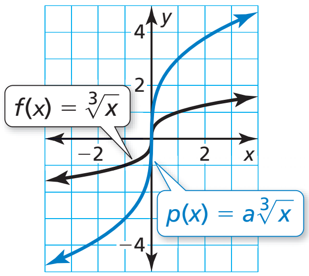


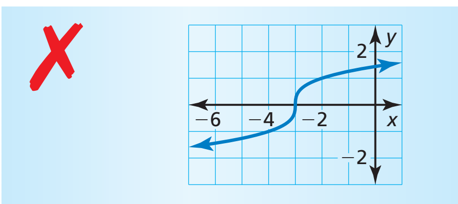
In Exercises 17–26, graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$. *(See Example 2.)*

17. $r(x) = -\sqrt[3]{x-2}$
18. $h(x) = -\sqrt[3]{x} + 3$
19. $k(x) = 5\sqrt[3]{x+1}$
20. $j(x) = 0.5\sqrt[3]{x-4}$
21. $g(x) = 4\sqrt[3]{x} - 3$
22. $m(x) = 3\sqrt[3]{x} + 7$
23. $n(x) = \sqrt[3]{-8x} - 1$
24. $v(x) = \sqrt[3]{5x} + 2$
25. $q(x) = \sqrt[3]{2(x+3)}$
26. $p(x) = \sqrt[3]{3(1-x)}$

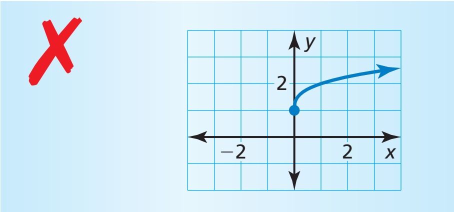
In Exercises 27–32, describe the transformations from the graph of $f(x) = \sqrt[3]{x}$ to the graph of the given function. Then graph the given function. *(See Example 3.)*

27. $g(x) = \sqrt[3]{x-4} + 2$
28. $n(x) = \sqrt[3]{x+1} - 3$
29. $j(x) = -5\sqrt[3]{x+3} + 2$
30. $k(x) = 6\sqrt[3]{x-9} - 5$
31. $v(x) = \frac{1}{3}\sqrt[3]{x-1} + 7$
32. $h(x) = -\frac{3}{2}\sqrt[3]{x+4} - 3$

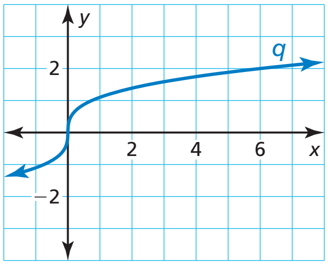
33. **ERROR ANALYSIS** Describe and correct the error in graphing the function $f(x) = \sqrt[3]{x-3}$.



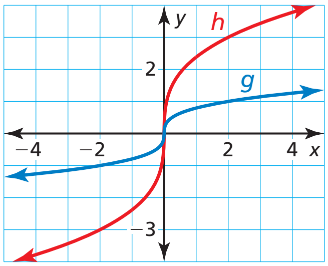
34. ERROR ANALYSIS Describe and correct the error in graphing the function $h(x) = \sqrt[3]{x} + 1$.



35. COMPARING FUNCTIONS The graph of cube root function q is shown. Compare the average rate of change of q to the average rate of change of $f(x) = 3\sqrt[3]{x}$ over the interval $x = 0$ to $x = 6$. *(See Example 4.)*



36. COMPARING FUNCTIONS The graphs of two cube root functions are shown. Compare the average rates of change of the two functions over the interval $x = -2$ to $x = 2$.

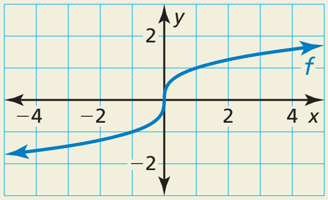


37. MODELING WITH MATHEMATICS For a drag race car that weighs 1600 kilograms, the velocity v (in kilometers per hour) reached by the end of a drag race can be modeled by the function $v = 23.8\sqrt[3]{p}$, where p is the car's power (in horsepower). Use a graphing calculator to graph the function. Estimate the power of a 1600-kilogram car that reaches a velocity of 220 kilometers per hour. *(See Example 5.)*

38. MODELING WITH MATHEMATICS The radius r of a sphere is given by the function $r = \sqrt[3]{\frac{3}{4\pi}V}$, where V is the volume of the sphere. Use a graphing calculator to graph the function. Estimate the volume of a spherical beach ball with a radius of 13 inches.

39. MAKING AN ARGUMENT Your friend says that all cube root functions are odd functions. Is your friend correct? Explain.

40. HOW DO YOU SEE IT? The graph represents the cube root function $f(x) = \sqrt[3]{x}$.



a. On what interval is f negative? positive?

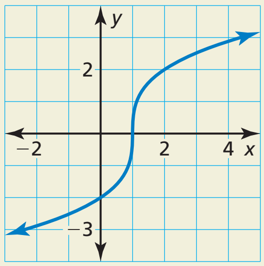
b. On what interval, if any, is f decreasing? increasing?

c. Does f have a maximum or minimum value? Explain.

d. Find the average rate of change of f over the interval $x = -1$ to $x = 1$.

41. PROBLEM SOLVING Write a cube root function that passes through the point (3, 4) and has an average rate of change of −1 over the interval $x = -5$ to $x = 2$.

42. THOUGHT PROVOKING Write the cube root function represented by the graph. Use a graphing calculator to check your answer.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Factor the polynomial. *(Section 7.6)*

43. $3x^2 + 12x - 36$ **44.** $2x^2 - 11x + 9$ **45.** $4x^2 + 7x - 15$

Solve the equation using square roots. *(Section 9.3)*

46. $x^2 - 36 = 0$ **47.** $5x^2 + 20 = 0$ **48.** $(x + 4)^2 = 81$ **49.** $25(x - 2)^2 = 9$