7.7 Factoring Special Products

Essential Question How can you recognize and factor special products?

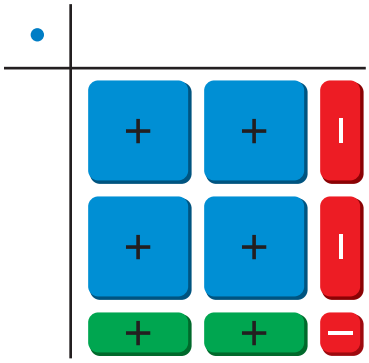
EXPLORATION 1 Factoring Special Products

Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying. State whether the product is a "special product" that you studied in Section 7.3.

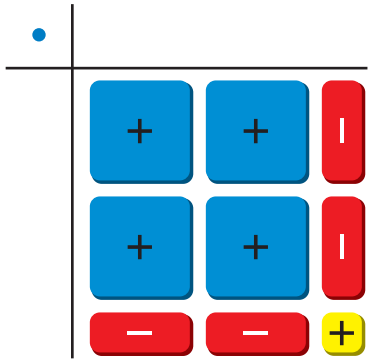
LOOKING FOR STRUCTURE

To be proficient in math, you need to see complicated things as single objects or as being composed of several objects.

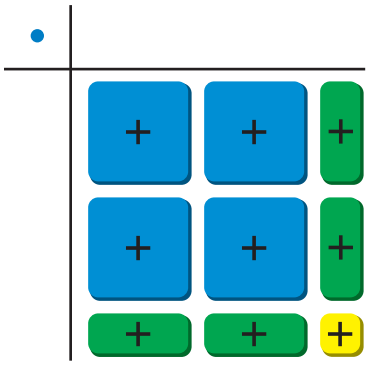
a. $4x^2 - 1 =$ _____



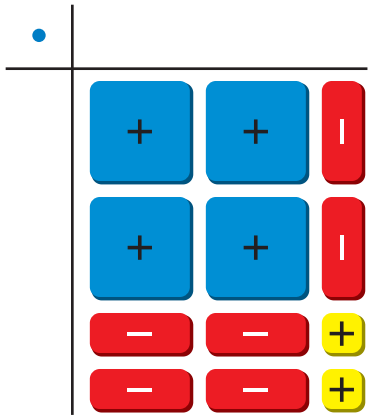
b. $4x^2 - 4x + 1 =$ _____



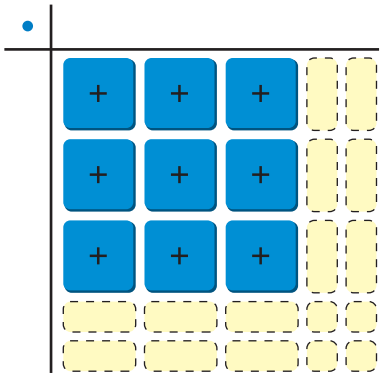
c. $4x^2 + 4x + 1 =$ _____



d. $4x^2 - 6x + 2 =$ _____



EXPLORATION 2 Factoring Special Products



Work with a partner. Use algebra tiles to complete the rectangular array at the left in three different ways, so that each way represents a different special product. Write each special product in standard form and in factored form.

Communicate Your Answer

3. How can you recognize and factor special products? Describe a strategy for recognizing which polynomials can be factored as special products.

4. Use the strategy you described in Question 3 to factor each polynomial.

a. $25x^2 + 10x + 1$ **b.** $25x^2 - 10x + 1$ **c.** $25x^2 - 1$

7.7 Lesson

Core Vocabulary

Previous
polynomial
trinomial

What You Will Learn

- Factor the difference of two squares.
- Factor perfect square trinomials.
- Use factoring to solve real-life problems.

Factoring the Difference of Two Squares

You can use special product patterns to factor polynomials.

Core Concept

Difference of Two Squares Pattern

Algebra	Example
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

EXAMPLE 1 Factoring the Difference of Two Squares

Factor (a) $x^2 - 25$ and (b) $4z^2 - 1$.

SOLUTION

a. $x^2 - 25 = x^2 - 5^2$ Write as $a^2 - b^2$.

$= (x + 5)(x - 5)$ Difference of two squares pattern

▶ So, $x^2 - 25 = (x + 5)(x - 5)$.

b. $4z^2 - 1 = (2z)^2 - 1^2$ Write as $a^2 - b^2$.

$= (2z + 1)(2z - 1)$ Difference of two squares pattern

▶ So, $4z^2 - 1 = (2z + 1)(2z - 1)$.

EXAMPLE 2 Evaluating a Numerical Expression

Use a special product pattern to evaluate the expression $54^2 - 48^2$.

SOLUTION

Notice that $54^2 - 48^2$ is a difference of two squares. So, you can rewrite the expression in a form that it is easier to evaluate using the difference of two squares pattern.

$54^2 - 48^2 = (54 + 48)(54 - 48)$ Difference of two squares pattern

$= 102(6)$ Simplify.

$= 612$ Multiply.

▶ So, $54^2 - 48^2 = 612$.

Monitoring Progress

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Factor the polynomial.

1. $x^2 - 36$ **2.** $100 - m^2$ **3.** $9n^2 - 16$ **4.** $16h^2 - 49$

Use a special product pattern to evaluate the expression.

5. $36^2 - 34^2$ **6.** $47^2 - 44^2$ **7.** $55^2 - 50^2$ **8.** $28^2 - 24^2$

Factoring Perfect Square Trinomials

Core Concept

Perfect Square Trinomial Pattern

Algebra	Example
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2 = (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2 = (x - 3)^2$

EXAMPLE 3 Factoring Perfect Square Trinomials

Factor each polynomial.

a. $n^2 + 8n + 16$ **b.** $4x^2 - 12x + 9$

SOLUTION

a. $n^2 + 8n + 16 = n^2 + 2(n)(4) + 4^2$ Write as $a^2 + 2ab + b^2$.

$= (n + 4)^2$ Perfect square trinomial pattern

▶ So, $n^2 + 8n + 16 = (n + 4)^2$.

b. $4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$ Write as $a^2 - 2ab + b^2$.

$= (2x - 3)^2$ Perfect square trinomial pattern

▶ So, $4x^2 - 12x + 9 = (2x - 3)^2$.

EXAMPLE 4 Solving a Polynomial Equation

Solve $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$.

SOLUTION

$x^2 + \frac{2}{3}x + \frac{1}{9} = 0$ Write equation.

$9x^2 + 6x + 1 = 0$ Multiply each side by 9.

$(3x)^2 + 2(3x)(1) + 1^2 = 0$ Write left side as $a^2 + 2ab + b^2$.

$(3x + 1)^2 = 0$ Perfect square trinomial pattern

$3x + 1 = 0$ Zero-Product Property

$x = -\frac{1}{3}$ Solve for x.

▶ The solution is $x = -\frac{1}{3}$.

LOOKING FOR STRUCTURE

Equations of the form $(x + a)^2 = 0$ always have repeated roots of $x = -a$.

Monitoring Progress

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Factor the polynomial.

9. $m^2 - 2m + 1$ **10.** $d^2 - 10d + 25$ **11.** $9z^2 + 36z + 36$

Solve the equation.

12. $a^2 + 6a + 9 = 0$ **13.** $w^2 - \frac{7}{3}w + \frac{49}{36} = 0$ **14.** $n^2 - 81 = 0$

Solving Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**

A bird picks up a golf ball and drops it while flying. The function represents the height y (in feet) of the golf ball t seconds after it is dropped. The ball hits the top of a 32-foot-tall pine tree. After how many seconds does the ball hit the tree?

SOLUTION

1. **Understand the Problem** You are given the height of the golf ball as a function of the amount of time after it is dropped and the height of the tree that the golf ball hits. You are asked to determine how many seconds it takes for the ball to hit the tree.

2. **Make a Plan** Use the function for the height of the golf ball. Substitute the height of the tree for y and solve for the time t.

3. **Solve the Problem** Substitute 32 for y and solve for t.

$$y = 81 - 16t^2 \qquad \text{Write equation.}$$
$$32 = 81 - 16t^2 \qquad \text{Substitute 32 for } y.$$
$$0 = 49 - 16t^2 \qquad \text{Subtract 32 from each side.}$$
$$0 = 7^2 - (4t)^2 \qquad \text{Write as } a^2 - b^2.$$
$$0 = (7 + 4t)(7 - 4t) \qquad \text{Difference of two squares pattern}$$
$$7 + 4t = 0 \quad \text{or} \quad 7 - 4t = 0 \qquad \text{Zero-Product Property}$$
$$t = -\tfrac{7}{4} \quad \text{or} \quad t = \tfrac{7}{4} \qquad \text{Solve for } t.$$

A negative time does not make sense in this situation.

▶ So, the golf ball hits the tree after $\frac{7}{4}$, or 1.75 seconds.

4. **Look Back** Check your solution, as shown, by substituting $t = \frac{7}{4}$ into the equation $32 = 81 - 16t^2$. Then verify that a time of $\frac{7}{4}$ seconds gives a height of 32 feet.

Check

$$32 = 81 - 16t^2$$
$$32 \stackrel{?}{=} 81 - 16\left(\tfrac{7}{4}\right)^2$$
$$32 \stackrel{?}{=} 81 - 16\left(\tfrac{49}{16}\right)$$
$$32 \stackrel{?}{=} 81 - 49$$
$$32 = 32 \checkmark$$

Monitoring Progress

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15. **WHAT IF?** The golf ball does not hit the pine tree. After how many seconds does the ball hit the ground?

7.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** Can you use the perfect square trinomial pattern to factor $y^2 + 16y + 64$? Explain.

2. **WHICH ONE DOESN'T BELONG?** Which polynomial does *not* belong with the other three? Explain your reasoning.

$n^2 - 4$ | $g^2 - 6g + 9$ | $r^2 + 12r + 36$ | $k^2 + 25$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, factor the polynomial. *(See Example 1.)*

3. $m^2 - 49$
4. $z^2 - 81$
5. $64 - 81d^2$
6. $25 - 4x^2$
7. $225a^2 - 36b^2$
8. $16x^2 - 169y^2$

In Exercises 9–14, use a special product pattern to evaluate the expression. *(See Example 2.)*

9. $12^2 - 9^2$
10. $19^2 - 11^2$
11. $78^2 - 72^2$
12. $54^2 - 52^2$
13. $53^2 - 47^2$
14. $39^2 - 36^2$

In Exercises 15–22, factor the polynomial. *(See Example 3.)*

15. $h^2 + 12h + 36$
16. $p^2 + 30p + 225$
17. $y^2 - 22y + 121$
18. $x^2 - 4x + 4$
19. $a^2 - 28a + 196$
20. $m^2 + 24m + 144$
21. $25n^2 + 20n + 4$
22. $49a^2 - 14a + 1$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in factoring the polynomial.**

23. ✗
$$n^2 - 64 = n^2 - 8^2$$
$$= (n - 8)^2$$

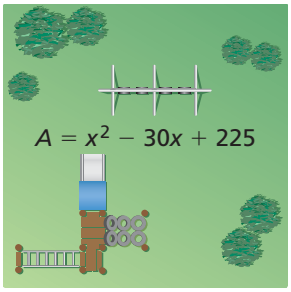
24. ✗
$$y^2 - 6y + 9 = y^2 - 2(y)(3) + 3^2$$
$$= (y - 3)(y + 3)$$

25. **MODELING WITH MATHEMATICS** The area (in square centimeters) of a square coaster can be represented by $d^2 + 8d + 16$.

   a. Write an expression that represents the side length of the coaster.

   b. Write an expression for the perimeter of the coaster.

26. **MODELING WITH MATHEMATICS** The polynomial represents the area (in square feet) of the square playground.



   a. Write a polynomial that represents the side length of the playground.

   b. Write an expression for the perimeter of the playground.

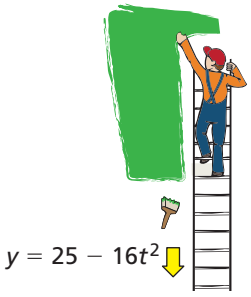
In Exercises 27–34, solve the equation. *(See Example 4.)*

27. $z^2 - 4 = 0$
28. $4x^2 = 49$
29. $k^2 - 16k + 64 = 0$
30. $s^2 + 20s + 100 = 0$
31. $n^2 + 9 = 6n$
32. $y^2 = 12y - 36$
33. $y^2 + \frac{1}{2}y = -\frac{1}{16}$
34. $-\frac{4}{3}x + \frac{4}{9} = -x^2$

In Exercises 35–40, factor the polynomial.

35. $3z^2 - 27$
36. $2m^2 - 50$
37. $4y^2 - 16y + 16$
38. $8k^2 + 80k + 200$
39. $50y^2 + 120y + 72$
40. $27m^2 - 36m + 12$

41. MODELING WITH MATHEMATICS While standing on a ladder, you drop a paintbrush. The function represents the height y (in feet) of the paintbrush t seconds after it is dropped. After how many seconds does the paintbrush land on the ground? *(See Example 5.)*



42. MODELING WITH MATHEMATICS The function represents the height y (in feet) of a grasshopper jumping straight up from the ground t seconds after the start of the jump. After how many seconds is the grasshopper 1 foot off the ground?

43. REASONING Tell whether the polynomial can be factored. If not, change the constant term so that the polynomial is a perfect square trinomial.

a. $w^2 + 18w + 84$ **b.** $y^2 - 10y + 23$

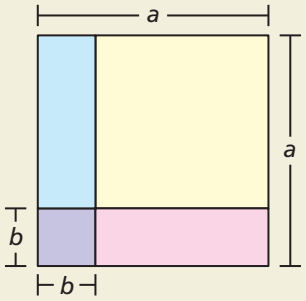
44. THOUGHT PROVOKING Use algebra tiles to factor each polynomial modeled by the tiles. Show your work.

a.

b.

45. COMPARING METHODS Describe two methods you can use to simplify $(2x - 5)^2 - (x - 4)^2$. Which one would you use? Explain.

46. HOW DO YOU SEE IT? The figure shows a large square with an area of a^2 that contains a smaller square with an area of b^2.



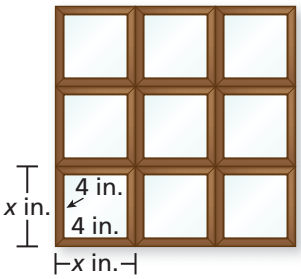
a. Describe the regions that represent $a^2 - b^2$. How can you rearrange these regions to show that $a^2 - b^2 = (a + b)(a - b)$?

b. How can you use the figure to show that $(a - b)^2 = a^2 - 2ab + b^2$?

47. PROBLEM SOLVING You hang nine identical square picture frames on a wall.

a. Write a polynomial that represents the area of the picture frames, not including the pictures.

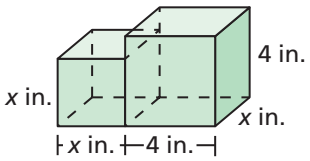
b. The area in part (a) is 81 square inches. What is the side length of one of the picture frames? Explain your reasoning.



48. MATHEMATICAL CONNECTIONS The composite solid is made up of a cube and a rectangular prism.

a. Write a polynomial that represents the volume of the composite solid.

b. The volume of the composite solid is equal to $25x$. What is the value of x? Explain your reasoning.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the prime factorization of the number. *(Skills Review Handbook)*

49. 50 **50.** 44 **51.** 85 **52.** 96

Graph the inequality in a coordinate plane. *(Section 5.6)*

53. $y \leq 4x - 1$ **54.** $y > -\frac{1}{2}x + 3$ **55.** $4y - 12 \geq 8x$ **56.** $3y + 3 < x$