7.6 Factoring $ax^2 + bx + c$

Essential Question

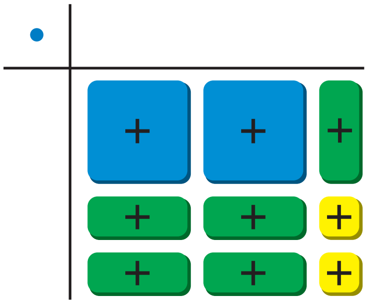
How can you use algebra tiles to factor the trinomial $ax^2 + bx + c$ into the product of two binomials?

EXPLORATION 1 Finding Binomial Factors

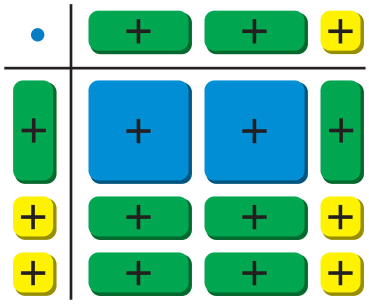
Work with a partner. Use algebra tiles to write each polynomial as the product of two binomials. Check your answer by multiplying.

Sample $2x^2 + 5x + 2$

Step 1 Arrange algebra tiles that model $2x^2 + 5x + 2$ into a rectangular array.



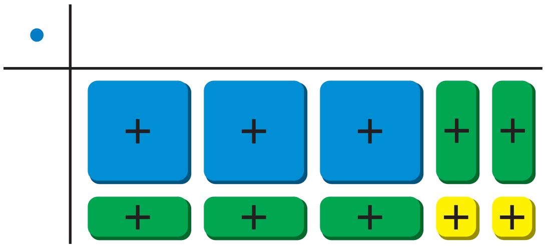
Step 2 Use additional algebra tiles to model the dimensions of the rectangle.



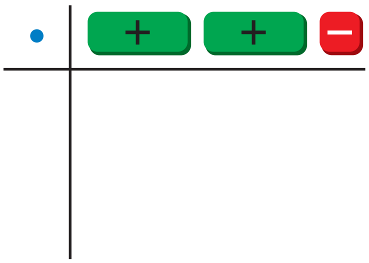
Step 3 Write the polynomial in factored form using the dimensions of the rectangle.

$$\text{Area} = 2x^2 + 5x + 2 = (\overbrace{x + 2}^{\text{width}})(\overbrace{2x + 1}^{\text{length}})$$

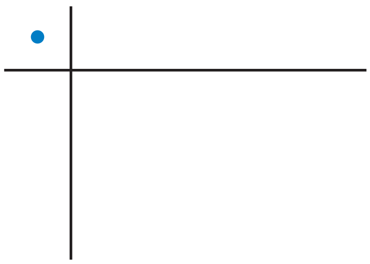
a. $3x^2 + 5x + 2 =$ ______



b. $4x^2 + 4x - 3 =$ ______



c. $2x^2 - 11x + 5 =$ ______



USING TOOLS STRATEGICALLY

To be proficient in math, you need to consider the available tools, including concrete models, when solving a mathematical problem.

Communicate Your Answer

2. How can you use algebra tiles to factor the trinomial $ax^2 + bx + c$ into the product of two binomials?
3. Is it possible to factor the trinomial $2x^2 + 2x + 1$? Explain your reasoning.

7.6 Lesson

Core Vocabulary

Previous
polynomial
greatest common factor (GCF)
Zero-Product Property

What You Will Learn

- Factor $ax^2 + bx + c$.
- Use factoring to solve real-life problems.

Factoring $ax^2 + bx + c$

In Section 7.5, you factored polynomials of the form $ax^2 + bx + c$, where $a = 1$. To factor polynomials of the form $ax^2 + bx + c$, where $a \neq 1$, first look for the GCF of the terms of the polynomial and then factor further if possible.

EXAMPLE 1 Factoring Out the GCF

Factor $5x^2 + 15x + 10$.

SOLUTION

Notice that the GCF of the terms $5x^2$, $15x$, and 10 is 5.

$5x^2 + 15x + 10 = 5(x^2 + 3x + 2)$ Factor out GCF.

$= 5(x + 1)(x + 2)$ Factor $x^2 + 3x + 2$.

▶ So, $5x^2 + 15x + 10 = 5(x + 1)(x + 2)$.

When there is no GCF, consider the possible factors of a and c.

EXAMPLE 2 Factoring $ax^2 + bx + c$ When ac Is Positive

Factor each polynomial.

a. $4x^2 + 13x + 3$ **b.** $3x^2 - 7x + 2$

SOLUTION

a. There is no GCF, so you need to consider the possible factors of a and c. Because b and c are both positive, the factors of c must be positive. Use a table to organize information about the factors of a and c.

STUDY TIP

You must consider the order of the factors of 3, because the middle terms formed by the possible factorizations are different.

Factors of 4	Factors of 3	Possible factorization	Middle term	
1, 4	1, 3	$(x + 1)(4x + 3)$	$3x + 4x = 7x$	✗
1, 4	3, 1	$(x + 3)(4x + 1)$	$x + 12x = 13x$	✓
2, 2	1, 3	$(2x + 1)(2x + 3)$	$6x + 2x = 8x$	✗

▶ So, $4x^2 + 13x + 3 = (x + 3)(4x + 1)$.

b. There is no GCF, so you need to consider the possible factors of a and c. Because b is negative and c is positive, both factors of c must be negative. Use a table to organize information about the factors of a and c.

Factors of 3	Factors of 2	Possible factorization	Middle term	
1, 3	$-1, -2$	$(x - 1)(3x - 2)$	$-2x - 3x = -5x$	✗
1, 3	$-2, -1$	$(x - 2)(3x - 1)$	$-x - 6x = -7x$	✓

▶ So, $3x^2 - 7x + 2 = (x - 2)(3x - 1)$.

EXAMPLE 3 Factoring $ax^2 + bx + c$ When ac Is Negative

Factor $2x^2 - 5x - 7$.

SOLUTION

There is no GCF, so you need to consider the possible factors of a and c. Because c is negative, the factors of c must have different signs. Use a table to organize information about the factors of a and c.

Factors of 2	Factors of −7	Possible factorization	Middle term	
1, 2	1, −7	$(x + 1)(2x - 7)$	$-7x + 2x = -5x$	✓
1, 2	7, −1	$(x + 7)(2x - 1)$	$-x + 14x = 13x$	✗
1, 2	−1, 7	$(x - 1)(2x + 7)$	$7x - 2x = 5x$	✗
1, 2	−7, 1	$(x - 7)(2x + 1)$	$x - 14x = -13x$	✗

► So, $2x^2 - 5x - 7 = (x + 1)(2x - 7)$.

STUDY TIP

When a is negative, factor -1 from each term of $ax^2 + bx + c$. Then factor the resulting trinomial as in the previous examples.

EXAMPLE 4 Factoring $ax^2 + bx + c$ When a Is Negative

Factor $-4x^2 - 8x + 5$.

SOLUTION

Step 1 Factor -1 from each term of the trinomial.

$$-4x^2 - 8x + 5 = -(4x^2 + 8x - 5)$$

Step 2 Factor the trinomial $4x^2 + 8x - 5$. Because c is negative, the factors of c must have different signs. Use a table to organize information about the factors of a and c.

Factors of 4	Factors of −5	Possible factorization	Middle term	
1, 4	1, −5	$(x + 1)(4x - 5)$	$-5x + 4x = -x$	✗
1, 4	5, −1	$(x + 5)(4x - 1)$	$-x + 20x = 19x$	✗
1, 4	−1, 5	$(x - 1)(4x + 5)$	$5x - 4x = x$	✗
1, 4	−5, 1	$(x - 5)(4x + 1)$	$x - 20x = -19x$	✗
2, 2	1, −5	$(2x + 1)(2x - 5)$	$-10x + 2x = -8x$	✗
2, 2	−1, 5	$(2x - 1)(2x + 5)$	$10x - 2x = 8x$	✓

► So, $-4x^2 - 8x + 5 = -(2x - 1)(2x + 5)$.

Monitoring Progress

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Factor the polynomial.

1. $8x^2 - 56x + 48$ **2.** $14x^2 + 31x + 15$ **3.** $2x^2 - 7x + 5$

4. $3x^2 - 14x + 8$ **5.** $4x^2 - 19x - 5$ **6.** $6x^2 + x - 12$

7. $-2y^2 - 5y - 3$ **8.** $-5m^2 + 6m - 1$ **9.** $-3x^2 - x + 2$

Solving Real-Life Problems

EXAMPLE 5 **Solving a Real-Life Problem**

The length of a rectangular game reserve is 1 mile longer than twice the width. The area of the reserve is 55 square miles. What is the width of the reserve?

SOLUTION

Use the formula for the area of a rectangle to write an equation for the area of the reserve. Let w represent the width. Then $2w + 1$ represents the length. Solve for w.

$w(2w + 1) = 55$	Area of the reserve
$2w^2 + w = 55$	Distributive Property
$2w^2 + w - 55 = 0$	Subtract 55 from each side.

Factor the left side of the equation. There is no GCF, so you need to consider the possible factors of a and c. Because c is negative, the factors of c must have different signs. Use a table to organize information about the factors of a and c.

Factors of 2	Factors of −55	Possible factorization	Middle term	
1, 2	1, −55	$(w + 1)(2w - 55)$	$-55w + 2w = -53w$	✗
1, 2	55, −1	$(w + 55)(2w - 1)$	$-w + 110w = 109w$	✗
1, 2	−1, 55	$(w - 1)(2w + 55)$	$55w - 2w = 53w$	✗
1, 2	−55, 1	$(w - 55)(2w + 1)$	$w - 110w = -109w$	✗
1, 2	5, −11	$(w + 5)(2w - 11)$	$-11w + 10w = -w$	✗
1, 2	11, −5	$(w + 11)(2w - 5)$	$-5w + 22w = 17w$	✗
1, 2	−5, 11	$(w - 5)(2w + 11)$	$11w - 10w = w$	✓
1, 2	−11, 5	$(w - 11)(2w + 5)$	$5w - 22w = -17w$	✗

So, you can rewrite $2w^2 + w - 55$ as $(w - 5)(2w + 11)$. Write the equation with the left side factored and continue solving for w.

$(w - 5)(2w + 11) = 0$	Rewrite equation with left side factored.
$w - 5 = 0$ or $2w + 11 = 0$	Zero-Product Property
$w = 5$ or $w = -\frac{11}{2}$	Solve for w.

A negative width does not make sense, so you should use the positive solution.

▶ So, the width of the reserve is 5 miles.

Check

Use mental math.

The width is 5 miles, so the length is $5(2) + 1 = 11$ miles and the area is $5(11) = 55$ square miles. ✓

Monitoring Progress

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10. WHAT IF? The area of the reserve is 136 square miles. How wide is the reserve?

7.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** What is the greatest common factor of the terms of $3y^2 - 21y + 36$?

2. **WRITING** Compare factoring $6x^2 - x - 2$ with factoring $x^2 - x - 2$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, factor the polynomial. *(See Example 1.)*

3. $3x^2 + 3x - 6$
4. $8v^2 + 8v - 48$
5. $4k^2 + 28k + 48$
6. $6y^2 - 24y + 18$
7. $7b^2 - 63b + 140$
8. $9r^2 - 36r - 45$

In Exercises 9–16, factor the polynomial.
(See Examples 2 and 3.)

9. $3h^2 + 11h + 6$
10. $8m^2 + 30m + 7$
11. $6x^2 - 5x + 1$
12. $10w^2 - 31w + 15$
13. $3n^2 + 5n - 2$
14. $4z^2 + 4z - 3$
15. $8g^2 - 10g - 12$
16. $18v^2 - 15v - 18$

In Exercises 17–22, factor the polynomial.
(See Example 4.)

17. $-3t^2 + 11t - 6$
18. $-7v^2 - 25v - 12$
19. $-4c^2 + 19c + 5$
20. $-8h^2 - 13h + 6$
21. $-15w^2 - w + 28$
22. $-22d^2 + 29d - 9$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in factoring the polynomial.**

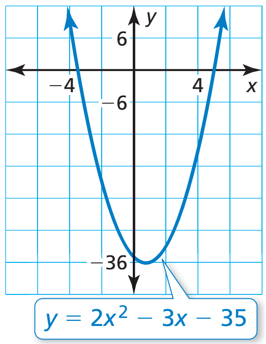
23. ✗
$$2x^2 - 2x - 24 = 2(x^2 - 2x - 24)$$
$$= 2(x - 6)(x + 4)$$

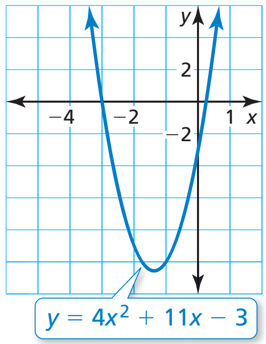
24. ✗
$$6x^2 - 7x - 3 = (3x - 3)(2x + 1)$$

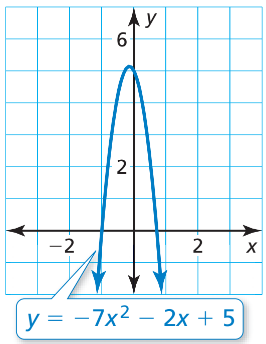
In Exercises 25–28, solve the equation.

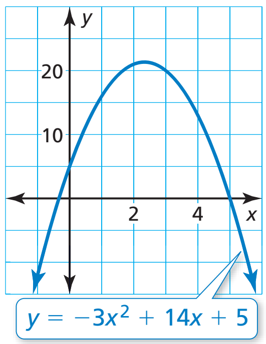
25. $5x^2 - 5x - 30 = 0$
26. $2k^2 - 5k - 18 = 0$
27. $-12n^2 - 11n = -15$
28. $14b^2 - 2 = -3b$

In Exercises 29–32, find the x-coordinates of the points where the graph crosses the x-axis.

29. 

30. 

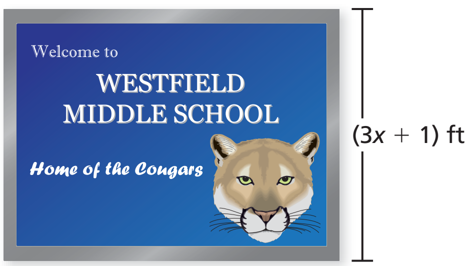
31. 

32. 

33. **MODELING WITH MATHEMATICS** The area (in square feet) of the school sign can be represented by $15x^2 - x - 2$.

   a. Write an expression that represents the length of the sign.

   b. Describe two ways to find the area of the sign when $x = 3$.



34. MODELING WITH MATHEMATICS The height h (in feet) above the water of a cliff diver is modeled by $h = -16t^2 + 8t + 80$, where t is the time (in seconds). How long is the diver in the air?

35. MODELING WITH MATHEMATICS The Parthenon in Athens, Greece, is an ancient structure that has a rectangular base. The length of the base of the Parthenon is 8 meters more than twice its width. The area of the base is about 2170 square meters. Find the length and width of the base. *(See Example 5.)*

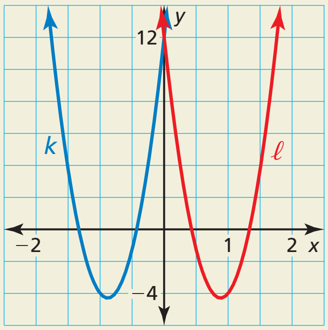
36. MODELING WITH MATHEMATICS The length of a rectangular birthday party invitation is 1 inch less than twice its width. The area of the invitation is 15 square inches. Will the invitation fit in the envelope shown without being folded? Explain.

$3\frac{5}{8}$ in.

$5\frac{1}{8}$ in.

37. OPEN-ENDED Write a binomial whose terms have a GCF of $3x$.

38. HOW DO YOU SEE IT? Without factoring, determine which of the graphs represents the function $g(x) = 21x^2 + 37x + 12$ and which represents the function $h(x) = 21x^2 - 37x + 12$. Explain your reasoning.

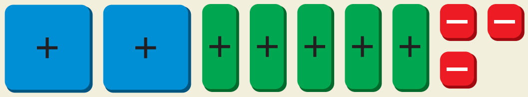


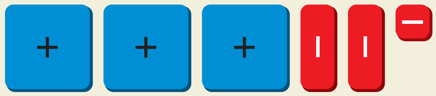
39. REASONING When is it not possible to factor $ax^2 + bx + c$, where $a \neq 1$? Give an example.

40. MAKING AN ARGUMENT Your friend says that to solve the equation $5x^2 + x - 4 = 2$, you should start by factoring the left side as $(5x - 4)(x + 1)$. Is your friend correct? Explain.

41. REASONING For what values of t can $2x^2 + tx + 10$ be written as the product of two binomials?

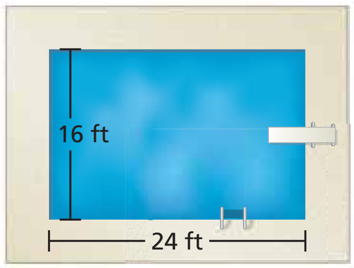
42. THOUGHT PROVOKING Use algebra tiles to factor each polynomial modeled by the tiles. Show your work.

a. 

b. 

43. MATHEMATICAL CONNECTIONS The length of a rectangle is 1 inch more than twice its width. The value of the area of the rectangle (in square inches) is 5 more than the value of the perimeter of the rectangle (in inches). Find the width.

44. PROBLEM SOLVING A rectangular swimming pool is bordered by a concrete patio. The width of the patio is the same on every side. The area of the surface of the pool is equal to the area of the patio. What is the width of the patio?



In Exercises 45–48, factor the polynomial.

45. $4k^2 + 7jk - 2j^2$

46. $6x^2 + 5xy - 4y^2$

47. $-6a^2 + 19ab - 14b^2$

48. $18m^3 + 39m^2n - 15mn^2$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the square root(s). *(Skills Review Handbook)*

49. $\pm\sqrt{64}$

50. $\sqrt{4}$

51. $-\sqrt{225}$

52. $\pm\sqrt{81}$

Solve the system of linear equations by substitution. Check your solution. *(Section 5.2)*

53. $y = 3 + 7x$
$y - x = -3$

54. $2x = y + 2$
$-x + 3y = 14$

55. $5x - 2y = 14$
$-7 = -2x + y$

56. $-x - 8 = -y$
$9y - 12 + 3x = 0$