5.7 Systems of Linear Inequalities

Essential Question How can you graph a system of linear inequalities?

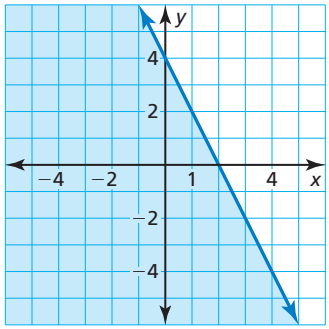
EXPLORATION 1 Graphing Linear Inequalities

Work with a partner. Match each linear inequality with its graph. Explain your reasoning.

$2x + y \leq 4$ Inequality 1

$2x - y \leq 0$ Inequality 2

A.



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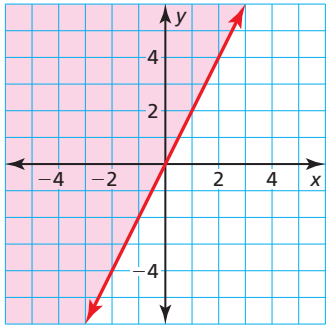


EXPLORATION 2 Graphing a System of Linear Inequalities

Work with a partner. Consider the linear inequalities given in Exploration 1.

$2x + y \leq 4$ Inequality 1

$2x - y \leq 0$ Inequality 2

a. Use two different colors to graph the inequalities in the same coordinate plane. What is the result?

b. Describe each of the shaded regions of the graph. What does the unshaded region represent?

MAKING SENSE OF PROBLEMS

To be proficient in math, you need to explain to yourself the meaning of a problem.

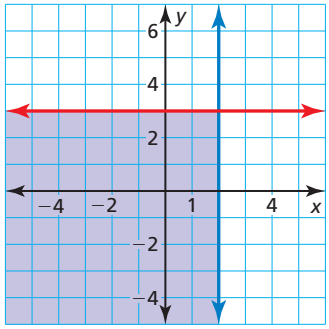
Communicate Your Answer

3. How can you graph a system of linear inequalities?

4. When graphing a system of linear inequalities, which region represents the solution of the system?

5. Do you think all systems of linear inequalities have a solution? Explain your reasoning.

6. Write a system of linear inequalities represented by the graph.



5.7 Lesson

Core Vocabulary

system of linear inequalities, *p. 274*

solution of a system of linear inequalities, *p. 274*

graph of a system of linear inequalities, *p. 275*

Previous

linear inequality in two variables

What You Will Learn

- Check solutions of systems of linear inequalities.
- Graph systems of linear inequalities.
- Write systems of linear inequalities.
- Use systems of linear inequalities to solve real-life problems.

Systems of Linear Inequalities

A **system of linear inequalities** is a set of two or more linear inequalities in the same variables. An example is shown below.

$y < x + 2$ Inequality 1

$y \geq 2x - 1$ Inequality 2

A **solution of a system of linear inequalities** in two variables is an ordered pair that is a solution of each inequality in the system.

EXAMPLE 1 Checking Solutions

Tell whether each ordered pair is a solution of the system of linear inequalities.

$y < 2x$ Inequality 1

$y \geq x + 1$ Inequality 2

a. $(3, 5)$ **b.** $(-2, 0)$

SOLUTION

a. Substitute 3 for x and 5 for y in each inequality.

Inequality 1	Inequality 2
$y < 2x$	$y \geq x + 1$
$5 \stackrel{?}{<} 2(3)$	$5 \stackrel{?}{\geq} 3 + 1$
$5 < 6$ ✓	$5 \geq 4$ ✓

▶ Because the ordered pair $(3, 5)$ is a solution of each inequality, it is a solution of the system.

b. Substitute -2 for x and 0 for y in each inequality.

Inequality 1	Inequality 2
$y < 2x$	$y \geq x + 1$
$0 \stackrel{?}{<} 2(-2)$	$0 \stackrel{?}{\geq} -2 + 1$
$0 \not< -4$ ✗	$0 \geq -1$ ✓

▶ Because $(-2, 0)$ is not a solution of each inequality, it is *not* a solution of the system.

Monitoring Progress

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Tell whether the ordered pair is a solution of the system of linear inequalities.

1. $(-1, 5)$; $y < 5$, $y > x - 4$

2. $(1, 4)$; $y \geq 3x + 1$, $y > x - 1$

Graphing Systems of Linear Inequalities

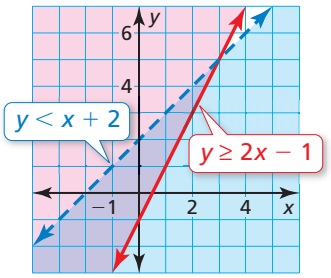
The **graph of a system of linear inequalities** is the graph of all the solutions of the system.

Core Concept

Graphing a System of Linear Inequalities

Step 1 Graph each inequality in the same coordinate plane.

Step 2 Find the intersection of the half-planes that are solutions of the inequalities. This intersection is the graph of the system.



Check

Verify that $(-3, 1)$ is a solution of each inequality.

Inequality 1

$y \leq 3$

$1 \leq 3$ ✓

Inequality 2

$y > x + 2$

$1 \stackrel{?}{>} -3 + 2$

$1 > -1$ ✓

EXAMPLE 2 Graphing a System of Linear Inequalities

Graph the system of linear inequalities.

$y \leq 3$ Inequality 1

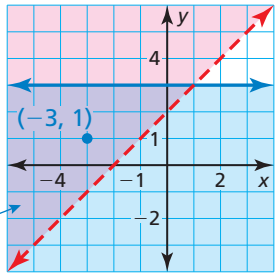
$y > x + 2$ Inequality 2

SOLUTION

Step 1 Graph each inequality.

Step 2 Find the intersection of the half-planes. One solution is $(-3, 1)$.

The solution is the purple-shaded region.



EXAMPLE 3 Graphing a System of Linear Inequalities: No Solution

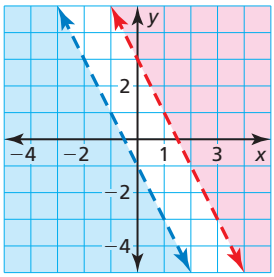
Graph the system of linear inequalities.

$2x + y < -1$ Inequality 1

$2x + y > 3$ Inequality 2

SOLUTION

Step 1 Graph each inequality.



Step 2 Find the intersection of the half-planes. Notice that the lines are parallel, and the half-planes do not intersect.

▶ So, the system has no solution.

Monitoring Progress

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Graph the system of linear inequalities.

3. $y \geq -x + 4$
$x + y \leq 0$

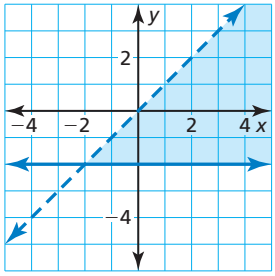
4. $y > 2x - 3$
$y \geq \frac{1}{2}x + 1$

5. $-2x + y < 4$
$2x + y > 4$

Writing Systems of Linear Inequalities

EXAMPLE 4 Writing a System of Linear Inequalities

Write a system of linear inequalities represented by the graph.



SOLUTION

Inequality 1 The horizontal boundary line passes through $(0, -2)$. So, an equation of the line is $y = -2$. The shaded region is *above* the *solid* boundary line, so the inequality is $y \geq -2$.

Inequality 2 The slope of the other boundary line is 1, and the y-intercept is 0. So, an equation of the line is $y = x$. The shaded region is *below* the *dashed* boundary line, so the inequality is $y < x$.

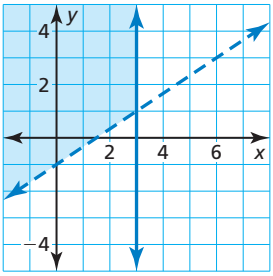
▶ The system of linear inequalities represented by the graph is

$y \geq -2$ Inequality 1

$y < x$. Inequality 2

EXAMPLE 5 Writing a System of Linear Inequalities

Write a system of linear inequalities represented by the graph.



SOLUTION

Inequality 1 The vertical boundary line passes through $(3, 0)$. So, an equation of the line is $x = 3$. The shaded region is to the *left* of the *solid* boundary line, so the inequality is $x \leq 3$.

Inequality 2 The slope of the other boundary line is $\frac{2}{3}$, and the y-intercept is -1. So, an equation of the line is $y = \frac{2}{3}x - 1$. The shaded region is *above* the *dashed* boundary line, so the inequality is $y > \frac{2}{3}x - 1$.

▶ The system of linear inequalities represented by the graph is

$x \leq 3$ Inequality 1

$y > \frac{2}{3}x - 1$. Inequality 2

Monitoring Progress

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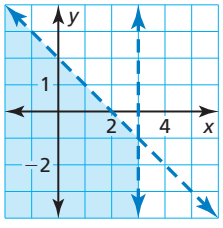
Write a system of linear inequalities represented by the graph.

6.



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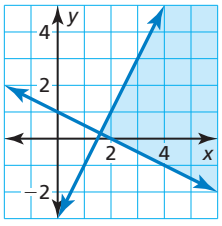


Solving Real-Life Problems

EXAMPLE 6 **Modeling with Mathematics**

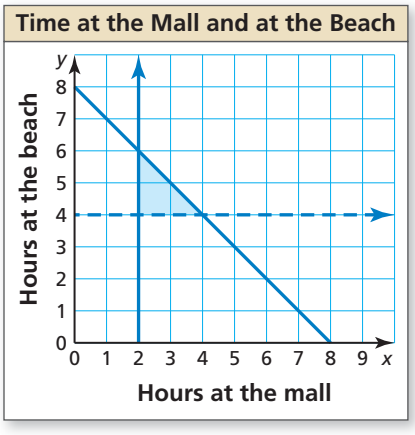
You have at most 8 hours to spend at the mall and at the beach. You want to spend at least 2 hours at the mall and more than 4 hours at the beach. Write and graph a system that represents the situation. How much time can you spend at each location?

SOLUTION

1. **Understand the Problem** You know the total amount of time you can spend at the mall and at the beach. You also know how much time you want to spend at each location. You are asked to write and graph a system that represents the situation and determine how much time you can spend at each location.
2. **Make a Plan** Use the given information to write a system of linear inequalities. Then graph the system and identify an ordered pair in the solution region.
3. **Solve the Problem** Let x be the number of hours at the mall and let y be the number of hours at the beach.

$$x + y \leq 8 \quad \text{at most 8 hours at the mall and at the beach}$$
$$x \geq 2 \quad \text{at least 2 hours at the mall}$$
$$y > 4 \quad \text{more than 4 hours at the beach}$$

Graph the system.



One ordered pair in the solution region is (2.5, 5).

So, you can spend 2.5 hours at the mall and 5 hours at the beach.

4. **Look Back** Check your solution by substituting it into the inequalities in the system, as shown.

Check

$x + y \leq 8$

$2.5 + 5 \stackrel{?}{\leq} 8$

$7.5 \leq 8$ ✓

$x \geq 2$

$2.5 \geq 2$ ✓

$y > 4$

$5 > 4$ ✓

Monitoring Progress

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8. Name another solution of Example 6.
9. **WHAT IF?** You want to spend at least 3 hours at the mall. How does this change the system? Is (2.5, 5) still a solution? Explain.

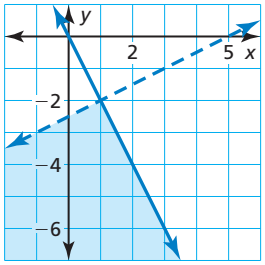
5.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** How can you verify that an ordered pair is a solution of a system of linear inequalities?

2. **WHICH ONE DOESN'T BELONG?** Use the graph shown. Which of the ordered pairs does *not* belong with the other three? Explain your reasoning.



(1, −2) (0, −4) (−1, −6) (2, −4)

Monitoring Progress and Modeling with Mathematics

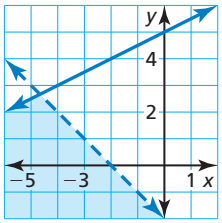
In Exercises 3–6, tell whether the ordered pair is a solution of the system of linear inequalities.

3. (−4, 3)

4. (−3, −1)

5. (−2, 5)

6. (1, 1)



In Exercises 7–10, tell whether the ordered pair is a solution of the system of linear inequalities. *(See Example 1.)*

7. (−5, 2); $y < 4$, $y > x + 3$

8. (1, −1); $y > -2$, $y > x - 5$

9. (0, 0); $y \leq x + 7$, $y \geq 2x + 3$

10. (4, −3); $y \leq -x + 1$, $y \leq 5x - 2$

In Exercises 11–20, graph the system of linear inequalities. *(See Examples 2 and 3.)*

11. $y > -3$, $y \geq 5x$

12. $y < -1$, $x > 4$

13. $y < -2$, $y > 2$

14. $y < x - 1$, $y \geq x + 1$

15. $y \geq -5$, $y - 1 < 3x$

16. $x + y > 4$, $y \geq \frac{3}{2}x - 9$

17. $x + y > 1$, $-x - y < -3$

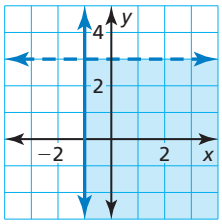
18. $2x + y \leq 5$, $y + 2 \geq -2x$

19. $x < 4$, $y > 1$, $y \geq -x + 1$

20. $x + y \leq 10$, $x - y \geq 2$, $y > 2$

In Exercises 21–26, write a system of linear inequalities represented by the graph. *(See Examples 4 and 5.)*

21.



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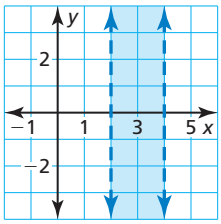

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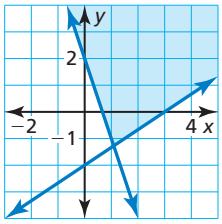

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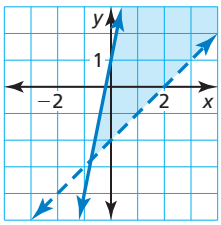

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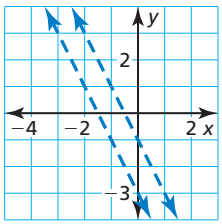

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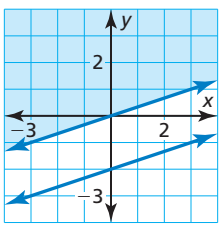
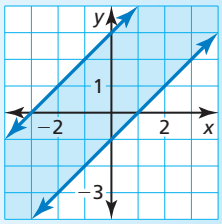


ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in graphing the system of linear inequalities.**

27.

$y \leq x - 1$

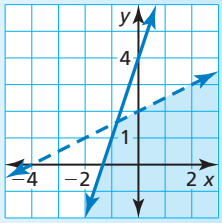
$y \geq x + 3$



28.

$y \leq 3x + 4$

$y > \frac{1}{2}x + 2$



29. **MODELING WITH MATHEMATICS** You can spend at most \$21 on fruit. Blueberries cost \$4 per pound, and strawberries cost \$3 per pound. You need at least 3 pounds of fruit to make muffins. *(See Example 6.)*

a. Write and graph a system of linear inequalities that represents the situation.

b. Identify and interpret a solution of the system.

c. Use the graph to determine whether you can buy 4 pounds of blueberries and 1 pound of strawberries.

30. **MODELING WITH MATHEMATICS** You earn \$10 per hour working as a manager at a grocery store. You are required to work at the grocery store at least 8 hours per week. You also teach music lessons for \$15 per hour. You need to earn at least \$120 per week, but you do not want to work more than 20 hours per week.

a. Write and graph a system of linear inequalities that represents the situation.

b. Identify and interpret a solution of the system.

c. Use the graph to determine whether you can work 8 hours at the grocery store and teach 1 hour of music lessons.

31. **MODELING WITH MATHEMATICS** You are fishing for surfperch and rockfish, which are species of bottomfish. Gaming laws allow you to catch no more than 15 surfperch per day, no more than 10 rockfish per day, and no more than 20 total bottomfish per day.

a. Write and graph a system of linear inequalities that represents the situation.

b. Use the graph to determine whether you can catch 11 surfperch and 9 rockfish in 1 day.

surfperch

rockfish

32. **REASONING** Describe the intersection of the half-planes of the system shown.

$$x - y \leq 4$$
$$x - y \geq 4$$

33. **MATHEMATICAL CONNECTIONS** The following points are the vertices of a shaded rectangle.

$(-1, 1), (6, 1), (6, -3), (-1, -3)$

a. Write a system of linear inequalities represented by the shaded rectangle.

b. Find the area of the rectangle.

34. **MATHEMATICAL CONNECTIONS** The following points are the vertices of a shaded triangle.

$(2, 5), (6, -3), (-2, -3)$

a. Write a system of linear inequalities represented by the shaded triangle.

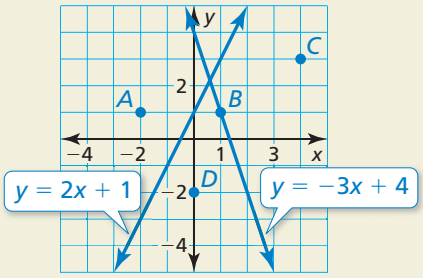
b. Find the area of the triangle.

35. **PROBLEM SOLVING** You plan to spend less than half of your monthly \$2000 paycheck on housing and savings. You want to spend at least 10% of your paycheck on savings and at most 30% of it on housing. How much money can you spend on savings and housing?

36. **PROBLEM SOLVING** On a road trip with a friend, you drive about 70 miles per hour, and your friend drives about 60 miles per hour. The plan is to drive less than 15 hours and at least 600 miles each day. Your friend will drive more hours than you. How many hours can you and your friend each drive in 1 day?

37. WRITING How are solving systems of linear inequalities and solving systems of linear equations similar? How are they different?

38. HOW DO YOU SEE IT? The graphs of two linear equations are shown.



Replace the equal signs with inequality symbols to create a system of linear inequalities that has point C as a solution, but not points A, B, and D. Explain your reasoning.

$y \square -3x + 4$

$y \square 2x + 1$

39. USING STRUCTURE Write a system of linear inequalities that is equivalent to $|y| < x$, where $x > 0$. Graph the system.

40. MAKING AN ARGUMENT Your friend says that a system of linear inequalities in which the boundary lines are parallel must have no solution. Is your friend correct? Explain.

41. CRITICAL THINKING Is it possible for the solution set of a system of linear inequalities to be all real numbers? Explain your reasoning.

OPEN-ENDED In Exercises 42–44, write a system of linear inequalities with the given characteristic.

42. All solutions are in Quadrant I.

43. All solutions have one positive coordinate and one negative coordinate.

44. There are no solutions.

45. OPEN-ENDED One inequality in a system is $-4x + 2y > 6$. Write another inequality so the system has (a) no solution and (b) infinitely many solutions.

46. THOUGHT PROVOKING You receive a gift certificate for a clothing store and plan to use it to buy T-shirts and sweatshirts. Describe a situation in which you can buy 9 T-shirts and 1 sweatshirt, but you cannot buy 3 T-shirts and 8 sweatshirts. Write and graph a system of linear inequalities that represents the situation.

47. CRITICAL THINKING Write a system of linear inequalities that has exactly one solution.

48. MODELING WITH MATHEMATICS You make necklaces and key chains to sell at a craft fair. The table shows the amounts of time and money it takes to make a necklace and a key chain, and the amounts of time and money you have available for making them.

	Necklace	Key chain	Available
Time to make (hours)	0.5	0.25	20
Cost to make (dollars)	2	3	120

a. Write and graph a system of four linear inequalities that represents the number x of necklaces and the number y of key chains that you can make.

b. Find the vertices (corner points) of the graph of the system.

c. You sell each necklace for \$10 and each key chain for \$8. The revenue R is given by the equation $R = 10x + 8y$. Find the revenue corresponding to each ordered pair in part (b). Which vertex results in the maximum revenue?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the product using exponents. *(Skills Review Handbook)*

49. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

50. $(-13) \cdot (-13) \cdot (-13)$

51. $x \cdot x \cdot x \cdot x \cdot x \cdot x$

Write an equation of the line with the given slope and y-intercept. *(Section 4.1)*

52. slope: 1, y-intercept: -6

53. slope: -3, y-intercept: 5

54. slope: $-\frac{1}{4}$, y-intercept: -1

55. slope: $\frac{4}{3}$, y-intercept: 0