# 5.5 Solving Equations by Graphing

**Essential Question** How can you use a system of linear equations to solve an equation with variables on both sides?

Previously, you learned how to use algebra to solve equations with variables on both sides. Another way is to use a system of linear equations.

## EXPLORATION 1 Solving an Equation by Graphing

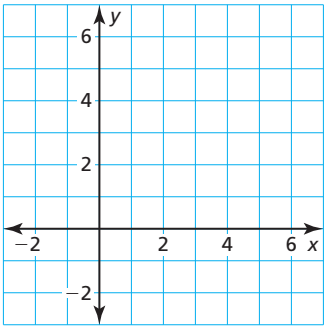
**Work with a partner.** Solve $2x - 1 = -\frac{1}{2}x + 4$ by graphing.

**a.** Use the left side to write a linear equation. Then use the right side to write another linear equation.

**b.** Graph the two linear equations from part (a). Find the $x$-value of the point of intersection. Check that the $x$-value is the solution of

$$2x - 1 = -\frac{1}{2}x + 4.$$

**c.** Explain why this "graphical method" works.




**USING TOOLS STRATEGICALLY**

To be proficient in math, you need to consider the available tools, which may include pencil and paper or a graphing calculator, when solving a mathematical problem.

## EXPLORATION 2 Solving Equations Algebraically and Graphically

**Work with a partner.** Solve each equation using two methods.

**Method 1** Use an algebraic method.

**Method 2** Use a graphical method.

Is the solution the same using both methods?

**a.** $\frac{1}{2}x + 4 = -\frac{1}{4}x + 1$

**b.** $\frac{2}{3}x + 4 = \frac{1}{3}x + 3$

**c.** $-\frac{2}{3}x - 1 = \frac{1}{3}x - 4$

**d.** $\frac{4}{5}x + \frac{7}{5} = 3x - 3$

**e.** $-x + 2.5 = 2x - 0.5$

**f.** $-3x + 1.5 = x + 1.5$

## Communicate Your Answer

**3.** How can you use a system of linear equations to solve an equation with variables on both sides?

**4.** Compare the algebraic method and the graphical method for solving a linear equation with variables on both sides. Describe the advantages and disadvantages of each method.

# 5.5 Lesson

## What You Will Learn

- Solve linear equations by graphing.
- Solve absolute value equations by graphing.
- Use linear equations to solve real-life problems.

### Core Vocabulary

***Previous***
absolute value equation

## Solving Linear Equations by Graphing

You can use a system of linear equations to solve an equation with variables on both sides.

### Core Concept

#### Solving Linear Equations by Graphing

**Step 1** To solve the equation $ax + b = cx + d$, write two linear equations.

$$ax + b = cx + d$$

$y = ax + b$ and $y = cx + d$

**Step 2** Graph the system of linear equations. The $x$-value of the solution of the system of linear equations is the solution of the equation $ax + b = cx + d$.

### EXAMPLE 1 Solving an Equation by Graphing

Solve $-x + 1 = 2x - 5$ by graphing. Check your solution.

**SOLUTION**

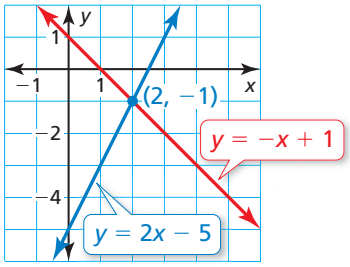
**Step 1** Write a system of linear equations using each side of the original equation.

$$-x + 1 = 2x - 5$$

$y = -x + 1$ $\qquad$ $y = 2x - 5$

**Step 2** Graph the system.

$y = -x + 1$ Equation 1
$y = 2x - 5$ Equation 2




The graphs intersect at $(2, -1)$.

So, the solution of the equation is $x = 2$.

**Check**

$$-x + 1 = 2x - 5$$
$$-(2) + 1 \stackrel{?}{=} 2(2) - 5$$
$$-1 = -1 \checkmark$$

## Monitoring Progress

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**Solve the equation by graphing. Check your solution.**

**1.** $\frac{1}{2}x - 3 = 2x$

**2.** $-4 + 9x = -3x + 2$

## Solving Absolute Value Equations by Graphing

### EXAMPLE 2 Solving an Absolute Value Equation by Graphing

Solve $|x + 1| = |2x - 4|$ by graphing. Check your solutions.

### SOLUTION

Recall that an absolute value equation of the form $|ax + b| = |cx + d|$ has two related equations.

$ax + b = cx + d$ Equation 1

$ax + b = -(cx + d)$ Equation 2

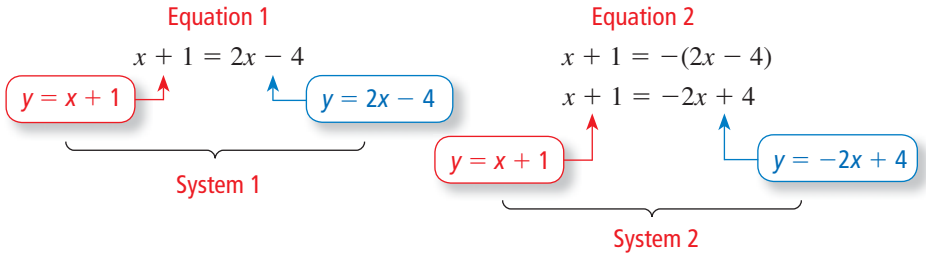
So, the related equations of $|x + 1| = |2x - 4|$ are as follows.

$x + 1 = 2x - 4$ Equation 1

$x + 1 = -(2x - 4)$ Equation 2

Apply the steps for solving an equation by graphing to each of the related equations.

**Step 1** Write a system of linear equations for each related equation.




**Check**

$|x + 1| = |2x - 4|$

$|5 + 1| \stackrel{?}{=} |2(5) - 4|$

$|6| \stackrel{?}{=} |6|$

$6 = 6$ ✓

$|x + 1| = |2x - 4|$

$|1 + 1| \stackrel{?}{=} |2(1) - 4|$

$|2| \stackrel{?}{=} |-2|$

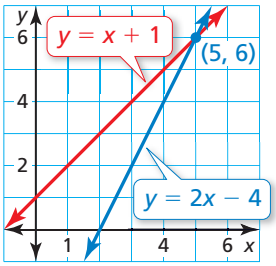
$2 = 2$ ✓

**Step 2** Graph each system.

System 1

$y = x + 1$

$y = 2x - 4$

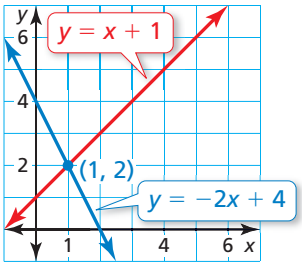



The graphs intersect at (5, 6).

System 2

$y = x + 1$

$y = -2x + 4$




The graphs intersect at (1, 2).

So, the solutions of the equation are $x = 5$ and $x = 1$.

## Monitoring Progress

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**Solve the equation by graphing. Check your solutions.**

**3.** $|2x + 2| = |x - 2|$

**4.** $|x - 6| = |-x + 4|$

## Solving Real-Life Problems

### EXAMPLE 3 Modeling with Mathematics


Your family needs to rent a car for a week while on vacation. Company A charges \$3.25 per mile plus a flat fee of \$125 per week. Company B charges \$3 per mile plus a flat fee of \$150 per week. After how many miles of travel are the total costs the same at both companies?

**SOLUTION**

1. **Understand the Problem** You know the costs of renting a car from two companies. You are asked to determine how many miles of travel will result in the same total costs at both companies.
2. **Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation by graphing.
3. **Solve the Problem**

**Words**

Company A: Cost per mile • Miles + Flat fee = Company B: Cost per mile • Miles + Flat fee

**Variable** Let $x$ be the number of miles traveled.

**Equation** $3.25x + 125 = 3x + 150$

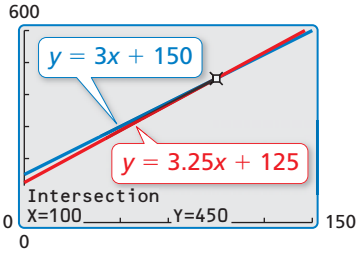
Solve the equation by graphing.

**Step 1** Write a system of linear equations using each side of the original equation.

$$3.25x + 125 = 3x + 150$$

$y = 3.25x + 125$ $\qquad$ $y = 3x + 150$

**Step 2** Use a graphing calculator to graph the system.




Because the graphs intersect at (100, 450), the solution of the equation is $x = 100$.

So, the total costs are the same after 100 miles.

4. **Look Back** One way to check your solution is to solve the equation algebraically, as shown.

**Check**

$$3.25x + 125 = 3x + 150$$
$$0.25x + 125 = 150$$
$$0.25x = 25$$
$$x = 100$$

## Monitoring Progress

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5. **WHAT IF?** Company C charges \$3.30 per mile plus a flat fee of \$115 per week. After how many miles are the total costs the same at Company A and Company C?

# 5.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

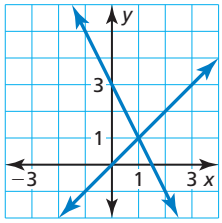
1. **REASONING** The graphs of the equations $y = 3x - 20$ and $y = -2x + 10$ intersect at the point $(6, -2)$. Without solving, find the solution of the equation $3x - 20 = -2x + 10$.

2. **WRITING** Explain how to rewrite the absolute value equation $|2x - 4| = |-5x + 1|$ as two systems of linear equations.

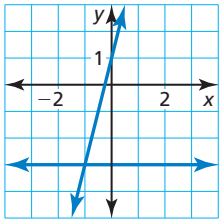
## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–6, use the graph to solve the equation. Check your solution.**

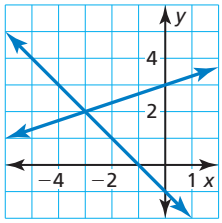
3. $-2x + 3 = x$



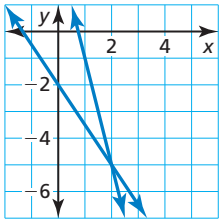

4. $-3 = 4x + 1$




5. $-x - 1 = \frac{1}{3}x + 3$




6. $-\frac{3}{2}x - 2 = -4x + 3$




**In Exercises 7–14, solve the equation by graphing. Check your solution.** *(See Example 1.)*

7. $x + 4 = -x$

8. $4x = x + 3$

9. $x + 5 = -2x - 4$

10. $-2x + 6 = 5x - 1$

11. $\frac{1}{2}x - 2 = 9 - 5x$

12. $-5 + \frac{1}{4}x = 3x + 6$

13. $5x - 7 = 2(x + 1)$

14. $-6(x + 4) = -3x - 6$

**In Exercises 15–20, solve the equation by graphing. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*.**

15. $3x - 1 = -x + 7$

16. $5x - 4 = 5x + 1$

17. $-4(2 - x) = 4x - 8$

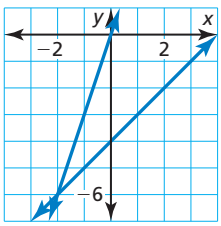
18. $-2x - 3 = 2(x - 2)$

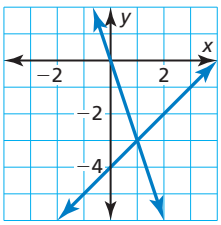
19. $-x - 5 = -\frac{1}{3}(3x + 5)$

20. $\frac{1}{2}(8x + 3) = 4x + \frac{3}{2}$

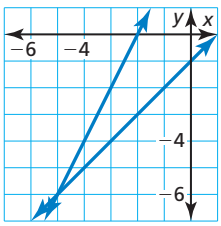
**In Exercises 21 and 22, use the graphs to solve the equation. Check your solutions.**

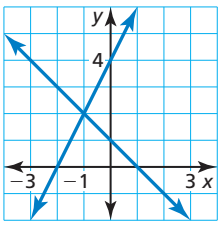
21. $|x - 4| = |3x|$




22. $|2x + 4| = |x - 1|$




**In Exercises 23–30, solve the equation by graphing. Check your solutions.** *(See Example 2.)*

23. $|2x| = |x + 3|$

24. $|2x - 6| = |x|$

25. $|-x + 4| = |2x - 2|$

26. $|x + 2| = |-3x + 6|$

27. $|x + 1| = |x - 5|$

28. $|2x + 5| = |-2x + 1|$

29. $|x - 3| = 2|x|$

30. $4|x + 2| = |2x + 7|$

**USING TOOLS** **In Exercises 31 and 32, use a graphing calculator to solve the equation.**

**31.** $0.7x + 0.5 = -0.2x - 1.3$

**32.** $2.1x + 0.6 = -1.4x + 6.9$

**33.** **MODELING WITH MATHEMATICS** You need to hire a catering company to serve meals to guests at a wedding reception. Company A charges \$500 plus \$20 per guest. Company B charges \$800 plus \$16 per guest. For how many guests are the total costs the same at both companies? *(See Example 3.)*

**34.** **MODELING WITH MATHEMATICS** Your dog is 16 years old in dog years. Your cat is 28 years old in cat years. For every human year, your dog ages by 7 dog years and your cat ages by 4 cat years. In how many human years will both pets be the same age in their respective types of years?

**35.** **MODELING WITH MATHEMATICS** You and a friend race across a field to a fence and back. Your friend has a 50-meter head start. The equations shown represent you and your friend's distances $d$ (in meters) from the fence $t$ seconds after the race begins. Find the time at which you catch up to your friend.

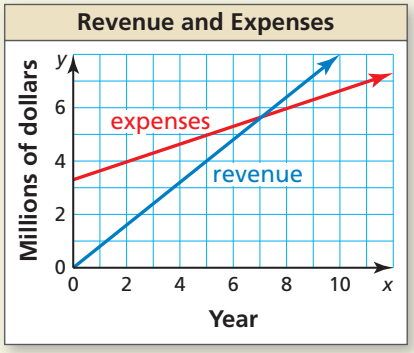
You: $d = |-5t + 100|$

Your friend: $d = \left|-3\frac{1}{3}t + 50\right|$

**36.** **MAKING AN ARGUMENT** The graphs of $y = -x + 4$ and $y = 2x - 8$ intersect at the point $(4, 0)$. So, your friend says the solution of the equation $-x + 4 = 2x - 8$ is $(4, 0)$. Is your friend correct? Explain.

**37.** **OPEN-ENDED** Find values for $m$ and $b$ so that the solution of the equation $mx + b = -2x - 1$ is $x = -3$.

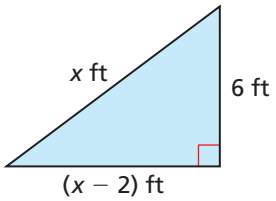
**38.** **HOW DO YOU SEE IT?** The graph shows the total revenue and expenses of a company $x$ years after it opens for business.




**a.** Estimate the point of intersection of the graphs.

**b.** Interpret your answer in part (a).

**39.** **MATHEMATICAL CONNECTIONS** The value of the perimeter of the triangle (in feet) is equal to the value of the area of the triangle (in square feet). Use a graph to find $x$.




**40.** **THOUGHT PROVOKING** A car has an initial value of \$20,000 and decreases in value at a rate of \$1500 per year. Describe a different car that will be worth the same amount as this car in exactly 5 years. Specify the initial value and the rate at which the value decreases.

**41.** **ABSTRACT REASONING** Use a graph to determine the sign of the solution of the equation $ax + b = cx + d$ in each situation.

**a.** $0 < b < d$ and $a < c$ **b.** $d < b < 0$ and $a < c$

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Graph the inequality.** *(Section 2.1)*

**42.** $y > 5$ **43.** $x \leq -2$ **44.** $n \geq 9$ **45.** $c < -6$

**Use the graphs of $f$ and $g$ to describe the transformation from the graph of $f$ to the graph of $g$.** *(Section 3.6)*

**46.** $f(x) = x - 5$; $g(x) = f(x + 2)$

**47.** $f(x) = 6x$; $g(x) = -f(x)$

**48.** $f(x) = -2x + 1$; $g(x) = f(4x)$

**49.** $f(x) = \frac{1}{2}x - 2$; $g(x) = f(x - 1)$