4.6 Arithmetic Sequences

**Essential Question** How can you use an arithmetic sequence to describe a pattern?

An arithmetic sequence is an ordered list of numbers in which the difference between each pair of consecutive terms, or numbers in the list, is the same.

**Exploration 1** Describing a Pattern

Work with a partner. Use the figures to complete the table. Plot the points given by your completed table. Describe the pattern of the y-values.

a. \( n = 1 \) \( n = 2 \) \( n = 3 \) \( n = 4 \) \( n = 5 \)

<table>
<thead>
<tr>
<th>Number of stars, ( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sides, ( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. \( n = 1 \) \( n = 2 \) \( n = 3 \) \( n = 4 \) \( n = 5 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of circles, ( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. \( n = 1 \) \( n = 2 \) \( n = 3 \) \( n = 4 \) \( n = 5 \)

<table>
<thead>
<tr>
<th>Number of rows, ( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of dots, ( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Communicate Your Answer**

2. How can you use an arithmetic sequence to describe a pattern? Give an example from real life.

3. In chemistry, water is called H\(_2\)O because each molecule of water has two hydrogen atoms and one oxygen atom. Describe the pattern shown below. Use the pattern to determine the number of atoms in 23 molecules.

\( n = 1 \) \( n = 2 \) \( n = 3 \) \( n = 4 \) \( n = 5 \)
4.6 Lesson

What You Will Learn

- Write the terms of arithmetic sequences.
- Graph arithmetic sequences.
- Write arithmetic sequences as functions.

Writing the Terms of Arithmetic Sequences

An arithmetic sequence is an ordered list of numbers. Each number in a sequence is called a term. Each term \(a_n\) has a specific position \(n\) in the sequence.

\[5, 10, 15, 20, 25, \ldots, a_n, \ldots\]

Terms of an arithmetic sequence

The next three terms are \(-35, -42, -49\).

Monitoring Progress

Write the next three terms of the arithmetic sequence.

1. \(-12, 0, 12, 24, \ldots\)
2. \(0.2, 0.6, 1, 1.4, \ldots\)
3. \(4, 3\frac{3}{4}, 3\frac{1}{2}, 3\frac{1}{4}, \ldots\)
Graphing Arithmetic Sequences

To graph a sequence, let a term’s position number \( n \) in the sequence be the \( x \)-value. The term \( a_n \) is the corresponding \( y \)-value. Plot the ordered pairs \((n, a_n)\).

**Example 2** Graphing an Arithmetic Sequence

Graph the arithmetic sequence 4, 8, 12, 16, . . . . What do you notice?

**Solution**

Make a table. Then plot the ordered pairs \((n, a_n)\).

<table>
<thead>
<tr>
<th>Position, ( n )</th>
<th>Term, ( a_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

The points lie on a line.

**Example 3** Identifying an Arithmetic Sequence from a Graph

Does the graph represent an arithmetic sequence? Explain.

**Solution**

Make a table to organize the ordered pairs. Then determine whether there is a common difference.

<table>
<thead>
<tr>
<th>Position, ( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term, ( a_n )</td>
<td>15</td>
<td>12</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

Each term is 3 less than the previous term. So, the common difference is –3.

Consecutive terms have a common difference of –3. So, the graph represents the arithmetic sequence 15, 12, 9, 6, . . . .

**Monitoring Progress**

Graph the arithmetic sequence. What do you notice?

4. 3, 6, 9, 12, . . .       5. 4, 2, 0, –2, . . .       6. 1, 0.8, 0.6, 0.4, . . .
7. Does the graph shown represent an arithmetic sequence? Explain.
Writing Arithmetic Sequences as Functions

Because consecutive terms of an arithmetic sequence have a common difference, the sequence has a constant rate of change. So, the points represented by any arithmetic sequence lie on a line. You can use the first term and the common difference to write a linear function that describes an arithmetic sequence. Let \( a_1 = 4 \) and \( d = 3 \).

<table>
<thead>
<tr>
<th>Position, ( n )</th>
<th>Term, ( a_n )</th>
<th>Written using ( a_1 ) and ( d )</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>first term, ( a_1 )</td>
<td>( a_1 )</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>second term, ( a_2 )</td>
<td>( a_1 + d )</td>
<td>4 + 3 = 7</td>
</tr>
<tr>
<td>3</td>
<td>third term, ( a_3 )</td>
<td>( a_1 + 2d )</td>
<td>4 + 2(3) = 10</td>
</tr>
<tr>
<td>4</td>
<td>fourth term, ( a_4 )</td>
<td>( a_1 + 3d )</td>
<td>4 + 3(3) = 13</td>
</tr>
<tr>
<td>( n )</td>
<td>( n )th term, ( a_n )</td>
<td>( a_1 + (n - 1)d )</td>
<td>4 + (n - 1)(3)</td>
</tr>
</tbody>
</table>

**STUDY TIP**

Notice that the equation in Example 4 is of the form \( y = mx + b \), where \( y \) is replaced by \( a_n \) and \( x \) is replaced by \( n \).
You can rewrite the equation for an arithmetic sequence with first term \(a_1\) and common difference \(d\) in function notation by replacing \(a_n\) with \(f(n)\).

\[
f(n) = a_1 + (n - 1)d
\]

The domain of the function is the set of positive integers.

**EXAMPLE 5** Writing Real-Life Functions

Online bidding for a purse increases by $5 for each bid after the $60 initial bid.

<table>
<thead>
<tr>
<th>Bid number</th>
<th>Bid amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$60</td>
</tr>
<tr>
<td>2</td>
<td>$65</td>
</tr>
<tr>
<td>3</td>
<td>$70</td>
</tr>
<tr>
<td>4</td>
<td>$75</td>
</tr>
</tbody>
</table>

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. The winning bid is $105. How many bids were there?

**SOLUTION**

a. The first term is 60, and the common difference is 5.

\[
f(n) = a_1 + (n - 1)d \quad \text{Function for an arithmetic sequence}
\]

\[
f(n) = 60 + (n - 1)5 \quad \text{Substitute 60 for } a_1 \text{ and 5 for } d.
\]

\[
f(n) = 5n + 55 \quad \text{Simplify.}
\]

The function \(f(n) = 5n + 55\) represents the arithmetic sequence.

b. Make a table. Then plot the ordered pairs \((n, a_n)\).

<table>
<thead>
<tr>
<th>Bid number, (n)</th>
<th>Bid amount, (a_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>65</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
</tr>
</tbody>
</table>

c. Use the function to find the value of \(n\) for which \(f(n) = 105\).

\[
f(n) = 5n + 55
\]

\[
105 = 5n + 55 \quad \text{Write the function.}
\]

\[
10 = 5n \quad \text{Substitute 105 for } f(n).
\]

\[
n = 2 \quad \text{Solve for } n.
\]

There were 10 bids.

**Monitoring Progress**

11. A carnival charges $2 for each game after you pay a $5 entry fee.

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. How many games can you play when you take $29 to the carnival?
4.6 Exercises

Vocabulary and Core Concept Check

1. **WRITING** Describe the graph of an arithmetic sequence.

2. **DIFFERENT WORDS, SAME QUESTION** Consider the arithmetic sequence represented by the graph. Which is different? Find “both” answers.

   - Find the slope of the linear function.
   - Find the difference between consecutive terms of the arithmetic sequence.
   - Find the difference between the terms $a_2$ and $a_4$.
   - Find the common difference of the arithmetic sequence.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, write the next three terms of the arithmetic sequence.

3. First term: 2
   Common difference: 13

4. First term: 18
   Common difference: −6

In Exercises 5–10, find the common difference of the arithmetic sequence.

5. 13, 18, 23, 28, . . .
6. 175, 150, 125, 100, . . .
7. $-16, -12, -8, -4, . . .$
8. $4, \frac{7}{3}, \frac{10}{3}, 3, . . .$
9. 6.5, 5, 3.5, 2 . . .
10. $-16, -7, 2, 11, . . .$

In Exercises 11–16, write the next three terms of the arithmetic sequence. (See Example 1.)

11. 19, 22, 25, 28, . . .
12. 1, 12, 23, 34, . . .
13. 16, 21, 26, 31, . . .
14. 60, 30, 0, −30, . . .
15. 1.3, 1, 0.7, 0.4, . . .
16. $\frac{5}{9}, \frac{1}{3}, \frac{1}{2}, \frac{1}{3}, . . .$

In Exercises 17–22, graph the arithmetic sequence. (See Example 2.)

17. 4, 12, 20, 28, . . .
18. $-15, 0, 15, 30, . . .$
19. $-1, -3, -5, -7, . . .$
20. 2, 19, 36, 53, . . .
21. $0, 4\frac{1}{2}, 9, 13\frac{1}{2}, . . .$
22. 6, 5.25, 4.5, 3.75, . . .

In Exercises 23–26, determine whether the graph represents an arithmetic sequence. Explain. (See Example 3.)

23.

24.

25.

26.

In Exercises 27–30, determine whether the sequence is arithmetic. If so, find the common difference.

27. 13, 26, 39, 52, . . .
28. 5, 9, 14, 20, . . .
29. 48, 24, 12, 6, . . .
30. 87, 81, 75, 69, . . .

31. **FINDING A PATTERN** Write a sequence that represents the number of smiley faces in each group. Is the sequence arithmetic? Explain.

214 Chapter 4 Writing Linear Functions
32. FINDING A PATTERN Write a sequence that represents the sum of the numbers in each roll. Is the sequence arithmetic? Explain.

In Exercises 33–38, write an equation for the \( n \)th term of the arithmetic sequence. Then find \( a_{10} \).

(See Example 4.)

33. \(-5, -4, -3, -2, \ldots\)

34. \(-6, -9, -12, -15, \ldots\)

35. \(\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots\)

36. \(100, 110, 120, 130, \ldots\)

37. \(10, 0, -10, -20, \ldots\)

38. \(\frac{3}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}, \ldots\)

39. ERROR ANALYSIS Describe and correct the error in finding the common difference of the arithmetic sequence.

The common difference is 1.

40. ERROR ANALYSIS Describe and correct the error in writing an equation for the \( n \)th term of the arithmetic sequence.

The common difference is 1.

41. NUMBER SENSE The first term of an arithmetic sequence is 3. The common difference of the sequence is 1.5 times the first term. Write the next three terms of the sequence. Then graph the sequence.

42. NUMBER SENSE The first row of a dominoes display has 10 dominoes. Each row after the first has two more dominoes than the row before it. Write the first five terms of the sequence that represents the number of dominoes in each row. Then graph the sequence.

43. REPEATED REASONING In Exercises 43 and 44, (a) draw the next three figures in the sequence and (b) describe the 20th figure in the sequence.

45. MODELING WITH MATHEMATICS The total number of babies born in a country each minute after midnight January 1st can be estimated by the sequence shown in the table. (See Example 5.)

<table>
<thead>
<tr>
<th>Minutes after midnight January 1st</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total babies born</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

a. Write a function that represents the arithmetic sequence.
b. Graph the function.
c. Estimate how many minutes after midnight January 1st it takes for 100 babies to be born.

46. MODELING WITH MATHEMATICS The amount of money a movie earns each week after its release can be approximated by the sequence shown in the graph.

a. Write a function that represents the arithmetic sequence.
b. In what week does the movie earn $16 million?
c. How much money does the movie earn overall?
**Mathematical Connections** In Exercises 47 and 48, each small square represents 1 square inch. Determine whether the areas of the figures form an arithmetic sequence. If so, write a function $f$ that represents the arithmetic sequence and find $f(30)$.

47.

![Image of a grid]

48.

![Image of a grid]

49. **Reasoning** Is the domain of an arithmetic sequence discrete or continuous? Is the range of an arithmetic sequence discrete or continuous?

50. **Making an Argument** Your friend says that the range of a function that represents an arithmetic sequence always contains only positive numbers or only negative numbers. Your friend claims this is true because the domain is the set of positive integers and the output values either constantly increase or constantly decrease. Is your friend correct? Explain.

51. **Open-Ended** Write the first four terms of two different arithmetic sequences with a common difference of $-3$. Write an equation for the $n$th term of each sequence.

52. **Thought Provoking** Describe an arithmetic sequence that models the numbers of people in a real-life situation.

53. **Repeated Reasoning** Firewood is stacked in a pile. The bottom row has 20 logs, and the top row has 14 logs. Each row has one more log than the row above it. How many logs are in the pile?

54. **How Do You See It?** The bar graph shows the costs of advertising in a magazine.

![Bar graph showing costs of advertising]

a. Does the graph represent an arithmetic sequence? Explain.

b. Explain how you would estimate the cost of a six-page advertisement in the magazine.

55. **Reasoning** Write a function $f$ that represents the arithmetic sequence shown in the mapping diagram.

![Mapping diagram with numbers]

56. **Problem Solving** A train stops at a station every 12 minutes starting at 6:00 A.M. You arrive at the station at 7:29 A.M. How long must you wait for the train?

57. **Abstract Reasoning** Let $x$ be a constant. Determine whether each sequence is an arithmetic sequence. Explain.

a. $x + 6, 3x + 6, 5x + 6, 7x + 6, \ldots$

b. $x + 1, 3x + 1, 9x + 1, 27x + 1, \ldots$

---

**Maintaining Mathematical Proficiency**

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution. (Section 2.2)

58. $x + 8 \geq -9$

59. $15 < b - 4$

60. $t - 21 < -12$

61. $7 + y \leq 3$

Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range. (Section 3.7)

62. $h(x) = 3|x|$

63. $v(x) = |x - 5|$

64. $g(x) = |x| + 1$

65. $r(x) = -2|x|$