

4.1

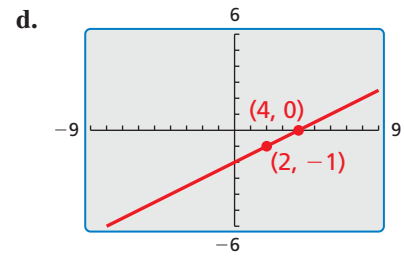
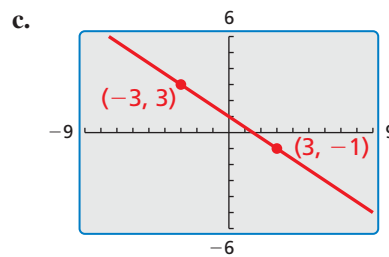
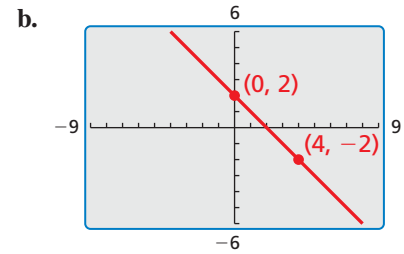
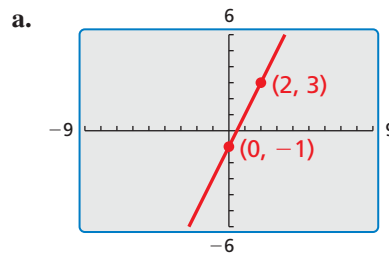
Writing Equations in Slope-Intercept Form

Essential Question Given the graph of a linear function, how can you write an equation of the line?

EXPLORATION 1 Writing Equations in Slope-Intercept Form

Work with a partner.

- Find the slope and y-intercept of each line.
- Write an equation of each line in slope-intercept form.
- Use a graphing calculator to verify your equation.



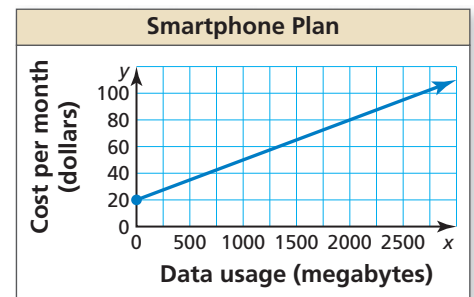
INTERPRETING MATHEMATICAL RESULTS

To be proficient in math, you need to routinely interpret your results in the context of the situation. The reason for studying mathematics is to enable you to model and solve real-life problems.

EXPLORATION 2 Mathematical Modeling

Work with a partner. The graph shows the cost of a smartphone plan.

- What is the y-intercept of the line? Interpret the y-intercept in the context of the problem.
- Approximate the slope of the line. Interpret the slope in the context of the problem.
- Write an equation that represents the cost as a function of data usage.



Communicate Your Answer

- Given the graph of a linear function, how can you write an equation of the line?
- Give an example of a graph of a linear function that is different from those above. Then use the graph to write an equation of the line.

4.1 Lesson

Core Vocabulary

linear model, p. 178

Previous

slope-intercept form

function

rate

What You Will Learn

- ▶ Write equations in slope-intercept form.
- ▶ Use linear equations to solve real-life problems.

Writing Equations in Slope-Intercept Form

EXAMPLE 1 Using Slopes and y-Intercepts to Write Equations

Write an equation of each line with the given slope and y-intercept.

- a. slope = -3 ; y-intercept = $\frac{1}{2}$ b. slope = 0 ; y-intercept = -2

SOLUTION

- a. $y = mx + b$ Write the slope-intercept form.
 $y = -3x + \frac{1}{2}$ Substitute -3 for m and $\frac{1}{2}$ for b .

▶ An equation is $y = -3x + \frac{1}{2}$.

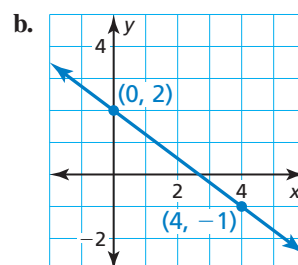
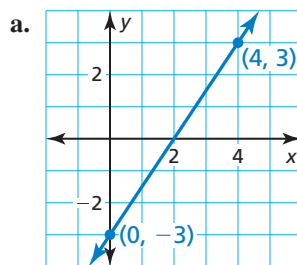
- b. $y = mx + b$ Write the slope-intercept form.
 $y = 0x + (-2)$ Substitute 0 for m and -2 for b .

$y = -2$ Simplify.

▶ An equation is $y = -2$.

EXAMPLE 2 Using Graphs to Write Equations

Write an equation of each line in slope-intercept form.



SOLUTION

- a. Find the slope and y-intercept.

Let $(x_1, y_1) = (0, -3)$ and $(x_2, y_2) = (4, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-3)}{4 - 0} = \frac{6}{4}, \text{ or } \frac{3}{2}$$

Because the line crosses the y-axis at $(0, -3)$, the y-intercept is -3 .

▶ So, the equation is $y = \frac{3}{2}x - 3$.

- b. Find the slope and y-intercept.

Let $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (4, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{4 - 0} = \frac{-3}{4}, \text{ or } -\frac{3}{4}$$

Because the line crosses the y-axis at $(0, 2)$, the y-intercept is 2 .

▶ So, the equation is $y = -\frac{3}{4}x + 2$.

STUDY TIP

You can use any two points on a line to find the slope.

STUDY TIP

After writing an equation, check that the given points are solutions of the equation.

EXAMPLE 3 Using Points to Write Equations

Write an equation of each line that passes through the given points.

a. $(-3, 5), (0, -1)$

b. $(0, -5), (8, -5)$

SOLUTION

a. Find the slope and y-intercept.

$$m = \frac{-1 - 5}{0 - (-3)} = -2$$

Because the line crosses the y-axis at $(0, -1)$, the y-intercept is -1 .

► So, an equation is $y = -2x - 1$.

b. Find the slope and y-intercept.

$$m = \frac{-5 - (-5)}{8 - 0} = 0$$

Because the line crosses the y-axis at $(0, -5)$, the y-intercept is -5 .

► So, an equation is $y = -5$.

REMEMBER

If f is a function and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x .

EXAMPLE 4 Writing a Linear Function

Write a linear function f with the values $f(0) = 10$ and $f(6) = 34$.

SOLUTION

Step 1 Write $f(0) = 10$ as $(0, 10)$ and $f(6) = 34$ as $(6, 34)$.

Step 2 Find the slope of the line that passes through $(0, 10)$ and $(6, 34)$.

$$m = \frac{34 - 10}{6 - 0} = \frac{24}{6}, \text{ or } 4$$

Step 3 Write an equation of the line. Because the line crosses the y-axis at $(0, 10)$, the y-intercept is 10.

$$y = mx + b \quad \text{Write the slope-intercept form.}$$

$$y = 4x + 10 \quad \text{Substitute 4 for } m \text{ and 10 for } b.$$

► A function is $f(x) = 4x + 10$.

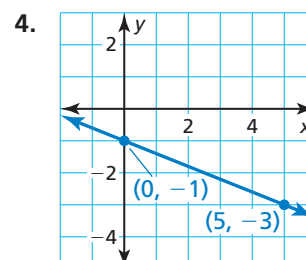
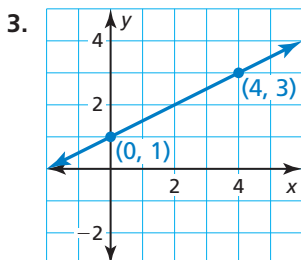
Monitoring Progress Help in English and Spanish at BigIdeasMath.com

Write an equation of the line with the given slope and y-intercept.

1. slope = 7; y-intercept = 2

2. slope = $\frac{1}{3}$; y-intercept = -1

Write an equation of the line in slope-intercept form.



5. Write an equation of the line that passes through $(0, -2)$ and $(4, 10)$.

6. Write a linear function g with the values $g(0) = 9$ and $g(8) = 7$.

Solving Real-Life Problems

A **linear model** is a linear function that models a real-life situation. When a quantity y changes at a constant rate with respect to a quantity x , you can use the equation $y = mx + b$ to model the relationship. The value of m is the constant rate of change, and the value of b is the initial, or starting, value of y .

EXAMPLE 5 Modeling with Mathematics



Excluding hydropower, U.S. power plants used renewable energy sources to generate 105 million megawatt hours of electricity in 2007. By 2012, the amount of electricity generated had increased to 219 million megawatt hours. Write a linear model that represents the number of megawatt hours generated by non-hydropower renewable energy sources as a function of the number of years since 2007. Use the model to predict the number of megawatt hours that will be generated in 2017.

SOLUTION

- Understand the Problem** You know the amounts of electricity generated in two distinct years. You are asked to write a linear model that represents the amount of electricity generated each year since 2007 and then predict a future amount.
- Make a Plan** Break the problem into parts and solve each part. Then combine the results to help you solve the original problem.

Part 1 Define the variables. Find the initial value and the rate of change.

Part 2 Write a linear model and predict the amount in 2017.

3. Solve the Problem

Part 1 Let x represent the time (in years) since 2007 and let y represent the number of megawatt hours (in millions). Because time x is defined in years since 2007, 2007 corresponds to $x = 0$ and 2012 corresponds to $x = 5$. Let $(x_1, y_1) = (0, 105)$ and $(x_2, y_2) = (5, 219)$. The initial value is the y -intercept b , which is 105. The rate of change is the slope m .

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{219 - 105}{5 - 0} = \frac{114}{5} = 22.8$$

Part 2

Megawatt hours (millions)	=	Initial value	+	Rate of change	•	Years since 2007
y		= 105		+ 22.8		• x

$$y = 105 + 22.8x \quad \text{Write the equation.}$$

2017 corresponds to $x = 10$. $\rightarrow y = 105 + 22.8(10)$ Substitute 10 for x .

$$y = 333 \quad \text{Simplify.}$$

► The linear model is $y = 22.8x + 105$. The model predicts non-hydropower renewable energy sources will generate 333 million megawatt hours in 2017.

- Look Back** To check that your model is correct, verify that $(0, 105)$ and $(5, 219)$ are solutions of the equation.

Monitoring Progress Help in English and Spanish at BigIdeasMath.com

- The corresponding data for electricity generated by hydropower are 248 million megawatt hours in 2007 and 277 million megawatt hours in 2012. Write a linear model that represents the number of megawatt hours generated by hydropower as a function of the number of years since 2007.

4.1 Exercises

Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** A linear function that models a real-life situation is called a _____.
- WRITING** Explain how you can use slope-intercept form to write an equation of a line given its slope and y-intercept.

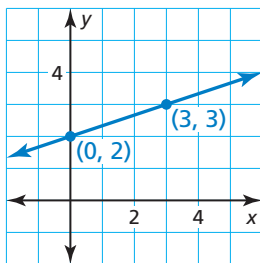
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write an equation of the line with the given slope and y-intercept. (See Example 1.)

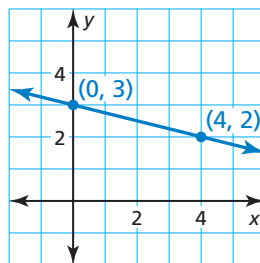
- | | |
|--|---|
| 3. slope: 2
y-intercept: 9 | 4. slope: 0
y-intercept: 5 |
| 5. slope: -3
y-intercept: 0 | 6. slope: -7
y-intercept: 1 |
| 7. slope: $\frac{2}{3}$
y-intercept: -8 | 8. slope: $-\frac{3}{4}$
y-intercept: -6 |

In Exercises 9–12, write an equation of the line in slope-intercept form. (See Example 2.)

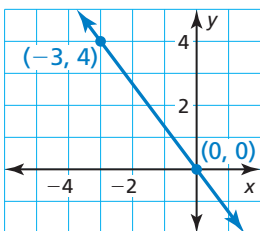
9.



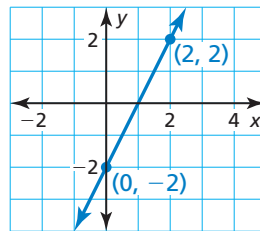
10.



11.



12.



In Exercises 13–18, write an equation of the line that passes through the given points. (See Example 3.)

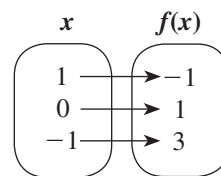
- | | |
|-----------------------|-----------------------|
| 13. (3, 1), (0, 10) | 14. (2, 7), (0, -5) |
| 15. (2, -4), (0, -4) | 16. (-6, 0), (0, -24) |
| 17. (0, 5), (-1.5, 1) | 18. (0, 3), (-5, 2.5) |

In Exercises 19–24, write a linear function f with the given values. (See Example 4.)

- | | |
|----------------------------|--------------------------|
| 19. $f(0) = 2, f(2) = 4$ | 20. $f(0) = 7, f(3) = 1$ |
| 21. $f(4) = -3, f(0) = -2$ | |
| 22. $f(5) = -1, f(0) = -5$ | |
| 23. $f(-2) = 6, f(0) = -4$ | |
| 24. $f(0) = 3, f(-6) = 3$ | |

In Exercises 25 and 26, write a linear function f with the given values.

25.



26.

x	$f(x)$
-4	-2
-2	-1
0	0

27. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line with a slope of 2 and a y-intercept of 7.

X $y = 7x + 2$

28. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line shown.

X slope = $\frac{1-4}{0-5} = \frac{-3}{-5} = \frac{3}{5}$
 $y = \frac{3}{5}x + 4$

A coordinate plane showing a line passing through the points (0, 4) and (5, 1). The x-axis is labeled from 0 to 6, and the y-axis is labeled from 0 to 4.

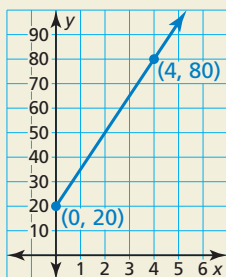
29. **MODELING WITH MATHEMATICS** In 1960, the world record for the men's mile was 3.91 minutes. In 1980, the record time was 3.81 minutes. (See Example 5.)
- Write a linear model that represents the world record (in minutes) for the men's mile as a function of the number of years since 1960.
 - Use the model to estimate the record time in 2000 and predict the record time in 2020.

30. **MODELING WITH MATHEMATICS** A recording studio charges musicians an initial fee of \$50 to record an album. Studio time costs an additional \$75 per hour.
- Write a linear model that represents the total cost of recording an album as a function of studio time (in hours).
 - Is it less expensive to purchase 12 hours of recording time at the studio or a \$750 music software program that you can use to record on your own computer? Explain.



31. **WRITING** A line passes through the points $(0, -2)$ and $(0, 5)$. Is it possible to write an equation of the line in slope-intercept form? Justify your answer.

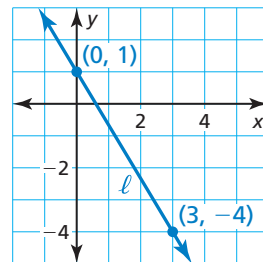
32. **THOUGHT PROVOKING** Describe a real-life situation involving a linear function whose graph passes through the points.



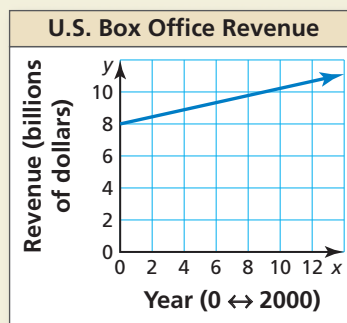
33. **REASONING** Recall that the standard form of a linear equation is $Ax + By = C$. Rewrite this equation in slope-intercept form. Use your answer to find the slope and y-intercept of the graph of the equation $-6x + 5y = 9$.

34. **MAKING AN ARGUMENT** Your friend claims that given $f(0)$ and any other value of a linear function f , you can write an equation in slope-intercept form that represents the function. Your cousin disagrees, claiming that the two points could lie on a vertical line. Who is correct? Explain.

35. **ANALYZING A GRAPH** Line ℓ is a reflection in the x -axis of line k . Write an equation that represents line k .



36. **HOW DO YOU SEE IT?** The graph shows the approximate U.S. box office revenues (in billions of dollars) from 2000 to 2012, where $x = 0$ represents the year 2000.



- Estimate the slope and y-intercept of the graph.
- Interpret your answers in part (a) in the context of the problem.
- How can you use your answers in part (a) to predict the U.S. box office revenue in 2018?

37. **ABSTRACT REASONING** Show that the equation of the line that passes through the points $(0, b)$ and $(1, b + m)$ is $y = mx + b$. Explain how you can be sure that the point $(-1, b - m)$ also lies on the line.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. (Section 1.3)

38. $3(x - 15) = x + 11$

39. $-4y - 10 = 4(y - 3)$

40. $2(3d + 3) = 7 + 6d$

41. $-5(4 - 3n) = 10(n - 2)$

Use intercepts to graph the linear equation. (Section 3.4)

42. $-4x + 2y = 16$

43. $3x + 5y = -15$

44. $x - 6y = 24$

45. $-7x - 2y = -21$