3.3 Function Notation

Essential Question How can you use function notation to represent a function?

The notation $f(x)$, called **function notation**, is another name for y. This notation is read as "the value of f at x" or "f of x." The parentheses do not imply multiplication. You can use letters other than f to name a function. The letters g, h, j, and k are often used to name functions.

EXPLORATION 1 Matching Functions with Their Graphs

Work with a partner. Match each function with its graph.

a. $f(x) = 2x - 3$

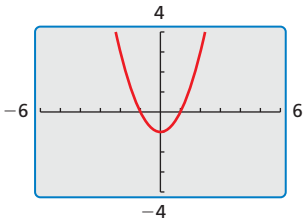
b. $g(x) = -x + 2$

c. $h(x) = x^2 - 1$

d. $j(x) = 2x^2 - 3$

ATTENDING TO PRECISION
To be proficient in math, you need to use clear definitions and state the meanings of the symbols you use.

A.



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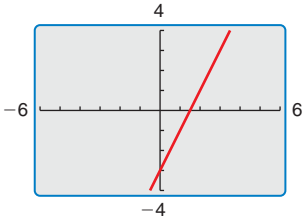

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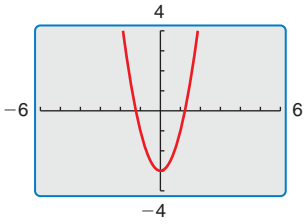
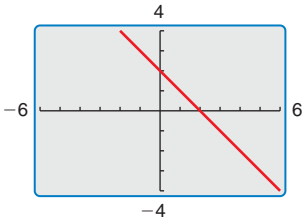


D.

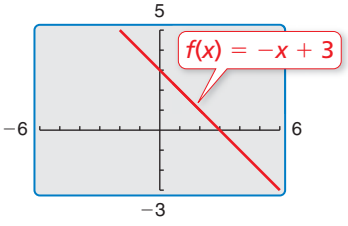


EXPLORATION 2 Evaluating a Function

Work with a partner. Consider the function

$$f(x) = -x + 3.$$

Locate the points $(x, f(x))$ on the graph. Explain how you found each point.



a. $(-1, f(-1))$

b. $(0, f(0))$

c. $(1, f(1))$

d. $(2, f(2))$

Communicate Your Answer

3. How can you use function notation to represent a function? How are standard notation and function notation similar? How are they different?

Standard Notation	*Function Notation*
$y = 2x + 5$	$f(x) = 2x + 5$

3.3 Lesson

Core Vocabulary

function notation, *p. 122*

Previous
linear function
quadrant

What You Will Learn

- Use function notation to evaluate and interpret functions.
- Use function notation to solve and graph functions.
- Solve real-life problems using function notation.

Using Function Notation to Evaluate and Interpret

You know that a linear function can be written in the form $y = mx + b$. By naming a linear function f, you can also write the function using **function notation**.

$f(x) = mx + b$ Function notation

The notation $f(x)$ is another name for y. If f is a function, and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x. You can use letters other than f to name a function, such as g or h.

READING

The notation $f(x)$ is read as "the value of f at x" or "f of x." It does not mean "f times x."

EXAMPLE 1 Evaluating a Function

Evaluate $f(x) = -4x + 7$ when $x = 2$ and $x = -2$.

SOLUTION

$f(x) = -4x + 7$	Write the function.	$f(x) = -4x + 7$
$f(2) = -4(2) + 7$	Substitute for x.	$f(-2) = -4(-2) + 7$
$= -8 + 7$	Multiply.	$= 8 + 7$
$= -1$	Add.	$= 15$

▶ When $x = 2$, $f(x) = -1$, and when $x = -2$, $f(x) = 15$.

EXAMPLE 2 Interpreting Function Notation

Let $f(t)$ be the outside temperature (°F) t hours after 6 A.M. Explain the meaning of each statement.

a. $f(0) = 58$ **b.** $f(6) = n$ **c.** $f(3) < f(9)$

SOLUTION

a. The initial value of the function is 58. So, the temperature at 6 A.M. is 58°F.

b. The output of f when $t = 6$ is n. So, the temperature at noon (6 hours after 6 A.M.) is n°F.

c. The output of f when $t = 3$ is less than the output of f when $t = 9$. So, the temperature at 9 A.M. (3 hours after 6 A.M.) is less than the temperature at 3 P.M. (9 hours after 6 A.M.).

Monitoring Progress

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Evaluate the function when $x = -4$, 0, and 3.

1. $f(x) = 2x - 5$ **2.** $g(x) = -x - 1$

3. WHAT IF? In Example 2, let $f(t)$ be the outside temperature (°F) t hours after 9 A.M. Explain the meaning of each statement.

a. $f(4) = 75$ **b.** $f(m) = 70$ **c.** $f(2) = f(9)$ d. $f(6) > f(0)$

Using Function Notation to Solve and Graph

EXAMPLE 3 Solving for the Independent Variable

For $h(x) = \frac{2}{3}x - 5$, find the value of x for which $h(x) = -7$.

SOLUTION

$h(x) = \frac{2}{3}x - 5$ Write the function.

$-7 = \frac{2}{3}x - 5$ Substitute –7 for $h(x)$.

$+5 \quad +5$ Add 5 to each side.

$-2 = \frac{2}{3}x$ Simplify.

$\frac{3}{2} \cdot (-2) = \frac{3}{2} \cdot \frac{2}{3}x$ Multiply each side by $\frac{3}{2}$.

$-3 = x$ Simplify.

▶ When $x = -3$, $h(x) = -7$.

EXAMPLE 4 Graphing a Linear Function in Function Notation

Graph $f(x) = 2x + 5$.

SOLUTION

Step 1 Make an input-output table to find ordered pairs.

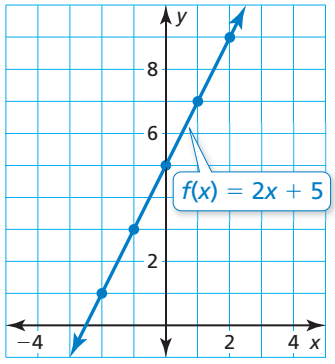
x	-2	-1	0	1	2
$f(x)$	1	3	5	7	9

Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points.

STUDY TIP

The graph of $y = f(x)$ consists of the points $(x, f(x))$.



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Find the value of x so that the function has the given value.

4. $f(x) = 6x + 9$; $f(x) = 21$

5. $g(x) = -\frac{1}{2}x + 3$; $g(x) = -1$

Graph the linear function.

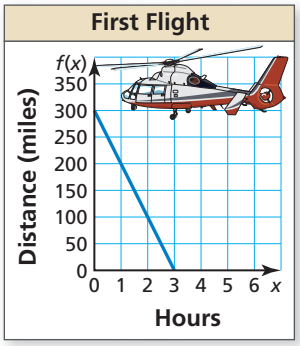
6. $f(x) = 3x - 2$

7. $g(x) = -x + 4$

8. $h(x) = -\frac{3}{4}x - 1$

Solving Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**



The graph shows the number of miles a helicopter is from its destination after x hours on its first flight. On its second flight, the helicopter travels 50 miles farther and increases its speed by 25 miles per hour. The function $f(x) = 350 - 125x$ represents the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

SOLUTION

1. **Understand the Problem** You are given a graph of the first flight and an equation of the second flight. You are asked to compare the flight times to determine which flight takes less time.

2. **Make a Plan** Graph the function that represents the second flight. Compare the graph to the graph of the first flight. The x-value that corresponds to $f(x) = 0$ represents the flight time.

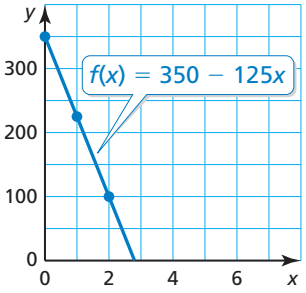
3. **Solve the Problem** Graph $f(x) = 350 - 125x$.

   Step 1 Make an input-output table to find the ordered pairs.

x	0	1	2	3
$f(x)$	350	225	100	−25

   Step 2 Plot the ordered pairs.

   Step 3 Draw a line through the points. Note that the function only makes sense when x and $f(x)$ are positive. So, only draw the line in the first quadrant.



   ▶ From the graph of the first flight, you can see that when $f(x) = 0$, $x = 3$. From the graph of the second flight, you can see that when $f(x) = 0$, x is slightly less than 3. So, the second flight takes less time.

4. **Look Back** You can check that your answer is correct by finding the value of x for which $f(x) = 0$.

$$f(x) = 350 - 125x \quad \text{Write the function.}$$
$$0 = 350 - 125x \quad \text{Substitute 0 for } f(x).$$
$$-350 = -125x \quad \text{Subtract 350 from each side.}$$
$$2.8 = x \quad \text{Divide each side by } -125.$$

   So, the second flight takes 2.8 hours, which is less than 3.

Monitoring Progress

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9. **WHAT IF?** Let $f(x) = 250 - 75x$ represent the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

3.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** When you write the function $y = 2x + 10$ as $f(x) = 2x + 10$, you are using _____.

2. **REASONING** Your height can be represented by a function h, where the input is your age. What does $h(14)$ represent?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, evaluate the function when $x = -2$, 0, and 5. *(See Example 1.)*

3. $f(x) = x + 6$
4. $g(x) = 3x$
5. $h(x) = -2x + 9$
6. $r(x) = -x - 7$
7. $p(x) = -3 + 4x$
8. $b(x) = 18 - 0.5x$
9. $v(x) = 12 - 2x - 5$
10. $n(x) = -1 - x + 4$

11. **INTERPRETING FUNCTION NOTATION** Let $c(t)$ be the number of customers in a restaurant t hours after 8 A.M. Explain the meaning of each statement. *(See Example 2.)*

    a. $c(0) = 0$
    b. $c(3) = c(8)$
    c. $c(n) = 29$
    d. $c(13) < c(12)$

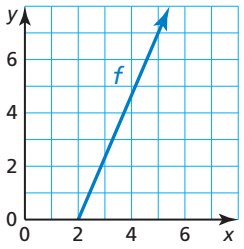
12. **INTERPRETING FUNCTION NOTATION** Let $H(x)$ be the percent of U.S. households with Internet use x years after 1980. Explain the meaning of each statement.

    a. $H(23) = 55$
    b. $H(4) = k$
    c. $H(27) \geq 61$
    d. $H(17) + H(21) \approx H(29)$

In Exercises 13–18, find the value of x so that the function has the given value. *(See Example 3.)*

13. $h(x) = -7x$; $h(x) = 63$
14. $t(x) = 3x$; $t(x) = 24$
15. $m(x) = 4x + 15$; $m(x) = 7$
16. $k(x) = 6x - 12$; $k(x) = 18$
17. $q(x) = \frac{1}{2}x - 3$; $q(x) = -4$
18. $j(x) = -\frac{4}{5}x + 7$; $j(x) = -5$

In Exercises 19 and 20, find the value of x so that $f(x) = 7$.

19.


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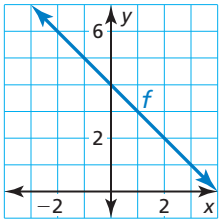


21. **MODELING WITH MATHEMATICS** The function $C(x) = 17.5x - 10$ represents the cost (in dollars) of buying x tickets to the orchestra with a \$10 coupon.

    a. How much does it cost to buy five tickets?
    b. How many tickets can you buy with \$130?

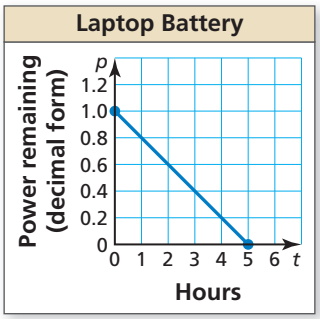
22. **MODELING WITH MATHEMATICS** The function $d(t) = 300{,}000t$ represents the distance (in kilometers) that light travels in t seconds.

    a. How far does light travel in 15 seconds?
    b. How long does it take light to travel 12 million kilometers?

In Exercises 23–28, graph the linear function. *(See Example 4.)*

23. $p(x) = 4x$
24. $h(x) = -5$
25. $d(x) = -\frac{1}{2}x - 3$
26. $w(x) = \frac{3}{5}x + 2$
27. $g(x) = -4 + 7x$
28. $f(x) = 3 - 6x$

29. PROBLEM SOLVING The graph shows the percent p (in decimal form) of battery power remaining in a laptop computer after t hours of use. A tablet computer initially has 75% of its battery power remaining and loses 12.5% per hour. Which computer's battery will last longer? Explain. *(See Example 5.)*



30. PROBLEM SOLVING The function $C(x) = 25x + 50$ represents the labor cost (in dollars) for Certified Remodeling to build a deck, where x is the number of hours of labor. The table shows sample labor costs from its main competitor, Master Remodeling. The deck is estimated to take 8 hours of labor. Which company would you hire? Explain.

Hours	Cost
2	$130
4	$160
6	$190

31. MAKING AN ARGUMENT Let $P(x)$ be the number of people in the U.S. who own a cell phone x years after 1990. Your friend says that $P(x + 1) > P(x)$ for any x because $x + 1$ is always greater than x. Is your friend correct? Explain.

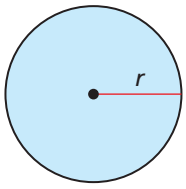
32. THOUGHT PROVOKING Let $B(t)$ be your bank account balance after t days. Describe a situation in which $B(0) < B(4) < B(2)$.

33. MATHEMATICAL CONNECTIONS Rewrite each geometry formula using function notation. Evaluate each function when $r = 5$ feet. Then explain the meaning of the result.

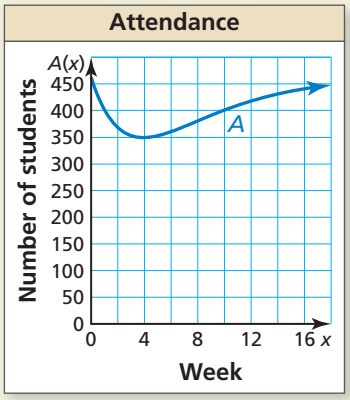
a. Diameter, $d = 2r$

b. Area, $A = \pi r^2$

c. Circumference, $C = 2\pi r$



34. HOW DO YOU SEE IT? The function $y = A(x)$ represents the attendance at a high school x weeks after a flu outbreak. The graph of the function is shown.



a. What happens to the school's attendance after the flu outbreak?

b. Estimate $A(13)$ and explain its meaning.

c. Use the graph to estimate the solution(s) of the equation $A(x) = 400$. Explain the meaning of the solution(s).

d. What was the least attendance? When did that occur?

e. How many students do you think are enrolled at this high school? Explain your reasoning.

35. INTERPRETING FUNCTION NOTATION Let f be a function. Use each statement to find the coordinates of a point on the graph of f.

a. $f(5)$ is equal to 9.

b. A solution of the equation $f(n) = -3$ is 5.

36. REASONING Given a function f, tell whether the statement

$$f(a + b) = f(a) + f(b)$$

is true or false for all inputs a and b. If it is false, explain why.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution. *(Section 2.5)*

37. $-2 \leq x - 11 \leq 6$

38. $5a < -35$ *or* $a - 14 > 1$

39. $-16 < 6k + 2 < 0$

40. $2d + 7 < -9$ *or* $4d - 1 > -3$

41. $5 \leq 3y + 8 < 17$

42. $4v + 9 \leq 5$ *or* $-3v \geq -6$