

2.2 Solving Inequalities Using Addition or Subtraction

Essential Question How can you use addition or subtraction to solve an inequality?

EXPLORATION 1 Quarterback Passing Efficiency

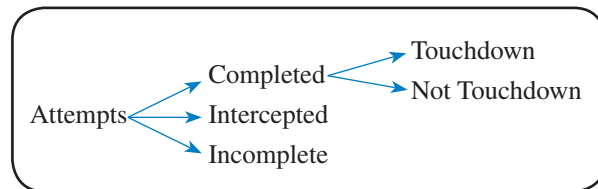
Work with a partner. The National Collegiate Athletic Association (NCAA) uses the following formula to rank the passing efficiencies P of quarterbacks.

$$P = \frac{8.4Y + 100C + 330T - 200N}{A}$$

Y = total length of all completed passes (in **Y**ards) C = **C**ompleted passes
 T = passes resulting in a **T**ouchdown N = **iN**tercepted passes
 A = **A**ttempted passes M = **incoM**plete passes

MODELING WITH MATHEMATICS

To be proficient in math, you need to identify and analyze important relationships and then draw conclusions, using tools such as diagrams, flowcharts, and formulas.



Determine whether each inequality must be true. Explain your reasoning.

- a. $T < C$ b. $C + N \leq A$ c. $N < A$ d. $A - C \geq M$

EXPLORATION 2 Finding Solutions of Inequalities

Work with a partner. Use the passing efficiency formula to create a passing record that makes each inequality true. Record your results in the table. Then describe the values of P that make each inequality true.

Attempts	Completions	Yards	Touchdowns	Interceptions

- a. $P < 0$
 b. $P + 100 \geq 250$
 c. $P - 250 > -80$

Communicate Your Answer

3. How can you use addition or subtraction to solve an inequality?
4. Solve each inequality.
- a. $x + 3 < 4$ b. $x - 3 \geq 5$
 c. $4 > x - 2$ d. $-2 \leq x + 1$

2.2 Lesson

Core Vocabulary

equivalent inequalities, p. 62

Previous
inequality

What You Will Learn

- ▶ Solve inequalities using addition.
- ▶ Solve inequalities using subtraction.
- ▶ Use inequalities to solve real-life problems.

Solving Inequalities Using Addition

Just as you used the properties of equality to produce equivalent equations, you can use the properties of inequality to produce equivalent inequalities. **Equivalent inequalities** are inequalities that have the same solutions.

Core Concept

Addition Property of Inequality

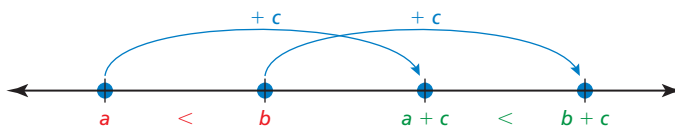
Words Adding the same number to each side of an inequality produces an equivalent inequality.

Numbers

$$\begin{array}{r} -3 < 2 & -3 \geq -10 \\ +4 & +4 & +3 & +3 \\ \hline 1 < 6 & 0 \geq -7 \end{array}$$

Algebra If $a > b$, then $a + c > b + c$. If $a \geq b$, then $a + c \geq b + c$.
If $a < b$, then $a + c < b + c$. If $a \leq b$, then $a + c \leq b + c$.

The diagram shows one way to visualize the Addition Property of Inequality when $c > 0$.



EXAMPLE 1 Solving an Inequality Using Addition

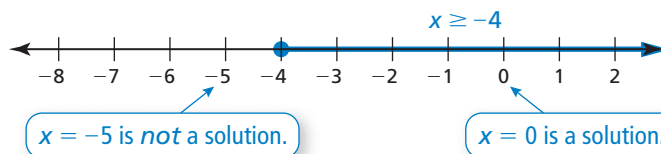
Solve $x - 6 \geq -10$. Graph the solution.

SOLUTION

$$\begin{array}{r} x - 6 \geq -10 & \text{Write the inequality.} \\ +6 & +6 & \text{Add 6 to each side.} \\ \hline x \geq -4 & \text{Simplify.} \end{array}$$

Addition Property of Inequality

- ▶ The solution is $x \geq -4$.



REMEMBER

To check this solution, substitute a few numbers to the left and right of -4 into the original inequality.

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Solve the inequality. Graph the solution.

1. $b - 2 > -9$

2. $m - 3 \leq 5$

3. $\frac{1}{4} > y - \frac{1}{4}$

Solving Inequalities Using Subtraction

Core Concept

Subtraction Property of Inequality

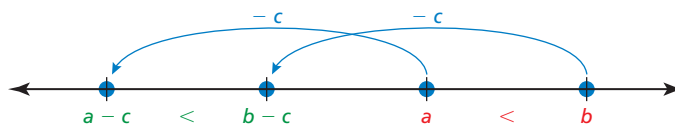
Words Subtracting the same number from each side of an inequality produces an equivalent inequality.

Numbers $-3 \leq 1$ $7 > -20$

$$\begin{array}{r} -5 \\ -8 \leq -4 \end{array} \quad \begin{array}{r} -5 \\ 0 > -27 \end{array} \quad \begin{array}{r} -7 \\ 0 > -27 \end{array}$$

Algebra If $a > b$, then $a - c > b - c$. If $a \geq b$, then $a - c \geq b - c$.
If $a < b$, then $a - c < b - c$. If $a \leq b$, then $a - c \leq b - c$.

The diagram shows one way to visualize the Subtraction Property of Inequality when $c > 0$.



EXAMPLE 2 Solving an Inequality Using Subtraction

Solve each inequality. Graph the solution.

a. $y + 8 \leq 5$

b. $-8 < 1.4 + m$

SOLUTION

a. $y + 8 \leq 5$

Write the inequality.

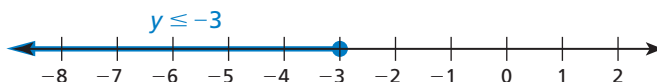
Subtraction Property of Inequality

$$\begin{array}{r} -8 \\ y \leq -3 \end{array}$$

Subtract 8 from each side.

Simplify.

▶ The solution is $y \leq -3$.



b. $-8 < 1.4 + m$

Write the inequality.

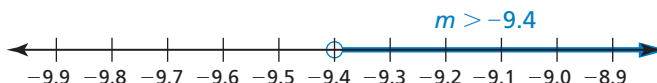
Subtraction Property of Inequality

$$\begin{array}{r} -1.4 \\ -9.4 < m \end{array}$$

Subtract 1.4 from each side.

Simplify.

▶ The solution is $m > -9.4$.



Monitoring Progress



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Solve the inequality. Graph the solution.

4. $k + 5 \leq -3$

5. $\frac{5}{6} \leq z + \frac{1}{6}$

6. $p + 0.7 > -2.3$

Solving Real-Life Problems

EXAMPLE 3 Modeling with Mathematics



A circuit overloads at 1800 watts of electricity. You plug a microwave oven that uses 1100 watts of electricity into the circuit.

- Write and solve an inequality that represents how many watts you can add to the circuit without overloading the circuit.
- In addition to the microwave oven, which of the following appliances can you plug into the circuit at the same time without overloading the circuit?

Appliance	Watts
Clock radio	50
Blender	300
Hot plate	1200
Toaster	800

SOLUTION

- Understand the Problem** You know that the microwave oven uses 1100 watts out of a possible 1800 watts. You are asked to write and solve an inequality that represents how many watts you can add without overloading the circuit. You also know the numbers of watts used by four other appliances. You are asked to identify the appliances you can plug in at the same time without overloading the circuit.
- Make a Plan** Use a verbal model to write an inequality. Then solve the inequality and identify other appliances that you can plug into the circuit at the same time without overloading the circuit.
- Solve the Problem**

Words Watts used by microwave oven + Additional watts < Overload wattage

Variable Let w be the additional watts you can add to the circuit.

Inequality 1100 + w < 1800

$$1100 + w < 1800 \quad \text{Write the inequality.}$$

Subtraction Property of Inequality

$$\xrightarrow{\quad} \quad \underline{-1100} \quad \underline{-1100} \quad \text{Subtract 1100 from each side.}$$

$$w < 700 \quad \text{Simplify.}$$

- ▶ You can add up to 700 watts to the circuit, which means that you can also plug in the clock radio and the blender.

- Look Back** You can check that your answer is correct by adding the numbers of watts used by the microwave oven, clock radio, and blender.

$$1100 + 50 + 300 = 1450$$

The circuit will not overload because the total wattage is less than 1800 watts.

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- The microwave oven uses only 1000 watts of electricity. Does this allow you to have both the microwave oven and the toaster plugged into the circuit at the same time? Explain your reasoning.

2.2 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- VOCABULARY** Why is the inequality $x \leq 6$ equivalent to the inequality $x - 5 \leq 6 - 5$?
- WRITING** Compare solving equations using addition with solving inequalities using addition.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, tell which number you would add to or subtract from each side of the inequality to solve it.

- $k + 11 < -3$
- $v - 2 > 14$
- $-1 \geq b - 9$
- $-6 \leq 17 + p$

In Exercises 7–20, solve the inequality. Graph the solution. (See Examples 1 and 2.)

- $x - 4 < -5$
- $1 \leq s - 8$
- $6 \geq m - 1$
- $c - 12 > -4$
- $r + 4 < 5$
- $-8 \leq 8 + y$
- $9 + w > 7$
- $15 \geq q + 3$
- $h - (-2) \geq 10$
- $-6 > t - (-13)$
- $j + 9 - 3 < 8$
- $1 - 12 + y \geq -5$
- $10 \geq 3p - 2p - 7$
- $18 - 5z + 6z > 3 + 6$

In Exercises 21–24, write the sentence as an inequality. Then solve the inequality.

- A number plus 8 is greater than 11.
- A number minus 3 is at least -5 .
- The difference of a number and 9 is fewer than 4.
- Six is less than or equal to the sum of a number and 15.
- MODELING WITH MATHEMATICS** You are riding a train. Your carry-on bag can weigh no more than 50 pounds. Your bag weighs 38 pounds. (See Example 3.)

- Write and solve an inequality that represents how much weight you can add to your bag.

- Can you add both a 9-pound laptop and a 5-pound pair of boots to your bag without going over the weight limit? Explain.

- MODELING WITH MATHEMATICS** You order the hardcover book shown from a website that offers free shipping on orders of \$25 or more. Write and solve an inequality that represents how much more you must spend to get free shipping.



- ERROR ANALYSIS** In Exercises 27 and 28, describe and correct the error in solving the inequality or graphing the solution.

27.
$$\begin{aligned} -17 < x - 14 \\ -17 + 14 < x - 14 + 14 \\ -3 < x \end{aligned}$$

28.
$$\begin{aligned} -10 + x &\geq -9 \\ -10 + 10 + x &\geq -9 \\ x &\geq -9 \end{aligned}$$

- PROBLEM SOLVING** An NHL hockey player has 59 goals so far in a season. What are the possible numbers of additional goals the player can score to match or break the NHL record of 92 goals in a season?

30. **MAKING AN ARGUMENT** In an aerial ski competition, you perform two acrobatic ski jumps. The scores on the two jumps are then added together.

Ski jump	Competitor's score	Your score
1	117.1	119.5
2	119.8	

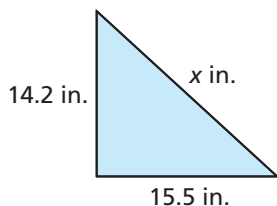
- a. Describe the score that you must earn on your second jump to beat your competitor.
- b. Your coach says that you will beat your competitor if you score 118.4 points. A teammate says that you only need 117.5 points. Who is correct? Explain.

31. **REASONING** Which of the following inequalities are equivalent to the inequality $x - b < 3$, where b is a constant? Justify your answer.

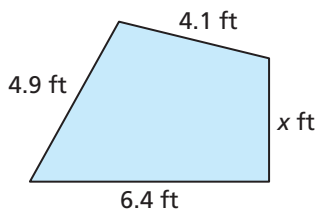
- (A) $x - b - 3 < 0$ (B) $0 > b - x + 3$
 (C) $x < 3 - b$ (D) $-3 < b - x$

MATHEMATICAL CONNECTIONS In Exercises 32 and 33, write and solve an inequality to find the possible values of x .

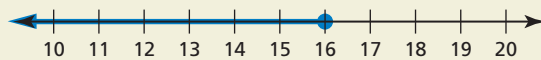
32. Perimeter < 51.3 inches



33. Perimeter ≤ 18.7 feet

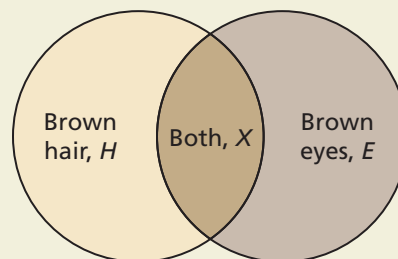


34. **THOUGHT PROVOKING** Write an inequality that has the solution shown in the graph. Describe a real-life situation that can be modeled by the inequality.



35. **WRITING** Is it possible to check all the numbers in the solution set of an inequality? When you solve the inequality $x - 11 \geq -3$, which numbers can you check to verify your solution? Explain your reasoning.

36. **HOW DO YOU SEE IT?** The diagram represents the numbers of students in a school with brown eyes, brown hair, or both.



Determine whether each inequality must be true. Explain your reasoning.

- a. $H \geq E$ b. $H + 10 \geq E$
 c. $H \geq X$ d. $H + 10 \geq X$
 e. $H > X$ f. $H + 10 > X$

37. **REASONING** Write and graph an inequality that represents the numbers that are *not* solutions of each inequality.

- a. $x + 8 < 14$
 b. $x - 12 \geq 5.7$

38. **PROBLEM SOLVING** Use the inequalities $c - 3 \geq d$, $b + 4 < a + 1$, and $a - 2 \leq d - 7$ to order a , b , c , and d from least to greatest.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the product or quotient. (*Skills Review Handbook*)

39. $7 \cdot (-9)$ 40. $-11 \cdot (-12)$ 41. $-27 \div (-3)$ 42. $20 \div (-5)$

Solve the equation. Check your solution. (*Section 1.1*)

43. $6x = 24$ 44. $-3y = -18$ 45. $\frac{s}{-8} = 13$ 46. $\frac{n}{4} = -7.3$