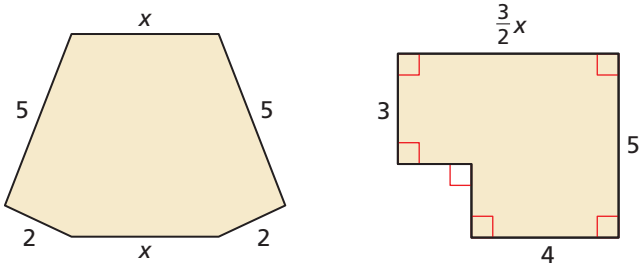
1.3 Solving Equations with Variables on Both Sides

Essential Question How can you solve an equation that has variables on both sides?

EXPLORATION 1 Perimeter

Work with a partner. The two polygons have the same perimeter. Use this information to write and solve an equation involving x. Explain the process you used to find the solution. Then find the perimeter of each polygon.



EXPLORATION 2 Perimeter and Area

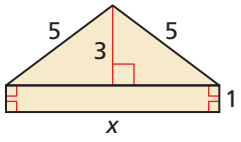
Work with a partner.

- Each figure has the unusual property that the value of its perimeter (in feet) is equal to the value of its area (in square feet). Use this information to write an equation for each figure.
- Solve each equation for x. Explain the process you used to find the solution.
- Find the perimeter and area of each figure.

LOOKING FOR STRUCTURE

To be proficient in math, you need to visualize complex things, such as composite figures, as being made up of simpler, more manageable parts.

a.



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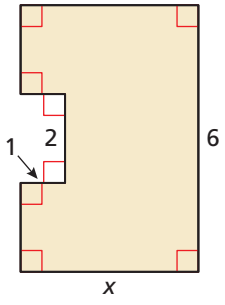

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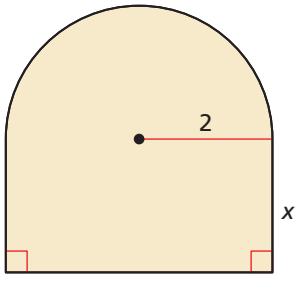

Communicate Your Answer

3. How can you solve an equation that has variables on both sides?

4. Write three equations that have the variable x on both sides. The equations should be different from those you wrote in Explorations 1 and 2. Have your partner solve the equations.

1.3 Lesson

Core Vocabulary

identity, *p. 21*

Previous

inverse operations

What You Will Learn

- Solve linear equations that have variables on both sides.
- Identify special solutions of linear equations.
- Use linear equations to solve real-life problems.

Solving Equations with Variables on Both Sides

Core Concept

Solving Equations with Variables on Both Sides

To solve an equation with variables on both sides, simplify one or both sides of the equation, if necessary. Then use inverse operations to collect the variable terms on one side, collect the constant terms on the other side, and isolate the variable.

EXAMPLE 1 Solving an Equation with Variables on Both Sides

Solve $10 - 4x = -9x$. Check your solution.

SOLUTION

$10 - 4x = -9x$ Write the equation.

$+ 4x \quad + 4x$ Add $4x$ to each side.

$10 = -5x$ Simplify.

$\frac{10}{-5} = \frac{-5x}{-5}$ Divide each side by -5.

$-2 = x$ Simplify.

Check

$10 - 4x = -9x$

$10 - 4(-2) \stackrel{?}{=} -9(-2)$

$18 = 18$ ✓

▶ The solution is $x = -2$.

EXAMPLE 2 Solving an Equation with Grouping Symbols

Solve $3(3x - 4) = \frac{1}{4}(32x + 56)$.

SOLUTION

$3(3x - 4) = \frac{1}{4}(32x + 56)$ Write the equation.

$9x - 12 = 8x + 14$ Distributive Property

$+ 12 \quad + 12$ Add 12 to each side.

$9x = 8x + 26$ Simplify.

$- 8x \quad - 8x$ Subtract $8x$ from each side.

$x = 26$ Simplify.

▶ The solution is $x = 26$.

Monitoring Progress

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Solve the equation. Check your solution.

1. $-2x = 3x + 10$ **2.** $\frac{1}{2}(6h - 4) = -5h + 1$ **3.** $-\frac{3}{4}(8n + 12) = 3(n - 3)$

Identifying Special Solutions of Linear Equations

Core Concept

Special Solutions of Linear Equations

Equations do not always have one solution. An equation that is true for all values of the variable is an **identity** and has *infinitely many solutions*. An equation that is not true for any value of the variable has *no solution*.

EXAMPLE 3 Identifying the Number of Solutions

Solve each equation.

a. $3(5x + 2) = 15x$ **b.** $-2(4y + 1) = -8y - 2$

SOLUTION

a.

$3(5x + 2) = 15x$	Write the equation.
$15x + 6 = 15x$	Distributive Property
$-15x \quad -15x$	Subtract $15x$ from each side.
$6 = 0$ ✗	Simplify.

▶ The statement $6 = 0$ is never true. So, the equation has no solution.

b.

$-2(4y + 1) = -8y - 2$	Write the equation.
$-8y - 2 = -8y - 2$	Distributive Property
$+8y \quad +8y$	Add $8y$ to each side.
$-2 = -2$	Simplify.

▶ The statement $-2 = -2$ is always true. So, the equation is an identity and has infinitely many solutions.

REASONING

The equation $15x + 6 = 15x$ is not true because the number $15x$ cannot be equal to 6 more than itself.

READING

All real numbers are solutions of an identity.

Monitoring Progress

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Solve the equation.

4. $4(1 - p) = -4p + 4$ **5.** $6m - m = \frac{5}{6}(6m - 10)$

6. $10k + 7 = -3 - 10k$ **7.** $3(2a - 2) = 2(3a - 3)$

Concept Summary

Steps for Solving Linear Equations

Here are several steps you can use to solve a linear equation. Depending on the equation, you may not need to use some steps.

Step 1 Use the Distributive Property to remove any grouping symbols.

Step 2 Simplify the expression on each side of the equation.

Step 3 Collect the variable terms on one side of the equation and the constant terms on the other side.

Step 4 Isolate the variable.

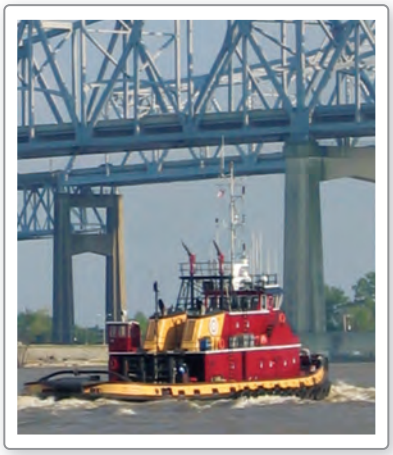
Step 5 Check your solution.

STUDY TIP

To check an identity, you can choose several different values of the variable.

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**



A boat leaves New Orleans and travels upstream on the Mississippi River for 4 hours. The return trip takes only 2.8 hours because the boat travels 3 miles per hour faster downstream due to the current. How far does the boat travel upstream?

SOLUTION

1. **Understand the Problem** You are given the amounts of time the boat travels and the difference in speeds for each direction. You are asked to find the distance the boat travels upstream.
2. **Make a Plan** Use the Distance Formula to write expressions that represent the problem. Because the distance the boat travels in both directions is the same, you can use the expressions to write an equation.
3. **Solve the Problem** Use the formula (distance) = (rate)(time).

Words Distance upstream = Distance downstream

Variable Let x be the speed (in miles per hour) of the boat traveling upstream.

Equation $\frac{x \text{ mi}}{1 \cancel{\text{h}}} \cdot 4 \cancel{\text{h}} = \frac{(x + 3) \text{ mi}}{1 \cancel{\text{h}}} \cdot 2.8 \cancel{\text{h}}$ (mi = mi) ✓

$$4x = 2.8(x + 3)$$ Write the equation.

$$4x = 2.8x + 8.4$$ Distributive Property

$$-2.8x \quad -2.8x$$ Subtract $2.8x$ from each side.

$$1.2x = 8.4$$ Simplify.

$$\frac{1.2x}{1.2} = \frac{8.4}{1.2}$$ Divide each side by 1.2.

$$x = 7$$ Simplify.

So, the boat travels 7 miles per hour upstream. To determine how far the boat travels upstream, multiply 7 miles per hour by 4 hours to obtain 28 miles.

4. **Look Back** To check that your solution is reasonable, use the formula for distance. Because the speed upstream is 7 miles per hour, the speed downstream is $7 + 3 = 10$ miles per hour. When you substitute each speed into the Distance Formula, you get the same distance for upstream and downstream.

Upstream

$$\text{Distance} = \frac{7 \text{ mi}}{1 \cancel{\text{h}}} \cdot 4 \cancel{\text{h}} = 28 \text{ mi}$$

Downstream

$$\text{Distance} = \frac{10 \text{ mi}}{1 \cancel{\text{h}}} \cdot 2.8 \cancel{\text{h}} = 28 \text{ mi}$$

Monitoring Progress

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8. A boat travels upstream on the Mississippi River for 3.5 hours. The return trip only takes 2.5 hours because the boat travels 2 miles per hour faster downstream due to the current. How far does the boat travel upstream?

1.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Is the equation $-2(4 - x) = 2x + 8$ an identity? Explain your reasoning.

2. **WRITING** Describe the steps in solving the linear equation $3(3x - 8) = 4x + 6$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–16, solve the equation. Check your solution. *(See Examples 1 and 2.)*

3. $15 - 2x = 3x$

4. $26 - 4s = 9s$

5. $5p - 9 = 2p + 12$

6. $8g + 10 = 35 + 3g$

7. $5t + 16 = 6 - 5t$

8. $-3r + 10 = 15r - 8$

9. $7 + 3x - 12x = 3x + 1$

10. $w - 2 + 2w = 6 + 5w$

11. $10(g + 5) = 2(g + 9)$

12. $-9(t - 2) = 4(t - 15)$

13. $\frac{2}{3}(3x + 9) = -2(2x + 6)$

14. $2(2t + 4) = \frac{3}{4}(24 - 8t)$

15. $10(2y + 2) - y = 2(8y - 8)$

16. $2(4x + 2) = 4x - 12(x - 1)$

17. **MODELING WITH MATHEMATICS** You and your friend drive toward each other. The equation $50h = 190 - 45h$ represents the number h of hours until you and your friend meet. When will you meet?

18. **MODELING WITH MATHEMATICS** The equation $1.5r + 15 = 2.25r$ represents the number r of movies you must rent to spend the same amount at each movie store. How many movies must you rent to spend the same amount at each movie store?

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In Exercises 19–24, solve the equation. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*. *(See Example 3.)*

19. $3t + 4 = 12 + 3t$

20. $6d + 8 = 14 + 3d$

21. $2(h + 1) = 5h - 7$

22. $12y + 6 = 6(2y + 1)$

23. $3(4g + 6) = 2(6g + 9)$

24. $5(1 + 2m) = \frac{1}{2}(8 + 20m)$

ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in solving the equation.**

25. ✗
$$5c - 6 = 4 - 3c$$
$$2c - 6 = 4$$
$$2c = 10$$
$$c = 5$$

26. ✗
$$6(2y + 6) = 4(9 + 3y)$$
$$12y + 36 = 36 + 12y$$
$$12y = 12y$$
$$0 = 0$$
The equation has no solution.

27. **MODELING WITH MATHEMATICS** Write and solve an equation to find the month when you would pay the same total amount for each Internet service.

	Installation fee	Price per month
Company A	$60.00	$42.95
Company B	$25.00	$49.95

28. **PROBLEM SOLVING** One serving of granola provides 4% of the protein you need daily. You must get the remaining 48 grams of protein from other sources. How many grams of protein do you need daily?

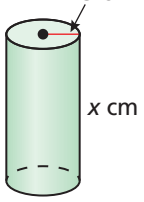
USING STRUCTURE **In Exercises 29 and 30, find the value of r.**

29. $8(x + 6) - 10 + r = 3(x + 12) + 5x$

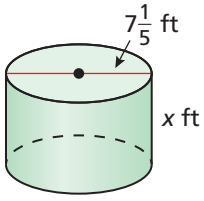
30. $4(x - 3) - r + 2x = 5(3x - 7) - 9x$

MATHEMATICAL CONNECTIONS **In Exercises 31 and 32, the value of the surface area of the cylinder is equal to the value of the volume of the cylinder. Find the value of x. Then find the surface area and volume of the cylinder.**

31.



32.



33. **MODELING WITH MATHEMATICS** A cheetah that is running 90 feet per second is 120 feet behind an antelope that is running 60 feet per second. How long will it take the cheetah to catch up to the antelope? *(See Example 4.)*

34. **MAKING AN ARGUMENT** A cheetah can run at top speed for only about 20 seconds. If an antelope is too far away for a cheetah to catch it in 20 seconds, the antelope is probably safe. Your friend claims the antelope in Exercise 33 will not be safe if the cheetah starts running 650 feet behind it. Is your friend correct? Explain.

REASONING **In Exercises 35 and 36, for what value of a is the equation an identity? Explain your reasoning.**

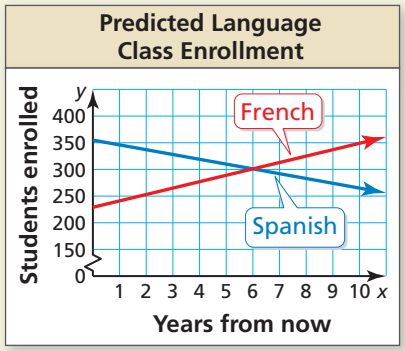
35. $a(2x + 3) = 9x + 15 + x$

36. $8x - 8 + 3ax = 5ax - 2a$

37. **REASONING** Two times the greater of two consecutive integers is 9 less than three times the lesser integer. What are the integers?

38. **HOW DO YOU SEE IT?** The table and the graph show information about students enrolled in Spanish and French classes at a high school.

	Students enrolled this year	Average rate of change
Spanish	355	9 fewer students each year
French	229	12 more students each year

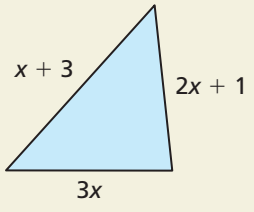


a. Use the graph to determine after how many years there will be equal enrollment in Spanish and French classes.

b. How does the equation $355 - 9x = 229 + 12x$ relate to the table and the graph? How can you use this equation to determine whether your answer in part (a) is reasonable?

39. **WRITING EQUATIONS** Give an example of a linear equation that has (a) no solution and (b) infinitely many solutions. Justify your answers.

40. **THOUGHT PROVOKING** Draw a different figure that has the same perimeter as the triangle shown. Explain why your figure has the same perimeter.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Order the values from least to greatest. *(Skills Review Handbook)*

41. $9, |-4|, -4, 5, |2|$

42. $|-32|, 22, -16, -|21|, |-10|$

43. $-18, |-24|, -19, |-18|, |22|$

44. $-|-3|, |0|, -1, |2|, -2$