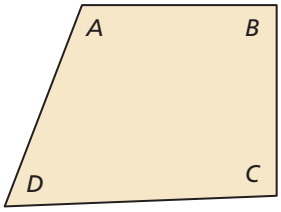
1.1 Solving Simple Equations

Essential Question How can you use simple equations to solve real-life problems?

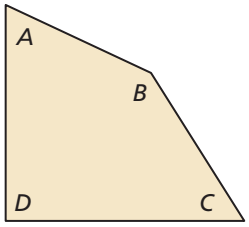
EXPLORATION 1 Measuring Angles

Work with a partner. Use a protractor to measure the angles of each quadrilateral. Copy and complete the table to organize your results. (The notation $m\angle A$ denotes the measure of angle A.) How precise are your measurements?

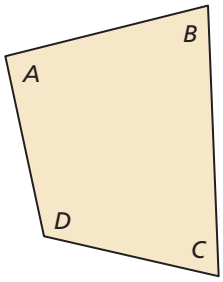
a.



b.



c.



Quadrilateral	$m\angle A$ (degrees)	$m\angle B$ (degrees)	$m\angle C$ (degrees)	$m\angle D$ (degrees)	$m\angle A + m\angle B + m\angle C + m\angle D$
a.					
b.					
c.					

UNDERSTANDING MATHEMATICAL TERMS

A **conjecture** is an unproven statement about a general mathematical concept. After the statement is proven, it is called a **rule** or a **theorem**.

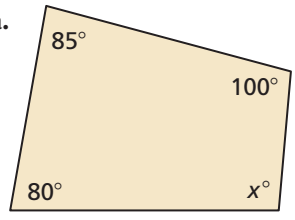
EXPLORATION 2 Making a Conjecture

Work with a partner. Use the completed table in Exploration 1 to write a conjecture about the sum of the angle measures of a quadrilateral. Draw three quadrilaterals that are different from those in Exploration 1 and use them to justify your conjecture.

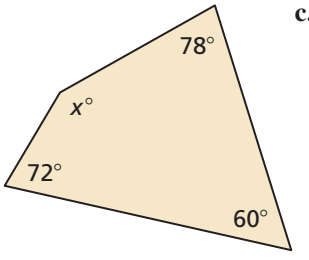
EXPLORATION 3 Applying Your Conjecture

Work with a partner. Use the conjecture you wrote in Exploration 2 to write an equation for each quadrilateral. Then solve the equation to find the value of x. Use a protractor to check the reasonableness of your answer.

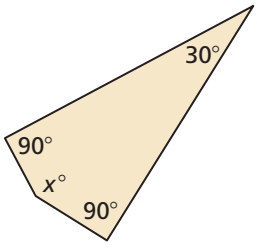
a.



b.



c.



Communicate Your Answer

4. How can you use simple equations to solve real-life problems?

5. Draw your own quadrilateral and cut it out. Tear off the four corners of the quadrilateral and rearrange them to affirm the conjecture you wrote in Exploration 2. Explain how this affirms the conjecture.

1.1 Lesson

What You Will Learn

- Solve linear equations using addition and subtraction.
- Solve linear equations using multiplication and division.
- Use linear equations to solve real-life problems.

Core Vocabulary

conjecture, *p. 3*
rule, *p. 3*
theorem, *p. 3*
equation, *p. 4*
linear equation in one variable, *p. 4*
solution, *p. 4*
inverse operations, *p. 4*
equivalent equations, *p. 4*

Previous
expression

Solving Linear Equations by Adding or Subtracting

An **equation** is a statement that two expressions are equal. A **linear equation in one variable** is an equation that can be written in the form $ax + b = 0$, where a and b are constants and $a \neq 0$. A **solution** of an equation is a value that makes the equation true.

Inverse operations are two operations that undo each other, such as addition and subtraction. When you perform the same inverse operation on each side of an equation, you produce an equivalent equation. **Equivalent equations** are equations that have the same solution(s).

Core Concept

Addition Property of Equality

Words Adding the same number to each side of an equation produces an equivalent equation.

Algebra If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality

Words Subtracting the same number from each side of an equation produces an equivalent equation.

Algebra If $a = b$, then $a - c = b - c$.

EXAMPLE 1 Solving Equations by Addition or Subtraction

Solve each equation. Justify each step. Check your answer.

a. $x - 3 = -5$ **b.** $0.9 = y + 2.8$

SOLUTION

a. $x - 3 = -5$ Write the equation.

Addition Property of Equality → $+ 3 \quad + 3$ Add 3 to each side.

$x = -2$ Simplify.

▶ The solution is $x = -2$.

Check

$x - 3 = -5$

$-2 - 3 \stackrel{?}{=} -5$

$-5 = -5$ ✓

b. $0.9 = y + 2.8$ Write the equation.

Subtraction Property of Equality → $- 2.8 \quad - 2.8$ Subtract 2.8 from each side.

$-1.9 = y$ Simplify.

▶ The solution is $y = -1.9$.

Check

$0.9 = y + 2.8$

$0.9 \stackrel{?}{=} -1.9 + 2.8$

$0.9 = 0.9$ ✓

Monitoring Progress

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Solve the equation. Justify each step. Check your solution.

1. $n + 3 = -7$ **2.** $g - \frac{1}{3} = -\frac{2}{3}$ **3.** $-6.5 = p + 3.9$

Solving Linear Equations by Multiplying or Dividing

Core Concept

Multiplication Property of Equality

Words Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Algebra If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.

Division Property of Equality

Words Dividing each side of an equation by the same nonzero number produces an equivalent equation.

Algebra If $a = b$, then $a \div c = b \div c$, $c \neq 0$.

REMEMBER

Multiplication and division are inverse operations.

EXAMPLE 2 Solving Equations by Multiplication or Division

Solve each equation. Justify each step. Check your answer.

a. $-\frac{n}{5} = -3$ **b.** $\pi x = -2\pi$ **c.** $1.3z = 5.2$

SOLUTION

a. $-\frac{n}{5} = -3$ Write the equation.

Multiplication Property of Equality → $-5 \cdot \left(-\frac{n}{5}\right) = -5 \cdot (-3)$ Multiply each side by -5.

$n = 15$ Simplify.

▶ The solution is $n = 15$.

Check

$-\frac{n}{5} = -3$

$-\frac{15}{5} \stackrel{?}{=} -3$

$-3 = -3$ ✓

b. $\pi x = -2\pi$ Write the equation.

Division Property of Equality → $\frac{\pi x}{\pi} = \frac{-2\pi}{\pi}$ Divide each side by π.

$x = -2$ Simplify.

▶ The solution is $x = -2$.

Check

$\pi x = -2\pi$

$\pi(-2) \stackrel{?}{=} -2\pi$

$-2\pi = -2\pi$ ✓

c. $1.3z = 5.2$ Write the equation.

Division Property of Equality → $\frac{1.3z}{1.3} = \frac{5.2}{1.3}$ Divide each side by 1.3.

$z = 4$ Simplify.

▶ The solution is $z = 4$.

Check

$1.3z = 5.2$

$1.3(4) \stackrel{?}{=} 5.2$

$5.2 = 5.2$ ✓

Monitoring Progress

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Solve the equation. Justify each step. Check your solution.

4. $\frac{y}{3} = -6$ **5.** $9\pi = \pi x$ **6.** $0.05w = 1.4$

Solving Real-Life Problems

Core Concept

Four-Step Approach to Problem Solving

1. **Understand the Problem** What is the unknown? What information is being given? What is being asked?
2. **Make a Plan** This plan might involve one or more of the problem-solving strategies shown on the next page.
3. **Solve the Problem** Carry out your plan. Check that each step is correct.
4. **Look Back** Examine your solution. Check that your solution makes sense in the original statement of the problem.

MODELING WITH MATHEMATICS

Mathematically proficient students routinely check that their solutions make sense in the context of a real-life problem.

EXAMPLE 3 Modeling with Mathematics

In the 2012 Olympics, Usain Bolt won the 200-meter dash with a time of 19.32 seconds. Write and solve an equation to find his average speed to the nearest hundredth of a meter per second.

REMEMBER

The formula that relates distance d, rate or speed r, and time t is

$$d = rt.$$

SOLUTION

1. **Understand the Problem** You know the winning time and the distance of the race. You are asked to find the average speed to the nearest hundredth of a meter per second.
2. **Make a Plan** Use the Distance Formula to write an equation that represents the problem. Then solve the equation.
3. **Solve the Problem**

$$d = r \cdot t \qquad \text{Write the Distance Formula.}$$

$$200 = r \cdot 19.32 \qquad \text{Substitute 200 for } d \text{ and 19.32 for } t.$$

$$\frac{200}{19.32} = \frac{19.32r}{19.32} \qquad \text{Divide each side by 19.32.}$$

$$10.35 \approx r \qquad \text{Simplify.}$$

REMEMBER

The symbol $\approx$ means "approximately equal to."

▶ Bolt's average speed was about 10.35 meters per second.

4. **Look Back** Round Bolt's average speed to 10 meters per second. At this speed, it would take

$$\frac{200 \text{ m}}{10 \text{ m/sec}} = 20 \text{ seconds}$$

to run 200 meters. Because 20 is close to 19.32, your solution is reasonable.

Monitoring Progress

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7. Suppose Usain Bolt ran 400 meters at the same average speed that he ran the 200 meters. How long would it take him to run 400 meters? Round your answer to the nearest hundredth of a second.

Core Concept

Common Problem-Solving Strategies

Use a verbal model.	Guess, check, and revise.
Draw a diagram.	Sketch a graph or number line.
Write an equation.	Make a table.
Look for a pattern.	Make a list.
Work backward.	Break the problem into parts.

EXAMPLE 4 Modeling with Mathematics

On January 22, 1943, the temperature in Spearfish, South Dakota, fell from 54°F at 9:00 A.M. to −4°F at 9:27 A.M. How many degrees did the temperature fall?

SOLUTION

1. **Understand the Problem** You know the temperature before and after the temperature fell. You are asked to find how many degrees the temperature fell.
2. **Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation.
3. **Solve the Problem**

Words Temperature at 9:27 A.M. = Temperature at 9:00 A.M. − Number of degrees the temperature fell

Variable Let T be the number of degrees the temperature fell.

Equation $-4 = 54 - T$

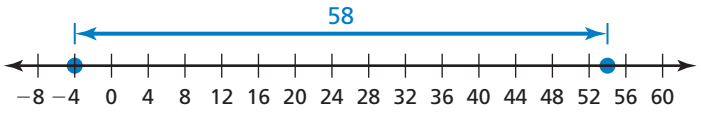
$$-4 = 54 - T \quad \text{Write the equation.}$$
$$-4 - 54 = 54 - 54 - T \quad \text{Subtract 54 from each side.}$$
$$-58 = -T \quad \text{Simplify.}$$
$$58 = T \quad \text{Divide each side by } -1.$$

▶ The temperature fell 58°F.

REMEMBER

The distance between two points on a number line is always positive.

4. **Look Back** The temperature fell from 54 degrees *above* 0 to 4 degrees *below* 0. You can use a number line to check that your solution is reasonable.



Monitoring Progress

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8. You thought the balance in your checking account was $68. When your bank statement arrives, you realize that you forgot to record a check. The bank statement lists your balance as $26. Write and solve an equation to find the amount of the check that you forgot to record.

1.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Which of the operations $+$, $-$, $\times$, and $\div$ are inverses of each other?
2. **VOCABULARY** Are the equations $-2x = 10$ and $-5x = 25$ equivalent? Explain.
3. **WRITING** Which property of equality would you use to solve the equation $14x = 56$? Explain.
4. **WHICH ONE DOESN'T BELONG?** Which expression does not belong with the other three? Explain your reasoning.

$8 = \frac{x}{2}$ $\quad$ $3 = x \div 4$ $\quad$ $x - 6 = 5$ $\quad$ $\frac{x}{3} = 9$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–14, solve the equation. Justify each step. Check your solution. *(See Example 1.)*

5. $x + 5 = 8$
6. $m + 9 = 2$
7. $y - 4 = 3$
8. $s - 2 = 1$
9. $w + 3 = -4$
10. $n - 6 = -7$
11. $-14 = p - 11$
12. $0 = 4 + q$
13. $r + (-8) = 10$
14. $t - (-5) = 9$

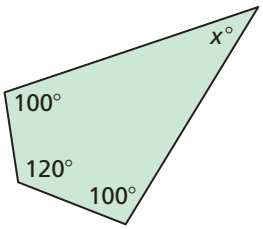
15. **MODELING WITH MATHEMATICS** A discounted amusement park ticket costs \$12.95 less than the original price p. Write and solve an equation to find the original price.

16. **MODELING WITH MATHEMATICS** You and a friend are playing a board game. Your final score x is 12 points less than your friend's final score. Write and solve an equation to find your final score.

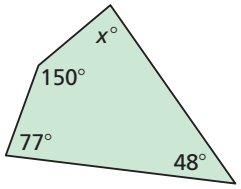
	ROUND 9	ROUND 10	FINAL SCORE
Your Friend	22	12	195
You	9	25	?

USING TOOLS **The sum of the angle measures of a quadrilateral is 360°. In Exercises 17–20, write and solve an equation to find the value of x. Use a protractor to check the reasonableness of your answer.**

17.



18.



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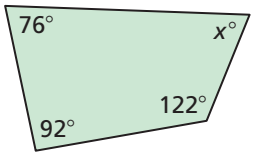
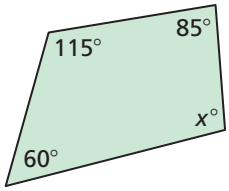


20.



In Exercises 21–30, solve the equation. Justify each step. Check your solution. *(See Example 2.)*

21. $5g = 20$
22. $4q = 52$
23. $p \div 5 = 3$
24. $y \div 7 = 1$
25. $-8r = 64$
26. $x \div (-2) = 8$
27. $\frac{x}{6} = 8$
28. $\frac{w}{-3} = 6$
29. $-54 = 9s$
30. $-7 = \frac{t}{7}$

In Exercises 31–38, solve the equation. Check your solution.

31. $\frac{3}{2} + t = \frac{1}{2}$

32. $b - \frac{3}{16} = \frac{5}{16}$

33. $\frac{3}{7}m = 6$

34. $-\frac{2}{5}y = 4$

35. $5.2 = a - 0.4$

36. $f + 3\pi = 7\pi$

37. $-108\pi = 6\pi j$

38. $x \div (-2) = 1.4$

ERROR ANALYSIS **In Exercises 39 and 40, describe and correct the error in solving the equation.**

39.

$$-0.8 + r = 12.6$$
$$r = 12.6 + (-0.8)$$
$$r = 11.8$$

40.

$$-\frac{m}{3} = -4$$
$$3 \cdot \left(-\frac{m}{3}\right) = 3 \cdot (-4)$$
$$m = -12$$

41. **ANALYZING RELATIONSHIPS** A baker orders 162 eggs. Each carton contains 18 eggs. Which equation can you use to find the number x of cartons? Explain your reasoning and solve the equation.

Ⓐ $162x = 18$ Ⓑ $\frac{x}{18} = 162$

Ⓒ $18x = 162$ Ⓓ $x + 18 = 162$

MODELING WITH MATHEMATICS **In Exercises 42–44, write and solve an equation to answer the question.** *(See Examples 3 and 4.)*

42. The temperature at 5 P.M. is 20°F. The temperature at 10 P.M. is −5°F. How many degrees did the temperature fall?

43. The length of an American flag is 1.9 times its width. What is the width of the flag?

9.5 ft

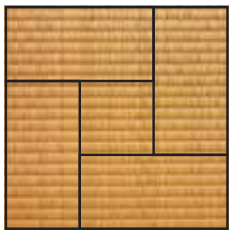
44. The balance of an investment account is $308 more than the balance 4 years ago. The current balance of the account is $4708. What was the balance 4 years ago?

45. **REASONING** Identify the property of equality that makes Equation 1 and Equation 2 equivalent.

Equation 1 $x - \frac{1}{2} = \frac{x}{4} + 3$

Equation 2 $4x - 2 = x + 12$

46. **PROBLEM SOLVING** Tatami mats are used as a floor covering in Japan. One possible layout uses four identical rectangular mats and one square mat, as shown. The area of the square mat is half the area of one of the rectangular mats.



Total area = 81 ft²

a. Write and solve an equation to find the area of one rectangular mat.

b. The length of a rectangular mat is twice the width. Use Guess, Check, and Revise to find the dimensions of one rectangular mat.

47. **PROBLEM SOLVING** You spend $30.40 on 4 CDs. Each CD costs the same amount and is on sale for 80% of the original price.

a. Write and solve an equation to find how much you spend on each CD.

b. The next day, the CDs are no longer on sale. You have $25. Will you be able to buy 3 more CDs? Explain your reasoning.

48. **ANALYZING RELATIONSHIPS** As c increases, does the value of x *increase*, *decrease*, or *stay the same* for each equation? Assume c is positive.

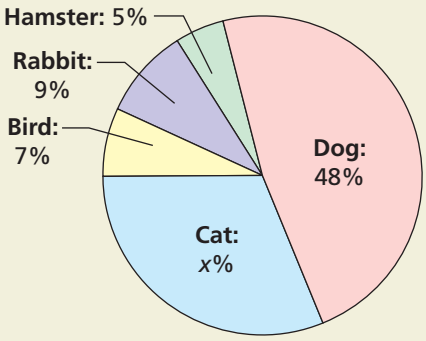
Equation	Value of x
$x - c = 0$	
$cx = 1$	
$cx = c$	
$\frac{x}{c} = 1$	

49. USING STRUCTURE Use the values -2, 5, 9, and 10 to complete each statement about the equation $ax = b - 5$.

a. When $a =$ ___ and $b =$ ___, x is a positive integer.

b. When $a =$ ___ and $b =$ ___, x is a negative integer.

50. HOW DO YOU SEE IT? The circle graph shows the percents of different animals sold at a local pet store in 1 year.



a. What percent is represented by the entire circle?

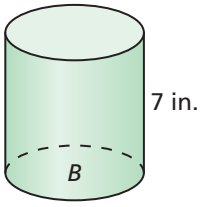
b. How does the equation $7 + 9 + 5 + 48 + x = 100$ relate to the circle graph? How can you use this equation to find the percent of cats sold?

51. REASONING One-sixth of the girls and two-sevenths of the boys in a school marching band are in the percussion section. The percussion section has 6 girls and 10 boys. How many students are in the marching band? Explain.

52. THOUGHT PROVOKING Write a real-life problem that can be modeled by an equation equivalent to the equation $5x = 30$. Then solve the equation and write the answer in the context of your real-life problem.

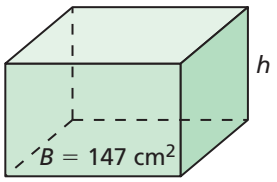
MATHEMATICAL CONNECTIONS In Exercises 53–56, find the height h or the area of the base B of the solid.

53.



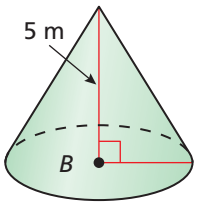
Volume $= 84\pi$ in.^{3}

54.



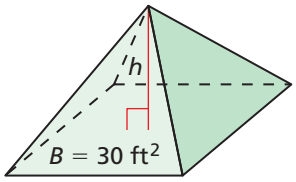
Volume $= 1323$ cm^3

55.



Volume $= 15\pi$ m^3

56.



Volume $= 35$ ft^3

57. MAKING AN ARGUMENT In baseball, a player's batting average is calculated by dividing the number of hits by the number of at-bats. The table shows Player A's batting average and number of at-bats for three regular seasons.

Season	Batting average	At-bats
2010	.312	596
2011	.296	446
2012	.295	599

a. How many hits did Player A have in the 2011 regular season? Round your answer to the nearest whole number.

b. Player B had 33 fewer hits in the 2011 season than Player A but had a greater batting average. Your friend concludes that Player B had more at-bats in the 2011 season than Player A. Is your friend correct? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the Distributive Property to simplify the expression. *(Skills Review Handbook)*

58. $8(y + 3)$

59. $\frac{5}{6}\left(x + \frac{1}{2} + 4\right)$

60. $5(m + 3 + n)$

61. $4(2p + 4q + 6)$

Copy and complete the statement. Round to the nearest hundredth, if necessary. *(Skills Review Handbook)*

62. $\frac{5 \text{ L}}{\text{min}} = \frac{\square \text{ L}}{\text{h}}$

63. $\frac{68 \text{ mi}}{\text{h}} \approx \frac{\square \text{ mi}}{\text{sec}}$

64. $\frac{7 \text{ gal}}{\text{min}} \approx \frac{\square \text{ qt}}{\text{sec}}$

65. $\frac{8 \text{ km}}{\text{min}} \approx \frac{\square \text{ mi}}{\text{h}}$