11 Circumference, Area, and Volume

Khafre's Pyramid *(p. 637)*

Gold Density *(p. 628)*

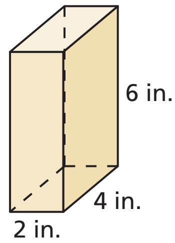
Basaltic Columns *(p. 615)*

Population Density *(p. 603)*

London Eye *(p. 599)*

Maintaining Mathematical Proficiency

Finding Surface Area



Example 1 **Find the surface area of the prism.**

$S = 2\ell w + 2\ell h + 2wh$ — Write formula for surface area of a rectangular prism.

$= 2(2)(4) + 2(2)(6) + 2(4)(6)$ — Substitute 2 for ℓ, 4 for w, and 6 for h.

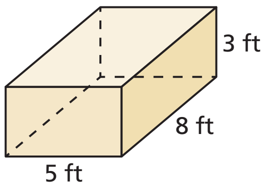
$= 16 + 24 + 48$ — Multiply.

$= 88$ — Add.

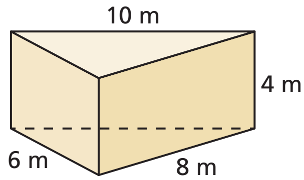
▶ The surface area is 88 square inches.

Find the surface area of the prism.

1.



2.



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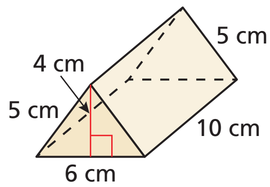


Finding a Missing Dimension

Example 2 **A rectangle has a perimeter of 10 meters and a length of 3 meters. What is the width of the rectangle?**

$P = 2\ell + 2w$ — Write formula for perimeter of a rectangle.

$10 = 2(3) + 2w$ — Substitute 10 for P and 3 for ℓ.

$10 = 6 + 2w$ — Multiply 2 and 3.

$4 = 2w$ — Subtract 6 from each side.

$2 = w$ — Divide each side by 2.

▶ The width is 2 meters.

Find the missing dimension.

4. A rectangle has a perimeter of 28 inches and a width of 5 inches. What is the length of the rectangle?

5. A triangle has an area of 12 square centimeters and a height of 12 centimeters. What is the base of the triangle?

6. A rectangle has an area of 84 square feet and a width of 7 feet. What is the length of the rectangle?

7. **ABSTRACT REASONING** Write an equation for the surface area of a prism with a length, width, and height of x inches. What solid figure does the prism represent?

Mathematical Practices

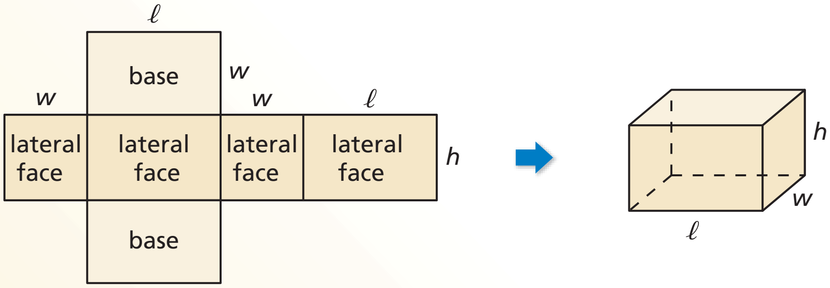
Mathematically proficient students create valid representations of problems.

Creating a Valid Representation

Core Concept

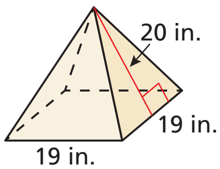
Nets for Three-Dimensional Figures

A **net** for a three-dimensional figure is a two-dimensional pattern that can be folded to form the three-dimensional figure.



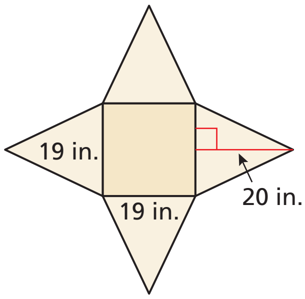
EXAMPLE 1 Drawing a Net for a Pyramid

Draw a net of the pyramid.



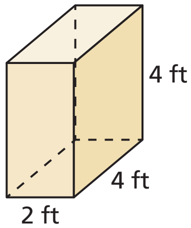
SOLUTION

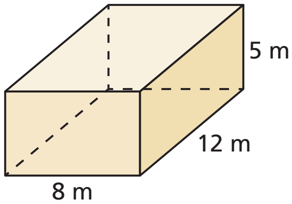
The pyramid has a square base. Its four lateral faces are congruent isosceles triangles.

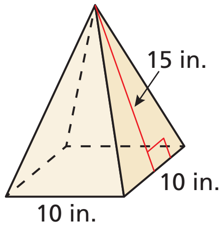


Monitoring Progress

Draw a net of the three-dimensional figure. Label the dimensions.

1. 

2. 

3. 

11.1 Circumference and Arc Length

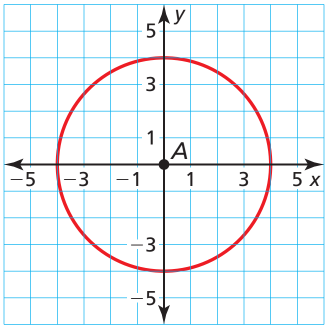
Essential Question

How can you find the length of a circular arc?

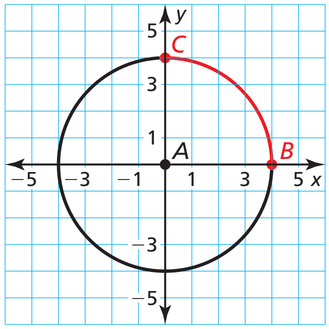
EXPLORATION 1 Finding the Length of a Circular Arc

Work with a partner. Find the length of each red circular arc.

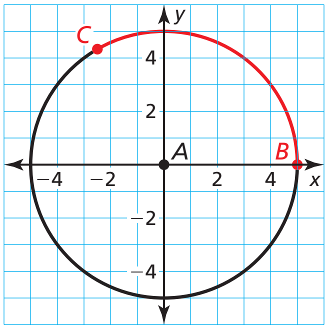
a. entire circle



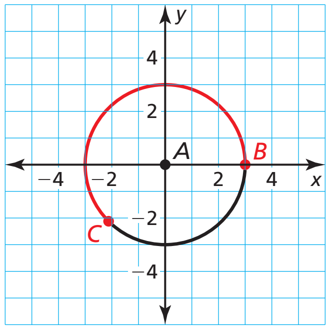
b. one-fourth of a circle



c. one-third of a circle

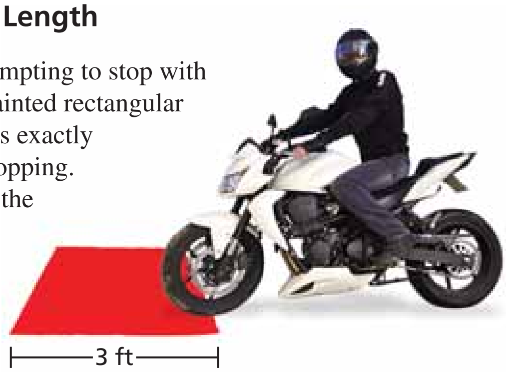


d. five-eighths of a circle



EXPLORATION 2 Using Arc Length

Work with a partner. The rider is attempting to stop with the front tire of the motorcycle in the painted rectangular box for a skills test. The front tire makes exactly one-half additional revolution before stopping. The diameter of the tire is 25 inches. Is the front tire still in contact with the painted box? Explain.



LOOKING FOR REGULARITY IN REPEATED REASONING

To be proficient in math, you need to notice if calculations are repeated and look both for general methods and for shortcuts.

Communicate Your Answer

3. How can you find the length of a circular arc?

4. A motorcycle tire has a diameter of 24 inches. Approximately how many inches does the motorcycle travel when its front tire makes three-fourths of a revolution?

11.1 Lesson

Core Vocabulary

circumference, *p. 594*
arc length, *p. 595*
radian, *p. 597*

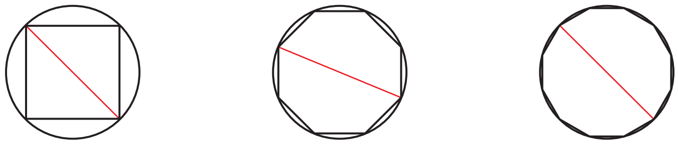
Previous
circle
diameter
radius

What You Will Learn

- Use the formula for circumference.
- Use arc lengths to find measures.
- Solve real-life problems.
- Measure angles in radians.

Using the Formula for Circumference

The **circumference** of a circle is the distance around the circle. Consider a regular polygon inscribed in a circle. As the number of sides increases, the polygon approximates the circle and the ratio of the perimeter of the polygon to the diameter of the circle approaches $\pi \approx 3.14159\ldots$.

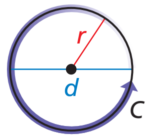


For all circles, the ratio of the circumference C to the diameter d is the same. This ratio is $\frac{C}{d} = \pi$. Solving for C yields the formula for the circumference of a circle, $C = \pi d$. Because $d = 2r$, you can also write the formula as $C = \pi(2r) = 2\pi r$.

Core Concept

Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.



$C = \pi d = 2\pi r$

EXAMPLE 1 Using the Formula for Circumference

Find each indicated measure.

a. circumference of a circle with a radius of 9 centimeters

b. radius of a circle with a circumference of 26 meters

ATTENDING TO PRECISION

You have sometimes used 3.14 to approximate the value of π. Throughout this chapter, you should use the π key on a calculator, then round to the hundredths place unless instructed otherwise.

SOLUTION

a.
$$\begin{aligned} C &= 2\pi r \\ &= 2 \cdot \pi \cdot 9 \\ &= 18\pi \\ &\approx 56.55 \end{aligned}$$

The circumference is about 56.55 centimeters.

b.
$$\begin{aligned} C &= 2\pi r \\ 26 &= 2\pi r \\ \frac{26}{2\pi} &= r \\ 4.14 &\approx r \end{aligned}$$

The radius is about 4.14 meters.

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1. Find the circumference of a circle with a diameter of 5 inches.

2. Find the diameter of a circle with a circumference of 17 feet.

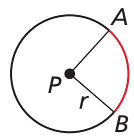
Using Arc Lengths to Find Measures

An **arc length** is a portion of the circumference of a circle. You can use the measure of the arc (in degrees) to find its length (in linear units).

Core Concept

Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.



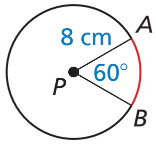
$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^\circ}, \text{ or}$$

$$\text{Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$$

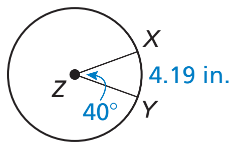
EXAMPLE 2 Using Arc Lengths to Find Measures

Find each indicated measure.

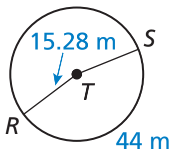
a. arc length of $\widehat{AB}$



b. circumference of $\odot Z$



c. $m\widehat{RS}$



SOLUTION

a. $\text{Arc length of } \widehat{AB} = \frac{60^\circ}{360^\circ} \cdot 2\pi(8)$

$\approx 8.38 \text{ cm}$

b. $\frac{\text{Arc length of } \widehat{XY}}{C} = \frac{m\widehat{XY}}{360^\circ}$

$\frac{4.19}{C} = \frac{40^\circ}{360^\circ}$

$\frac{4.19}{C} = \frac{1}{9}$

$37.71 \text{ in.} = C$

c. $\frac{\text{Arc length of } \widehat{RS}}{2\pi r} = \frac{m\widehat{RS}}{360^\circ}$

$\frac{44}{2\pi(15.28)} = \frac{m\widehat{RS}}{360^\circ}$

$360^\circ \cdot \frac{44}{2\pi(15.28)} = m\widehat{RS}$

$165^\circ \approx m\widehat{RS}$

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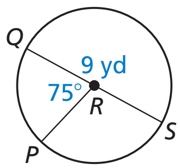
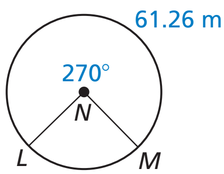
Find the indicated measure.

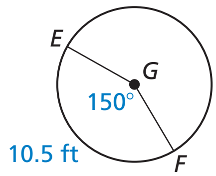
3. arc length of $\widehat{PQ}$



4. circumference of $\odot N$



5. radius of $\odot G$



Solving Real-Life Problems

EXAMPLE 3 Using Circumference to Find Distance Traveled

The dimensions of a car tire are shown. To the nearest foot, how far does the tire travel when it makes 15 revolutions?

SOLUTION

Step 1 Find the diameter of the tire.

$$d = 15 + 2(5.5) = 26 \text{ in.}$$

Step 2 Find the circumference of the tire.

$$C = \pi d = \pi \cdot 26 = 26\pi \text{ in.}$$

Step 3 Find the distance the tire travels in 15 revolutions. In one revolution, the tire travels a distance equal to its circumference. In 15 revolutions, the tire travels a distance equal to 15 times its circumference.

Distance traveled	=	Number of revolutions	•	Circumference

$$= 15 \cdot 26\pi \approx 1225.2 \text{ in.}$$

COMMON ERROR

Always pay attention to units. In Example 3, you need to convert units to get a correct answer.

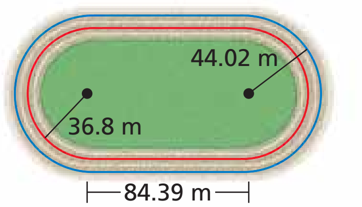
Step 4 Use unit analysis. Change 1225.2 inches to feet.

$$1225.2 \text{ in.} \cdot \frac{1 \text{ ft}}{12 \text{ in.}} = 102.1 \text{ ft}$$

▶ The tire travels approximately 102 feet.

EXAMPLE 4 Using Arc Length to Find Distances

The curves at the ends of the track shown are 180° arcs of circles. The radius of the arc for a runner on the red path shown is 36.8 meters. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.



SOLUTION

The path of the runner on the red path is made of two straight sections and two semicircles. To find the total distance, find the sum of the lengths of each part.

Distance	=	2 • Length of each straight section	+	2 • Length of each semicircle

$$= 2(84.39) + 2\left(\tfrac{1}{2} \cdot 2\pi \cdot 36.8\right)$$

$$\approx 400.0$$

▶ The runner on the red path travels about 400.0 meters.

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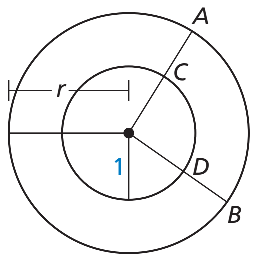
6. A car tire has a diameter of 28 inches. How many revolutions does the tire make while traveling 500 feet?

7. In Example 4, the radius of the arc for a runner on the blue path is 44.02 meters, as shown in the diagram. About how far does this runner travel to go once around the track? Round to the nearest tenth of a meter.

Measuring Angles in Radians

Recall that in a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°. To see why, consider the diagram.



A circle of radius 1 has circumference 2π, so the arc length of $\overparen{CD}$ is $\frac{m\overparen{CD}}{360°} \cdot 2\pi$.

Recall that all circles are similar and corresponding lengths of similar figures are proportional. Because $m\overparen{AB} = m\overparen{CD}$, $\overparen{AB}$ and $\overparen{CD}$ are corresponding arcs. So, you can write the following proportion.

$$\frac{\text{Arc length of } \overparen{AB}}{\text{Arc length of } \overparen{CD}} = \frac{r}{1}$$

$$\text{Arc length of } \overparen{AB} = r \cdot \text{Arc length of } \overparen{CD}$$

$$\text{Arc length of } \overparen{AB} = r \cdot \frac{m\overparen{CD}}{360°} \cdot 2\pi$$

This form of the equation shows that the arc length associated with a central angle is *proportional to the radius* of the circle. The constant of proportionality, $\frac{m\overparen{CD}}{360°} \cdot 2\pi$, is defined to be the **radian** measure of the central angle associated with the arc.

In a circle of radius 1, the radian measure of a given central angle can be thought of as the length of the arc associated with the angle. The radian measure of a complete circle (360°) is exactly 2π radians, because the circumference of a circle of radius 1 is exactly 2π. You can use this fact to convert from degree measure to radian measure and vice versa.

Core Concept

Converting between Degrees and Radians

Degrees to radians

Multiply degree measure by

$$\frac{2\pi \text{ radians}}{360°}, \text{ or } \frac{\pi \text{ radians}}{180°}.$$

Radians to degrees

Multiply radian measure by

$$\frac{360°}{2\pi \text{ radians}}, \text{ or } \frac{180°}{\pi \text{ radians}}.$$

EXAMPLE 5 Converting between Degree and Radian Measure

a. Convert 45° to radians.

b. Convert $\frac{3\pi}{2}$ radians to degrees.

SOLUTION

a. $45° \cdot \frac{\pi \text{ radians}}{180°} = \frac{\pi}{4}$ radian

▶ So, $45° = \frac{\pi}{4}$ radian.

b. $\frac{3\pi}{2}$ radians $\cdot \frac{180°}{\pi \text{ radians}} = 270°$

▶ So, $\frac{3\pi}{2}$ radians $= 270°$.

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8. Convert 15° to radians.

9. Convert $\frac{4\pi}{3}$ radians to degrees.

11.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The circumference of a circle with diameter d is $C =$ ______.

2. **WRITING** Describe the difference between an arc measure and an arc length.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, find the indicated measure. *(See Examples 1 and 2.)*

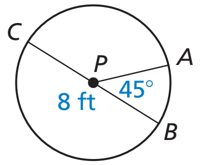
3. circumference of a circle with a radius of 6 inches

4. diameter of a circle with a circumference of 63 feet

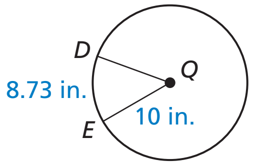
5. radius of a circle with a circumference of 28π

6. exact circumference of a circle with a diameter of 5 inches

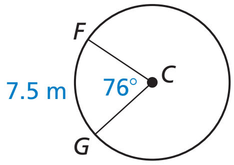
7. arc length of $\widehat{AB}$



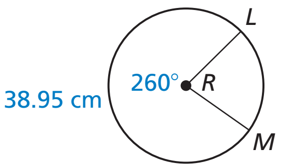
8. $m\widehat{DE}$



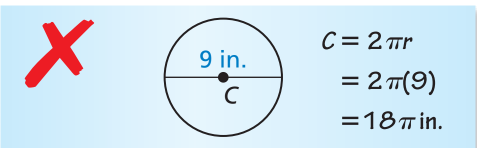
9. circumference of $\odot C$



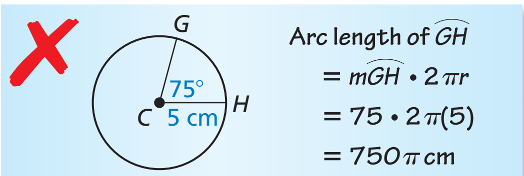
10. radius of $\odot R$



11. **ERROR ANALYSIS** Describe and correct the error in finding the circumference of $\odot C$.



12. **ERROR ANALYSIS** Describe and correct the error in finding the length of $\widehat{GH}$.

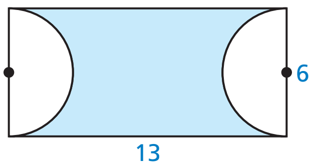


13. **PROBLEM SOLVING** A measuring wheel is used to calculate the length of a path. The diameter of the wheel is 8 inches. The wheel makes 87 complete revolutions along the length of the path. To the nearest foot, how long is the path? *(See Example 3.)*

14. **PROBLEM SOLVING** You ride your bicycle 40 meters. How many complete revolutions does the front wheel make?

In Exercises 15–18, find the perimeter of the shaded region. *(See Example 4.)*

15.



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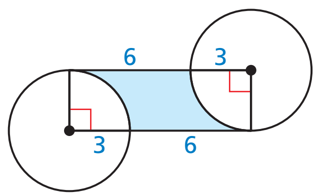
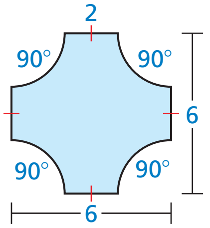


17.



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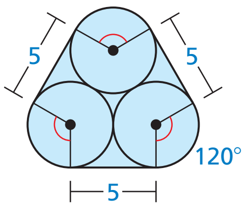


In Exercises 19–22, convert the angle measure. *(See Example 5.)*

19. Convert 70° to radians.

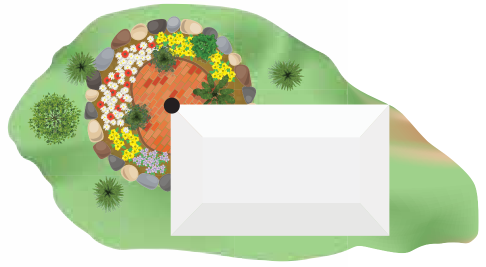
20. Convert 300° to radians.

21. Convert $\frac{11\pi}{12}$ radians to degrees.

22. Convert $\frac{\pi}{8}$ radian to degrees.

23. **PROBLEM SOLVING** The London Eye is a Ferris wheel in London, England, that travels at a speed of 0.26 meter per second. How many minutes does it take the London Eye to complete one full revolution?

24. **PROBLEM SOLVING** You are planning to plant a circular garden adjacent to one of the corners of a building, as shown. You can use up to 38 feet of fence to make a border around the garden. What radius (in feet) can the garden have? Choose all that apply. Explain your reasoning.



(A) 7 (B) 8 (C) 9 (D) 10

In Exercises 25 and 26, find the circumference of the circle with the given equation. Write the circumference in terms of π.

25. $x^2 + y^2 = 16$

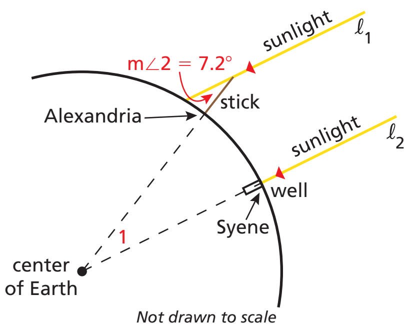
26. $(x + 2)^2 + (y - 3)^2 = 9$

27. **USING STRUCTURE** A semicircle has endpoints $(-2, 5)$ and $(2, 8)$. Find the arc length of the semicircle.

28. **REASONING** $\widehat{EF}$ is an arc on a circle with radius r. Let $x°$ be the measure of $\widehat{EF}$. Describe the effect on the length of $\widehat{EF}$ if you (a) double the radius of the circle, and (b) double the measure of $\widehat{EF}$.

29. **MAKING AN ARGUMENT** Your friend claims that it is possible for two arcs with the same measure to have different arc lengths. Is your friend correct? Explain your reasoning.

30. **PROBLEM SOLVING** Over 2000 years ago, the Greek scholar Eratosthenes estimated Earth's circumference by assuming that the Sun's rays were parallel. He chose a day when the Sun shone straight down into a well in the city of Syene. At noon, he measured the angle the Sun's rays made with a vertical stick in the city of Alexandria. Eratosthenes assumed that the distance from Syene to Alexandria was equal to about 575 miles. Explain how Eratosthenes was able to use this information to estimate Earth's circumference. Then estimate Earth's circumference.



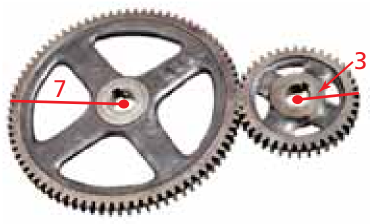
31. **ANALYZING RELATIONSHIPS** In ⊙C, the ratio of the length of $\widehat{PQ}$ to the length of $\widehat{RS}$ is 2 to 1. What is the ratio of $m\angle PCQ$ to $m\angle RCS$?

(A) 4 to 1 (B) 2 to 1

(C) 1 to 4 (D) 1 to 2

32. **ANALYZING RELATIONSHIPS** A 45° arc in ⊙C and a 30° arc in ⊙P have the same length. What is the ratio of the radius r_1 of ⊙C to the radius r_2 of ⊙P? Explain your reasoning.

33. **PROBLEM SOLVING** How many revolutions does the smaller gear complete during a single revolution of the larger gear?



34. **USING STRUCTURE** Find the circumference of each circle.

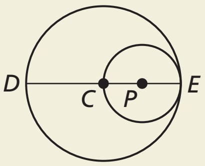
a. a circle circumscribed about a right triangle whose legs are 12 inches and 16 inches long

b. a circle circumscribed about a square with a side length of 6 centimeters

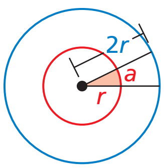
c. a circle inscribed in an equilateral triangle with a side length of 9 inches

35. **REWRITING A FORMULA** Write a formula in terms of the measure θ (theta) of the central angle (in radians) that can be used to find the length of an arc of a circle. Then use this formula to find the length of an arc of a circle with a radius of 4 inches and a central angle of $\frac{3\pi}{4}$ radians.

36. **HOW DO YOU SEE IT?** Compare the circumference of $\odot P$ to the length of $\widehat{DE}$. Explain your reasoning.



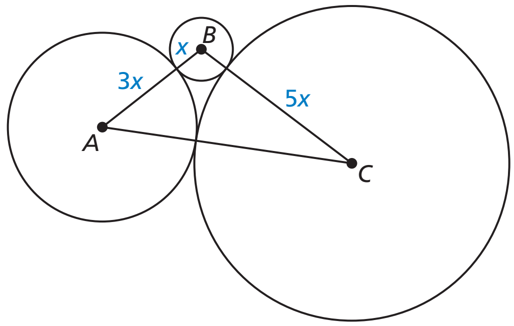
37. **MAKING AN ARGUMENT** In the diagram, the measure of the red shaded angle is 30°. The arc length a is 2. Your classmate claims that it is possible to find the circumference of the blue circle without finding the radius of either circle. Is your classmate correct? Explain your reasoning.



38. **MODELING WITH MATHEMATICS** What is the measure (in radians) of the angle formed by the hands of a clock at each time? Explain your reasoning.

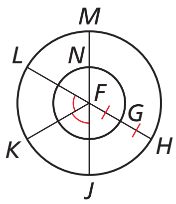
a. 1:30 P.M. **b.** 3:15 P.M.

39. **MATHEMATICAL CONNECTIONS** The sum of the circumferences of circles A, B, and C is 63π. Find AC.

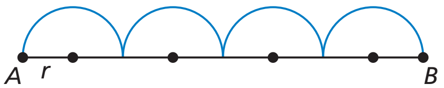


40. **THOUGHT PROVOKING** Is π a rational number? Compare the rational number $\frac{355}{113}$ to π. Find a different rational number that is even closer to π.

41. **PROOF** The circles in the diagram are concentric and $\overline{FG} \cong \overline{GH}$. Prove that $\widehat{JK}$ and $\widehat{NG}$ have the same length.



42. **REPEATED REASONING** $\overline{AB}$ is divided into four congruent segments, and semicircles with radius r are drawn.



a. What is the sum of the four arc lengths?

b. What would the sum of the arc lengths be if $\overline{AB}$ was divided into 8 congruent segments? 16 congruent segments? n congruent segments? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the area of the polygon with the given vertices. *(Section 1.4)*

43. $X(2, 4)$, $Y(8, -1)$, $Z(2, -1)$

44. $L(-3, 1)$, $M(4, 1)$, $N(4, -5)$, $P(-3, -5)$

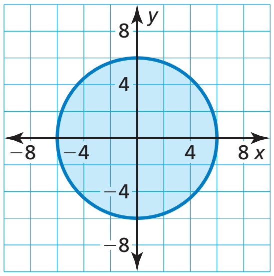
11.2 Areas of Circles and Sectors

Essential Question How can you find the area of a sector of a circle?

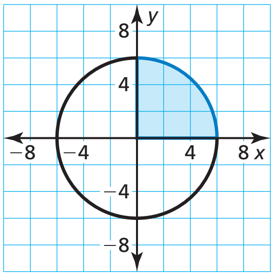
EXPLORATION 1 Finding the Area of a Sector of a Circle

Work with a partner. A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. Find the area of each shaded circle or sector of a circle.

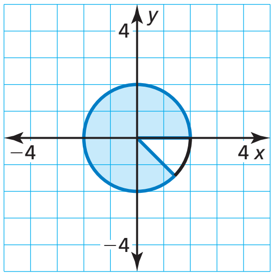
a. entire circle



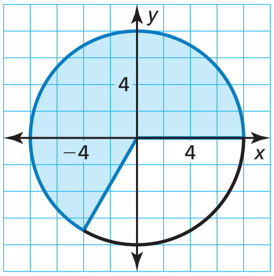
b. one-fourth of a circle



c. seven-eighths of a circle



d. two-thirds of a circle



REASONING ABSTRACTLY

To be proficient in math, you need to explain to yourself the meaning of a problem and look for entry points to its solution.

EXPLORATION 2 Finding the Area of a Circular Sector

Work with a partner. A center pivot irrigation system consists of 400 meters of sprinkler equipment that rotates around a central pivot point at a rate of once every 3 days to irrigate a circular region with a diameter of 800 meters. Find the area of the sector that is irrigated by this system in one day.

Communicate Your Answer

3. How can you find the area of a sector of a circle?

4. In Exploration 2, find the area of the sector that is irrigated in 2 hours.

11.2 Lesson

Core Vocabulary

population density, *p. 603*
sector of a circle, *p. 604*

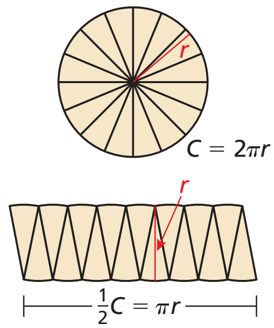
Previous
circle
radius
diameter
intercepted arc

What You Will Learn

- Use the formula for the area of a circle.
- Use the formula for population density.
- Find areas of sectors.
- Use areas of sectors.

Using the Formula for the Area of a Circle

You can divide a circle into congruent sections and rearrange the sections to form a figure that approximates a parallelogram. Increasing the number of congruent sections increases the figure's resemblance to a parallelogram.



The base of the parallelogram that the figure approaches is half of the circumference, so $b = \frac{1}{2}C = \frac{1}{2}(2\pi r) = \pi r$. The height is the radius, so $h = r$. So, the area of the parallelogram is $A = bh = (\pi r)(r) = \pi r^2$.

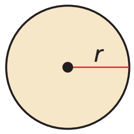
Core Concept

Area of a Circle

The area of a circle is

$$A = \pi r^2$$

where r is the radius of the circle.



EXAMPLE 1 Using the Formula for the Area of a Circle

Find each indicated measure.

a. area of a circle with a radius of 2.5 centimeters

b. diameter of a circle with an area of 113.1 square centimeters

SOLUTION

a.

$A = \pi r^2$	Formula for area of a circle
$= \pi \cdot (2.5)^2$	Substitute 2.5 for *r*.
$= 6.25\pi$	Simplify.
≈ 19.63	Use a calculator.

- The area of the circle is about 19.63 square centimeters.

b.

$A = \pi r^2$	Formula for area of a circle
$113.1 = \pi r^2$	Substitute 113.1 for *A*.
$\frac{113.1}{\pi} = r^2$	Divide each side by π.
$6 \approx r$	Find the positive square root of each side.

- The radius is about 6 centimeters, so the diameter is about 12 centimeters.

Monitoring Progress

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1. Find the area of a circle with a radius of 4.5 meters.

2. Find the radius of a circle with an area of 176.7 square feet.

Using the Formula for Population Density

The **population density** of a city, county, or state is a measure of how many people live within a given area.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}}$$

Population density is usually given in terms of square miles but can be expressed using other units, such as city blocks.

EXAMPLE 2 Using the Formula for Population Density

a. About 430,000 people live in a 5-mile radius of a city's town hall. Find the population density in people per square mile.

b. A region with a 3-mile radius has a population density of about 6195 people per square mile. Find the number of people who live in the region.

SOLUTION

a. Step 1 Find the area of the region.

$$A = \pi r^2 = \pi \cdot 5^2 = 25\pi$$

The area of the region is $25\pi \approx 78.54$ square miles.

Step 2 Find the population density.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} \quad \text{Formula for population density}$$

$$= \frac{430{,}000}{25\pi} \quad \text{Substitute.}$$

$$\approx 5475 \quad \text{Use a calculator.}$$

▶ The population density is about 5475 people per square mile.

b. Step 1 Find the area of the region.

$$A = \pi r^2 = \pi \cdot 3^2 = 9\pi$$

The area of the region is $9\pi \approx 28.27$ square miles.

Step 2 Let x represent the number of people who live in the region. Find the value of x.

$$\text{Population density} = \frac{\text{number of people}}{\text{area of land}} \quad \text{Formula for population density}$$

$$6195 \approx \frac{x}{9\pi} \quad \text{Substitute.}$$

$$175{,}159 \approx x \quad \text{Multiply and use a calculator.}$$

▶ The number of people who live in the region is about 175,159.

Monitoring Progress

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3. About 58,000 people live in a region with a 2-mile radius. Find the population density in people per square mile.

4. A region with a 3-mile radius has a population density of about 1000 people per square mile. Find the number of people who live in the region.

Finding Areas of Sectors

A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram below, sector APB is bounded by $\overline{AP}$, $\overline{BP}$, and $\overset{\frown}{AB}$.

ANALYZING RELATIONSHIPS

The area of a sector is a fractional part of the area of a circle. The area of a sector formed by a 45° arc is $\frac{45°}{360°}$, or $\frac{1}{8}$ of the area of the circle.

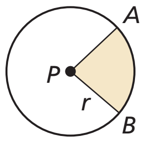
Core Concept

Area of a Sector

The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.

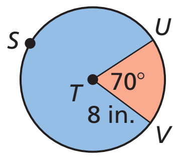
$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360°}, \text{ or}$$

$$\text{Area of sector } APB = \frac{m\overset{\frown}{AB}}{360°} \cdot \pi r^2$$



EXAMPLE 3 **Finding Areas of Sectors**

Find the areas of the sectors formed by $\angle UTV$.



SOLUTION

Step 1 Find the measures of the minor and major arcs.

Because $m\angle UTV = 70°$, $m\overset{\frown}{UV} = 70°$ and $m\overset{\frown}{USV} = 360° - 70° = 290°$.

Step 2 Find the areas of the small and large sectors.

Area of small sector $= \frac{m\overset{\frown}{UV}}{360°} \cdot \pi r^2$ — Formula for area of a sector

$= \frac{70°}{360°} \cdot \pi \cdot 8^2$ — Substitute.

≈ 39.10 — Use a calculator.

Area of large sector $= \frac{m\overset{\frown}{USV}}{360°} \cdot \pi r^2$ — Formula for area of a sector

$= \frac{290°}{360°} \cdot \pi \cdot 8^2$ — Substitute.

≈ 161.97 — Use a calculator.

▶ The areas of the small and large sectors are about 39.10 square inches and about 161.97 square inches, respectively.

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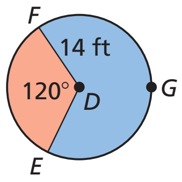
Find the indicated measure.

5. area of red sector

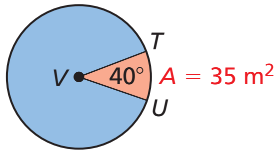
6. area of blue sector



Using Areas of Sectors

EXAMPLE 4 Using the Area of a Sector

Find the area of $\odot V$.



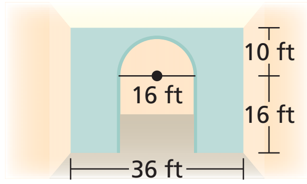
SOLUTION

Area of sector TVU	$= \dfrac{m\overset{\frown}{TU}}{360^\circ} \cdot \text{Area of } \odot V$	Formula for area of a sector
35	$= \dfrac{40^\circ}{360^\circ} \cdot \text{Area of } \odot V$	Substitute.
315	$= \text{Area of } \odot V$	Solve for area of $\odot V$.

▶ The area of $\odot V$ is 315 square meters.

EXAMPLE 5 Finding the Area of a Region

A rectangular wall has an entrance cut into it. You want to paint the wall. To the nearest square foot, what is the area of the region you need to paint?



SOLUTION

The area you need to paint is the area of the rectangle minus the area of the entrance. The entrance can be divided into a semicircle and a square.

Area of wall = Area of rectangle − (Area of semicircle + Area of square)

$$= 36(26) - \left[\frac{180^\circ}{360^\circ} \cdot (\pi \cdot 8^2) + 16^2\right]$$

$$= 936 - (32\pi + 256)$$

$$\approx 579.47$$

▶ The area you need to paint is about 579 square feet.

COMMON ERROR

Use the radius (8 feet), not the diameter (16 feet), when you calculate the area of the semicircle.

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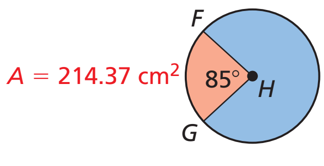
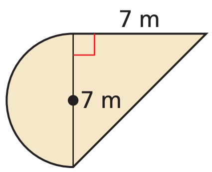
7. Find the area of $\odot H$.



8. Find the area of the figure.



9. If you know the area and radius of a sector of a circle, can you find the measure of the intercepted arc? Explain.

11.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

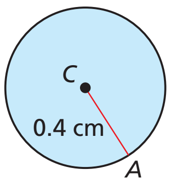
1. **VOCABULARY** A(n) __________ of a circle is the region bounded by two radii of the circle and their intercepted arc.

2. **WRITING** The arc measure of a sector in a given circle is doubled. Will the area of the sector also be doubled? Explain your reasoning.

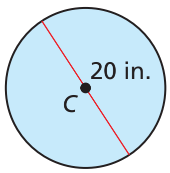
Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, find the indicated measure. *(See Example 1.)*

3. area of ⊙*C*



4. area of ⊙*C*



5. area of a circle with a radius of 5 inches

6. area of a circle with a diameter of 16 feet

7. radius of a circle with an area of 89 square feet

8. radius of a circle with an area of 380 square inches

9. diameter of a circle with an area of 12.6 square inches

10. diameter of a circle with an area of 676π square centimeters

In Exercises 11–14, find the indicated measure. *(See Example 2.)*

11. About 210,000 people live in a region with a 12-mile radius. Find the population density in people per square mile.

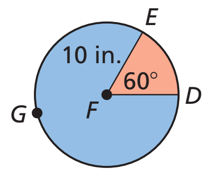
12. About 650,000 people live in a region with a 6-mile radius. Find the population density in people per square mile.

13. A region with a 4-mile radius has a population density of about 6366 people per square mile. Find the number of people who live in the region.

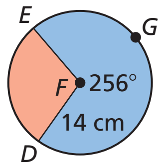
14. About 79,000 people live in a circular region with a population density of about 513 people per square mile. Find the radius of the region.

In Exercises 15–18, find the areas of the sectors formed by ∠*DFE*. *(See Example 3.)*

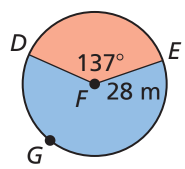
15.



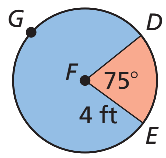
16.



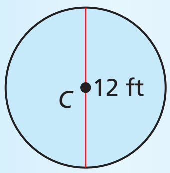
17.



18.



19. **ERROR ANALYSIS** Describe and correct the error in finding the area of the circle.

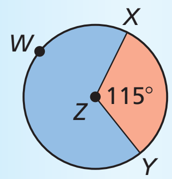


$A = \pi(12)^2$

$= 144\pi$

$\approx 452.39 \text{ ft}^2$

20. **ERROR ANALYSIS** Describe and correct the error in finding the area of sector *XZY* when the area of ⊙*Z* is 255 square feet.



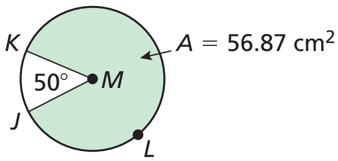
Let *n* be the area of sector XZY.

$\frac{n}{360} = \frac{115}{255}$

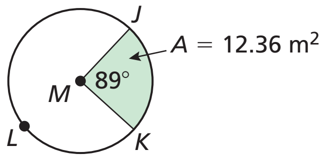
$n \approx 162.35 \text{ ft}^2$

In Exercises 21 and 22, the area of the shaded sector is shown. Find the indicated measure. *(See Example 4.)*

21. area of $\odot M$

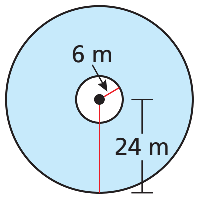


22. radius of $\odot M$



In Exercises 23–28, find the area of the shaded region. *(See Example 5.)*

23.



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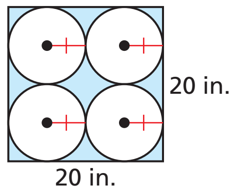

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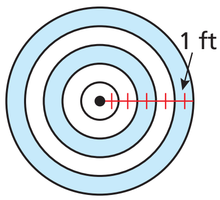

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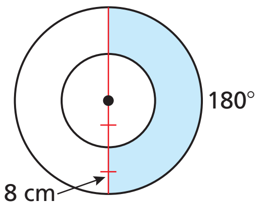
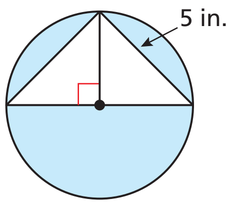
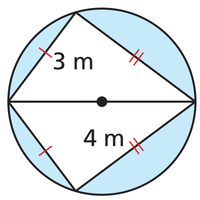


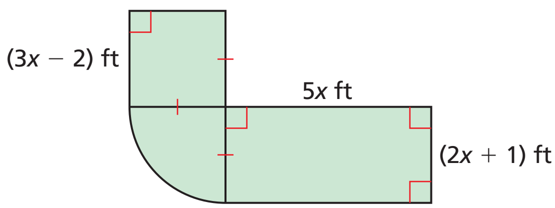
27.



28.

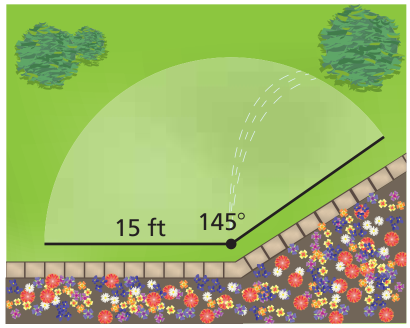


29. **PROBLEM SOLVING** The diagram shows the shape of a putting green at a miniature golf course. One part of the green is a sector of a circle. Find the area of the putting green.



30. **MAKING AN ARGUMENT** Your friend claims that if the radius of a circle is doubled, then its area doubles. Is your friend correct? Explain your reasoning.

31. **MODELING WITH MATHEMATICS** The diagram shows the area of a lawn covered by a water sprinkler.



a. What is the area of the lawn that is covered by the sprinkler?

b. The water pressure is weakened so that the radius is 12 feet. What is the area of the lawn that will be covered?

32. **MODELING WITH MATHEMATICS** The diagram shows a projected beam of light from a lighthouse.

a. What is the area of water that can be covered by the light from the lighthouse?

b. What is the area of land that can be covered by the light from the lighthouse?

33. **ANALYZING RELATIONSHIPS** Look back at the Perimeters of Similar Polygons Theorem (Theorem 8.1) and the Areas of Similar Polygons Theorem (Theorem 8.2) in Section 8.1. How would you rewrite these theorems to apply to circles? Explain your reasoning.

34. **ANALYZING RELATIONSHIPS** A square is inscribed in a circle. The same square is also circumscribed about a smaller circle. Draw a diagram that represents this situation. Then find the ratio of the area of the larger circle to the area of the smaller circle.

35. CONSTRUCTION The table shows how students get to school.

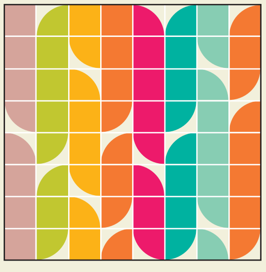
Method	Percent of students
bus	65%
walk	25%
other	10%

a. Explain why a circle graph is appropriate for the data.

b. You will represent each method by a sector of a circle graph. Find the central angle to use for each sector. Then construct the graph using a radius of 2 inches.

c. Find the area of each sector in your graph.

36. HOW DO YOU SEE IT? The outermost edges of the pattern shown form a square. If you know the dimensions of the outer square, is it possible to compute the total colored area? Explain.



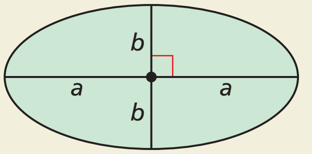
37. ABSTRACT REASONING A circular pizza with a 12-inch diameter is enough for you and 2 friends. You want to buy pizzas for yourself and 7 friends. A 10-inch diameter pizza with one topping costs \$6.99 and a 14-inch diameter pizza with one topping costs \$12.99. How many 10-inch and 14-inch pizzas should you buy in each situation? Explain.

a. You want to spend as little money as possible.

b. You want to have three pizzas, each with a different topping, and spend as little money as possible.

c. You want to have as much of the thick outer crust as possible.

38. THOUGHT PROVOKING You know that the area of a circle is πr^2. Find the formula for the area of an *ellipse*, shown below.



39. MULTIPLE REPRESENTATIONS Consider a circle with a radius of 3 inches.

a. Complete the table, where x is the measure of the arc and y is the area of the corresponding sector. Round your answers to the nearest tenth.

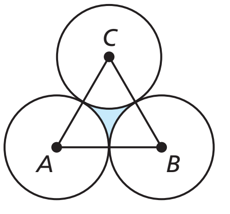
x	30°	60°	90°	120°	150°	180°
y						

b. Graph the data in the table.

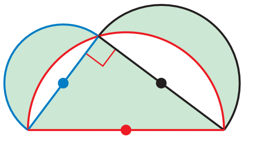
c. Is the relationship between x and y linear? Explain.

d. If parts (a)–(c) were repeated using a circle with a radius of 5 inches, would the areas in the table change? Would your answer to part (c) change? Explain your reasoning.

40. CRITICAL THINKING Find the area between the three congruent tangent circles. The radius of each circle is 6 inches.



41. PROOF Semicircles with diameters equal to three sides of a right triangle are drawn, as shown. Prove that the sum of the areas of the two shaded crescents equals the area of the triangle.

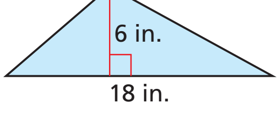


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the area of the figure. *(Skills Review Handbook)*

42.



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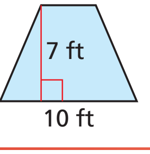

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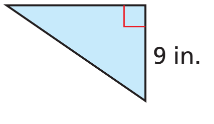

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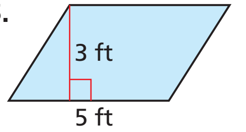
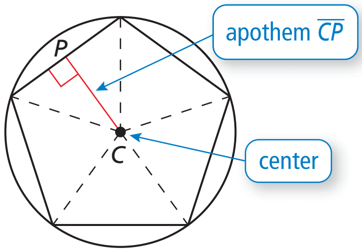


11.3 Areas of Polygons

Essential Question How can you find the area of a regular polygon?

The **center of a regular polygon** is the center of its circumscribed circle.

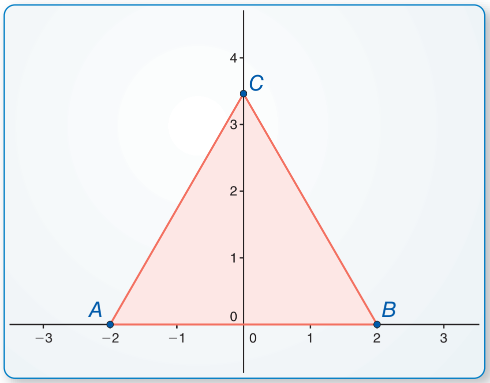
The distance from the center to any side of a regular polygon is called the **apothem of a regular polygon**.



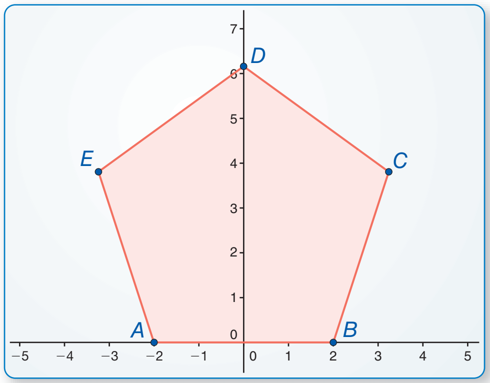
EXPLORATION 1 Finding the Area of a Regular Polygon

Work with a partner. Use dynamic geometry software to construct each regular polygon with side lengths of 4, as shown. Find the apothem and use it to find the area of the polygon. Describe the steps that you used.

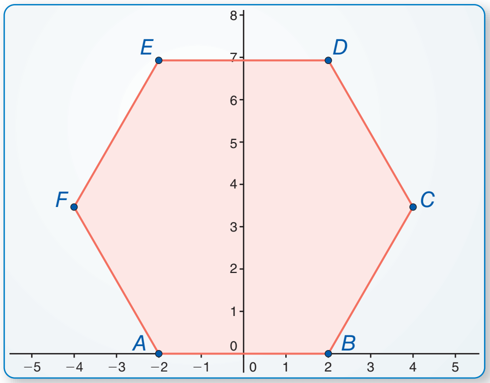
a.



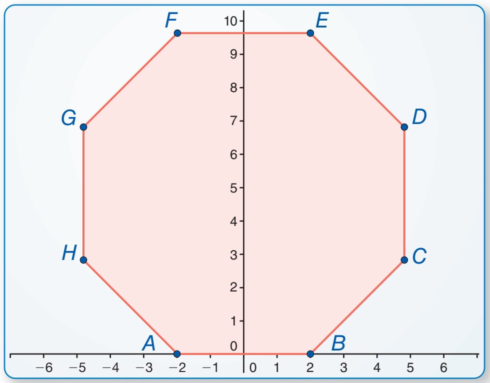
b.



c.



d.



EXPLORATION 2 Writing a Formula for Area

Work with a partner. Generalize the steps you used in Exploration 1 to develop a formula for the area of a regular polygon.

REASONING ABSTRACTLY

To be proficient in math, you need to know and flexibly use different properties of operations and objects.

Communicate Your Answer

3. How can you find the area of a regular polygon?

4. Regular pentagon *ABCDE* has side lengths of 6 meters and an apothem of approximately 4.13 meters. Find the area of *ABCDE*.

11.3 Lesson

Core Vocabulary

center of a regular polygon, *p. 611*
radius of a regular polygon, *p. 611*
apothem of a regular polygon, *p. 611*
central angle of a regular polygon, *p. 611*

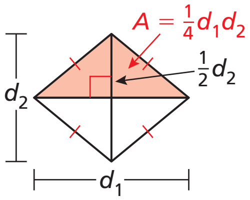
Previous
rhombus
kite

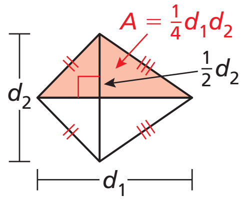
What You Will Learn

- Find areas of rhombuses and kites.
- Find angle measures in regular polygons.
- Find areas of regular polygons.

Finding Areas of Rhombuses and Kites

You can divide a rhombus or kite with diagonals d_1 and d_2 into two congruent triangles with base d_1, height $\frac{1}{2}d_2$, and area $\frac{1}{2}d_1\left(\frac{1}{2}d_2\right) = \frac{1}{4}d_1d_2$. So, the area of a rhombus or kite is $2\left(\frac{1}{4}d_1d_2\right) = \frac{1}{2}d_1d_2$.

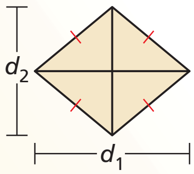


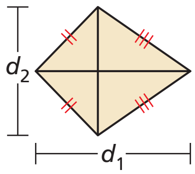


Core Concept

Area of a Rhombus or Kite

The area of a rhombus or kite with diagonals d_1 and d_2 is $\frac{1}{2}d_1d_2$.

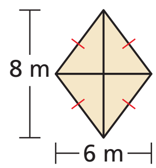




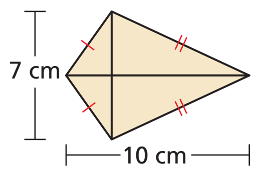
EXAMPLE 1 Finding the Area of a Rhombus or Kite

Find the area of each rhombus or kite.

a.



b.



SOLUTION

a. $A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(6)(8)$

$= 24$

▶ So, the area is 24 square meters.

b. $A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(10)(7)$

$= 35$

▶ So, the area is 35 square centimeters.

Monitoring Progress

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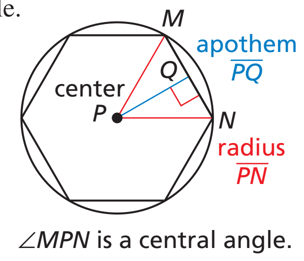
1. Find the area of a rhombus with diagonals $d_1 = 4$ feet and $d_2 = 5$ feet.
2. Find the area of a kite with diagonals $d_1 = 12$ inches and $d_2 = 9$ inches.

Finding Angle Measures in Regular Polygons



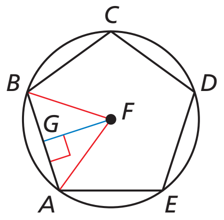
The diagram shows a regular polygon inscribed in a circle. The **center of a regular polygon** and the **radius of a regular polygon** are the center and the radius of its circumscribed circle.

The distance from the center to any side of a regular polygon is called the **apothem of a regular polygon**. The apothem is the height to the base of an isosceles triangle that has two radii as legs. The word "apothem" refers to a segment as well as a length. For a given regular polygon, think of *an* apothem as a segment and *the* apothem as a length.

A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon. To find the measure of each central angle, divide 360° by the number of sides.

EXAMPLE 2 Finding Angle Measures in a Regular Polygon

In the diagram, $ABCDE$ is a regular pentagon inscribed in $\odot F$. Find each angle measure.



a. $m\angle AFB$ **b.** $m\angle AFG$ **c.** $m\angle GAF$

ANALYZING RELATIONSHIPS

$\overline{FG}$ is an altitude of an isosceles triangle, so it is also a median and angle bisector of the isosceles triangle.

SOLUTION

a. $\angle AFB$ is a central angle, so $m\angle AFB = \frac{360°}{5} = 72°$.

b. $\overline{FG}$ is an apothem, which makes it an altitude of isosceles $\triangle AFB$.

So, $\overline{FG}$ bisects $\angle AFB$ and $m\angle AFG = \frac{1}{2}m\angle AFB = 36°$.

c. By the Triangle Sum Theorem (Theorem 5.1), the sum of the angle measures of right $\triangle GAF$ is 180°.

$$m\angle GAF = 180° - 90° - 36°$$
$$= 54°$$

So, $m\angle GAF = 54°$.

Monitoring Progress

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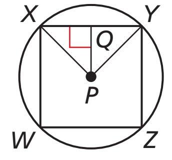
In the diagram, *WXYZ* is a square inscribed in $\odot P$.



3. Identify the center, a radius, an apothem, and a central angle of the polygon.

4. Find $m\angle XPY$, $m\angle XPQ$, and $m\angle PXQ$.

Finding Areas of Regular Polygons

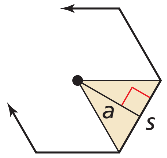
You can find the area of any regular n-gon by dividing it into congruent triangles.

A = Area of one triangle • Number of triangles

$= \left(\frac{1}{2} \cdot s \cdot a\right) \cdot n$ — Base of triangle is s and height of triangle is a. Number of triangles is n.

$= \frac{1}{2} \cdot a \cdot (n \cdot s)$ — Commutative and Associative Properties of Multiplication

$= \frac{1}{2}a \cdot P$ — There are n congruent sides of length s, so perimeter P is $n \cdot s$.



READING DIAGRAMS

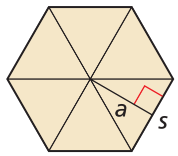
In this book, a point shown inside a regular polygon marks the center of the circle that can be circumscribed about the polygon.

Core Concept

Area of a Regular Polygon

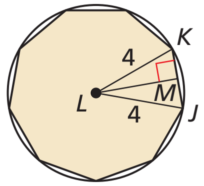
The area of a regular n-gon with side length s is one-half the product of the apothem a and the perimeter P.

$$A = \frac{1}{2}aP, \text{ or } A = \frac{1}{2}a \cdot ns$$



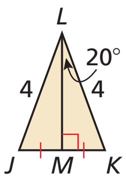
EXAMPLE 3 Finding the Area of a Regular Polygon

A regular nonagon is inscribed in a circle with a radius of 4 units. Find the area of the nonagon.



SOLUTION

The measure of central $\angle JLK$ is $\frac{360^\circ}{9}$, or 40°. Apothem $\overline{LM}$ bisects the central angle, so $m\angle KLM$ is 20°. To find the lengths of the legs, use trigonometric ratios for right $\triangle KLM$.



$$\sin 20^\circ = \frac{MK}{LK} \qquad \cos 20^\circ = \frac{LM}{LK}$$

$$\sin 20^\circ = \frac{MK}{4} \qquad \cos 20^\circ = \frac{LM}{4}$$

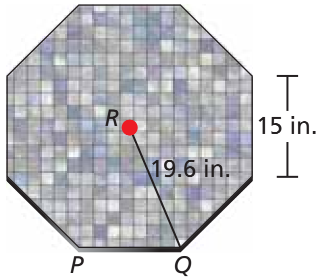
$$4 \sin 20^\circ = MK \qquad 4 \cos 20^\circ = LM$$

The regular nonagon has side length $s = 2(MK) = 2(4 \sin 20^\circ) = 8 \sin 20^\circ$, and apothem $a = LM = 4 \cos 20^\circ$.

▶ So, the area is $A = \frac{1}{2}a \cdot ns = \frac{1}{2}(4 \cos 20^\circ) \cdot (9)(8 \sin 20^\circ) \approx 46.3$ square units.

EXAMPLE 4 Finding the Area of a Regular Polygon

You are decorating the top of a table by covering it with small ceramic tiles. The tabletop is a regular octagon with 15-inch sides and a radius of about 19.6 inches. What is the area you are covering?



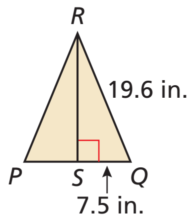
SOLUTION

Step 1 Find the perimeter P of the tabletop.
An octagon has 8 sides, so $P = 8(15) = 120$ inches.

Step 2 Find the apothem a. The apothem is height RS of $\triangle PQR$.

Because $\triangle PQR$ is isosceles, altitude $\overline{RS}$ bisects $\overline{QP}$.

So, $QS = \frac{1}{2}(QP) = \frac{1}{2}(15) = 7.5$ inches.



To find RS, use the Pythagorean Theorem (Theorem 9.1) for $\triangle RQS$.

$$a = RS = \sqrt{19.6^2 - 7.5^2} = \sqrt{327.91} \approx 18.108$$

Step 3 Find the area A of the tabletop.

$A = \frac{1}{2}aP$	Formula for area of a regular polygon
$= \frac{1}{2}(\sqrt{327.91})(120)$	Substitute.
≈ 1086.5	Simplify.

▶ The area you are covering with tiles is about 1086.5 square inches.

Monitoring Progress

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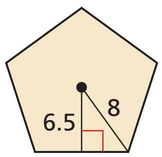
Find the area of the regular polygon.

5.



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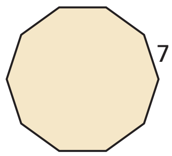


11.3 Exercises

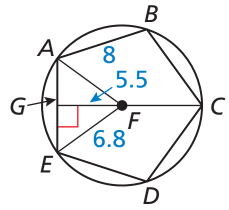
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how to find the measure of a central angle of a regular polygon.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

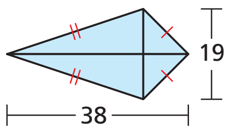
Find the radius of $\odot F$.	Find the apothem of polygon *ABCDE*.
Find *AF*.	Find the radius of polygon *ABCDE*.



Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the area of the kite or rhombus. *(See Example 1.)*

3.



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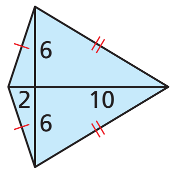

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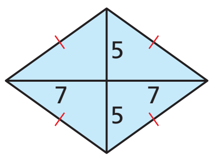

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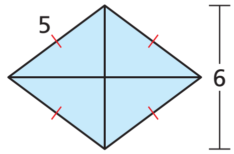
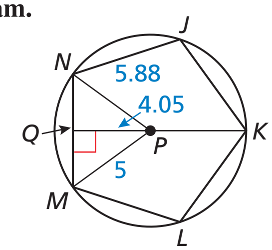


In Exercises 7–10, use the diagram.



7. Identify the center of polygon *JKLMN*.

8. Identify a central angle of polygon *JKLMN*.

9. What is the radius of polygon *JKLMN?*

10. What is the apothem of polygon *JKLMN*?

In Exercises 11–14, find the measure of a central angle of a regular polygon with the given number of sides. Round answers to the nearest tenth of a degree, if necessary.

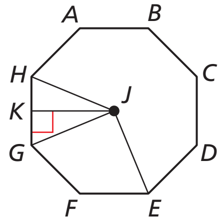
11. 10 sides

12. 18 sides

13. 24 sides

14. 7 sides

In Exercises 15–18, find the given angle measure for regular octagon *ABCDEFGH*. *(See Example 2.)*



15. $m\angle GJH$

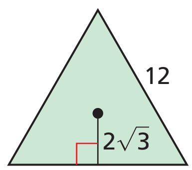
16. $m\angle GJK$

17. $m\angle KGJ$

18. $m\angle EJH$

In Exercises 19–24, find the area of the regular polygon. *(See Examples 3 and 4.)*

19.



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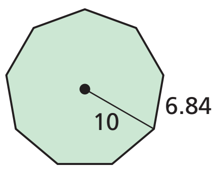

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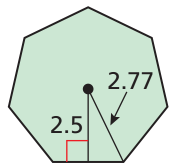
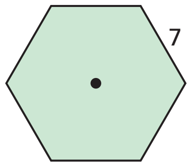


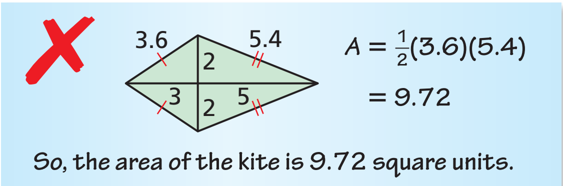
22.



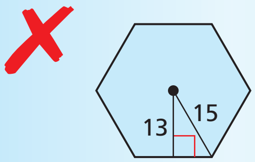
23. an octagon with a radius of 11 units

24. a pentagon with an apothem of 5 units

25. **ERROR ANALYSIS** Describe and correct the error in finding the area of the kite.



26. ERROR ANALYSIS Describe and correct the error in finding the area of the regular hexagon.



$s = \sqrt{15^2 - 13^2} \approx 7.5$

$A = \frac{1}{2}a \cdot ns$

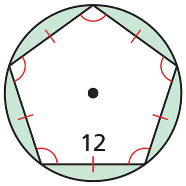
$\approx \frac{1}{2}(13)(6)(7.5)$

$= 292.5$

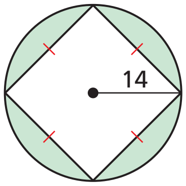
So, the area of the hexagon is about 292.5 square units.

In Exercises 27–30, find the area of the shaded region.

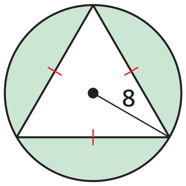
27.



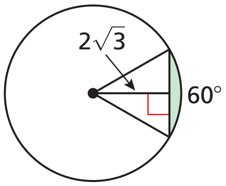
28.



29.

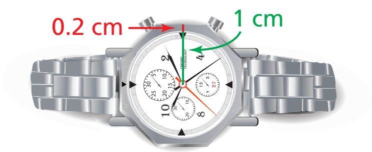


30.



31. MODELING WITH MATHEMATICS Basaltic columns are geological formations that result from rapidly cooling lava. Giant's Causeway in Ireland contains many hexagonal basaltic columns. Suppose the top of one of the columns is in the shape of a regular hexagon with a radius of 8 inches. Find the area of the top of the column to the nearest square inch.

32. MODELING WITH MATHEMATICS A watch has a circular surface on a background that is a regular octagon. Find the area of the octagon. Then find the area of the silver border around the circular face.



CRITICAL THINKING In Exercises 33–35, tell whether the statement is *true* or *false*. Explain your reasoning.

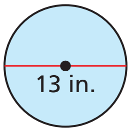
33. The area of a regular n-gon of a fixed radius r increases as n increases.

34. The apothem of a regular polygon is always less than the radius.

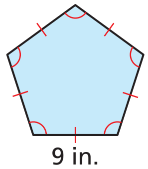
35. The radius of a regular polygon is always less than the side length.

36. REASONING Predict which figure has the greatest area and which has the least area. Explain your reasoning. Check by finding the area of each figure.

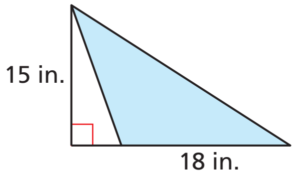
(A)



(B)



(C)



37. USING EQUATIONS Find the area of a regular pentagon inscribed in a circle whose equation is given by $(x - 4)^2 + (y + 2)^2 = 25$.

38. REASONING What happens to the area of a kite if you double the length of one of the diagonals? if you double the length of both diagonals? Justify your answer.

MATHEMATICAL CONNECTIONS In Exercises 39 and 40, write and solve an equation to find the indicated lengths. Round decimal answers to the nearest tenth.

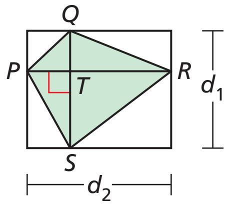
39. The area of a kite is 324 square inches. One diagonal is twice as long as the other diagonal. Find the length of each diagonal.

40. One diagonal of a rhombus is four times the length of the other diagonal. The area of the rhombus is 98 square feet. Find the length of each diagonal.

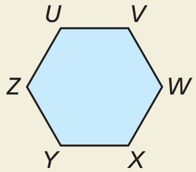
41. REASONING The perimeter of a regular nonagon, or 9-gon, is 18 inches. Is this enough information to find the area? If so, find the area and explain your reasoning. If not, explain why not.

42. MAKING AN ARGUMENT Your friend claims that it is possible to find the area of any rhombus if you only know the perimeter of the rhombus. Is your friend correct? Explain your reasoning.

43. PROOF Prove that the area of any quadrilateral with perpendicular diagonals is $A = \frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of the diagonals.



44. HOW DO YOU SEE IT? Explain how to find the area of the regular hexagon by dividing the hexagon into equilateral triangles.



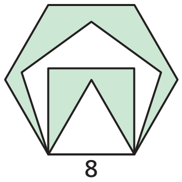
45. REWRITING A FORMULA Rewrite the formula for the area of a rhombus for the special case of a square with side length s. Show that this is the same as the formula for the area of a square, $A = s^2$.

46. REWRITING A FORMULA Use the formula for the area of a regular polygon to show that the area of an equilateral triangle can be found by using the formula $A = \frac{1}{4}s^2\sqrt{3}$, where s is the side length.

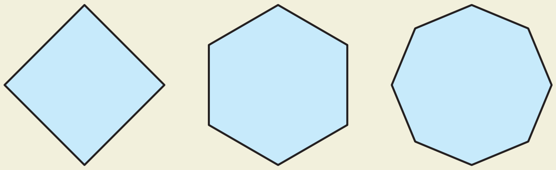
47. CRITICAL THINKING The area of a regular pentagon is 72 square centimeters. Find the length of one side.

48. CRITICAL THINKING The area of a dodecagon, or 12-gon, is 140 square inches. Find the apothem of the polygon.

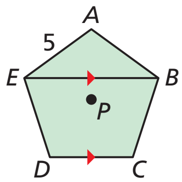
49. USING STRUCTURE In the figure, an equilateral triangle lies inside a square inside a regular pentagon inside a regular hexagon. Find the approximate area of the entire shaded region to the nearest whole number.



50. THOUGHT PROVOKING The area of a regular n-gon is given by $A = \frac{1}{2}aP$. As n approaches infinity, what does the n-gon approach? What does P approach? What does a approach? What can you conclude from your three answers? Explain your reasoning.



51. COMPARING METHODS Find the area of regular pentagon $ABCDE$ by using the formula $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \bullet ns$. Then find the area by adding the areas of smaller polygons. Check that both methods yield the same area. Which method do you prefer? Explain your reasoning.



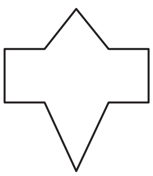
52. USING STRUCTURE Two regular polygons both have n sides. One of the polygons is inscribed in, and the other is circumscribed about, a circle of radius r. Find the area between the two polygons in terms of n and r.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the figure has *line symmetry*, *rotational symmetry*, *both*, or *neither*. If the figure has line symmetry, determine the number of lines of symmetry. If the figure has rotational symmetry, describe any rotations that map the figure onto itself. *(Section 4.2 and Section 4.3)*

53.



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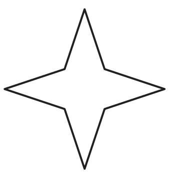

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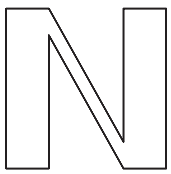

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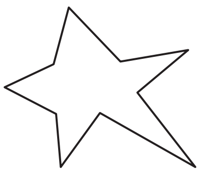
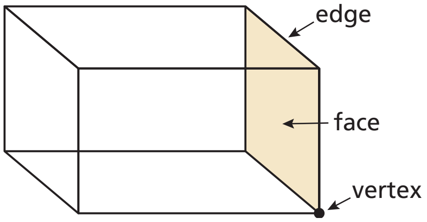


11.4 Three-Dimensional Figures

Essential Question What is the relationship between the numbers of vertices V, edges E, and faces F of a polyhedron?

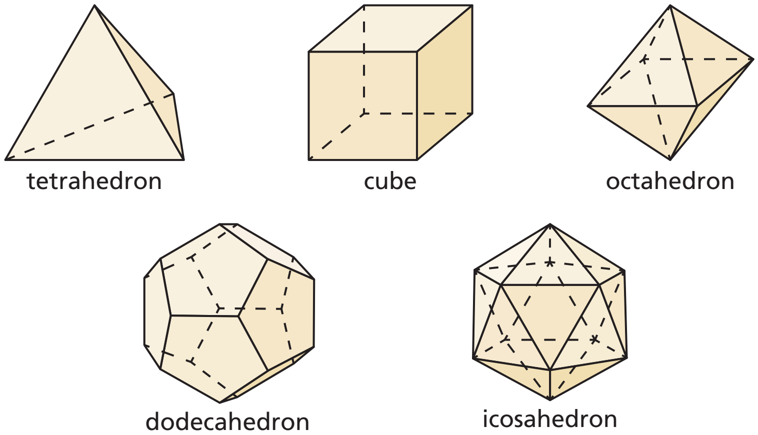
A **polyhedron** is a solid that is bounded by polygons, called **faces**.

- Each *vertex* is a point.
- Each *edge* is a segment of a line.
- Each *face* is a portion of a plane.



EXPLORATION 1 Analyzing a Property of Polyhedra

Work with a partner. The five *Platonic solids* are shown below. Each of these solids has congruent regular polygons as faces. Complete the table by listing the numbers of vertices, edges, and faces of each Platonic solid.



Solid	Vertices, V	Edges, E	Faces, F
tetrahedron			
cube			
octahedron			
dodecahedron			
icosahedron			

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to reason inductively about data.

Communicate Your Answer

2. What is the relationship between the numbers of vertices V, edges E, and faces F of a polyhedron? (*Note*: Swiss mathematician Leonhard Euler (1707–1783) discovered a formula that relates these quantities.)

3. Draw three polyhedra that are different from the Platonic solids given in Exploration 1. Count the numbers of vertices, edges, and faces of each polyhedron. Then verify that the relationship you found in Question 2 is valid for each polyhedron.

11.4 Lesson

Core Vocabulary

polyhedron, *p. 618*
face, *p. 618*
edge, *p. 618*
vertex, *p. 618*
cross section, *p. 619*
solid of revolution, *p. 620*
axis of revolution, *p. 620*

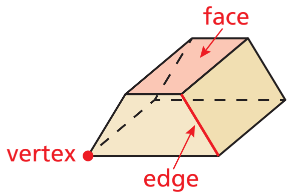
Previous
solid
prism
pyramid
cylinder
cone
sphere
base

What You Will Learn

- Classify solids.
- Describe cross sections.
- Sketch and describe solids of revolution.

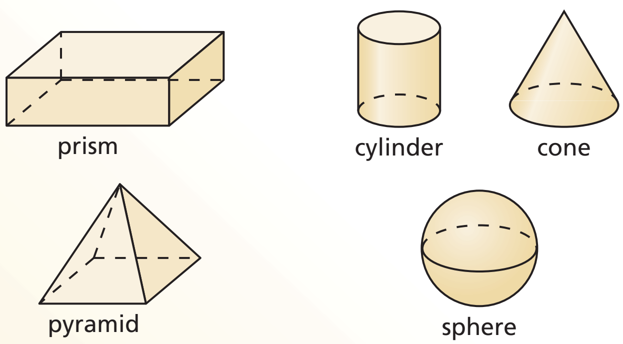
Classifying Solids

A three-dimensional figure, or solid, is bounded by flat or curved surfaces that enclose a single region of space. A **polyhedron** is a solid that is bounded by polygons, called **faces**. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.

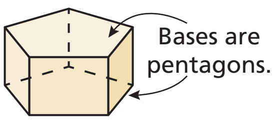


Core Concept

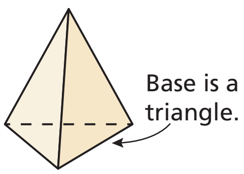
Types of Solids



Pentagonal prism



Triangular pyramid

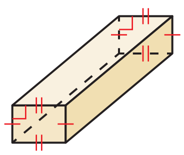


To name a prism or a pyramid, use the shape of the *base.* The two bases of a prism are congruent polygons in parallel planes. For example, the bases of a pentagonal prism are pentagons. The base of a pyramid is a polygon. For example, the base of a triangular pyramid is a triangle.

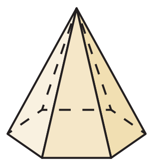
EXAMPLE 1 Classifying Solids

Tell whether each solid is a polyhedron. If it is, name the polyhedron.

a.



b.



c.

SOLUTION

a. The solid is formed by polygons, so it is a polyhedron. The two bases are congruent rectangles, so it is a rectangular prism.

b. The solid is formed by polygons, so it is a polyhedron. The base is a hexagon, so it is a hexagonal pyramid.

c. The cone has a curved surface, so it is not a polyhedron.

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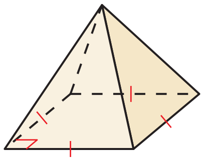
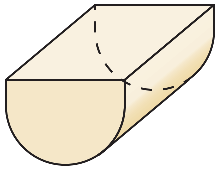
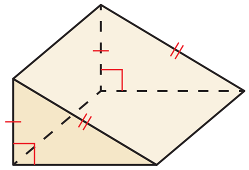
Tell whether the solid is a polyhedron. If it is, name the polyhedron.

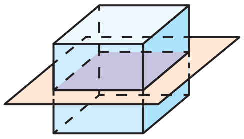
1. 

2. 

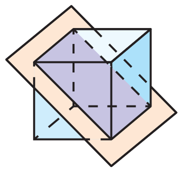
3. 

Describing Cross Sections

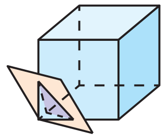
Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, three different cross sections of a cube are shown below.



square



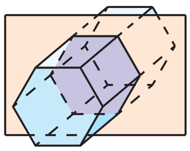
rectangle

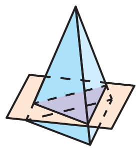


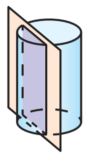
triangle

EXAMPLE 2 Describing Cross Sections

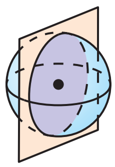
Describe the shape formed by the intersection of the plane and the solid.

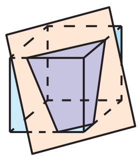
a. 

b. 

c. 

d. 

e. 

f. 

SOLUTION

a. The cross section is a hexagon.

b. The cross section is a triangle.

c. The cross section is a rectangle.

d. The cross section is a circle.

e. The cross section is a circle.

f. The cross section is a trapezoid.

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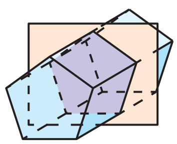
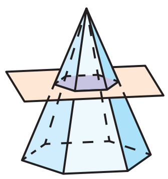
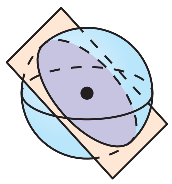
Describe the shape formed by the intersection of the plane and the solid.

4. 

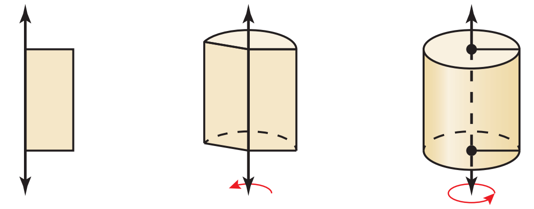
5. 

6. 

Sketching and Describing Solids of Revolution

A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis. The line around which the shape is rotated is called the **axis of revolution**.

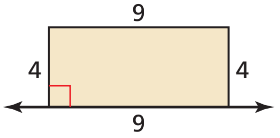
For example, when you rotate a rectangle around a line that contains one of its sides, the solid of revolution that is produced is a cylinder.



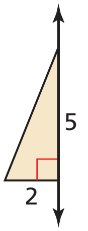
EXAMPLE 3 Sketching and Describing Solids of Revolution

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

a.

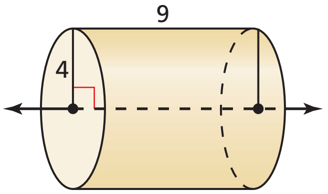


b.



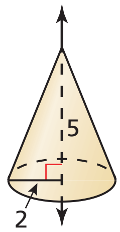
SOLUTION

a.



▶ The solid is a cylinder with a height of 9 and a base radius of 4.

b.



▶ The solid is a cone with a height of 5 and a base radius of 2.

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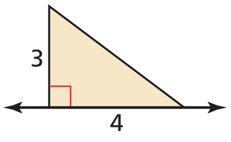
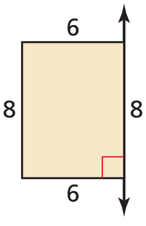
Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

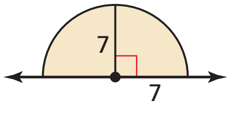
7.



8.



9.



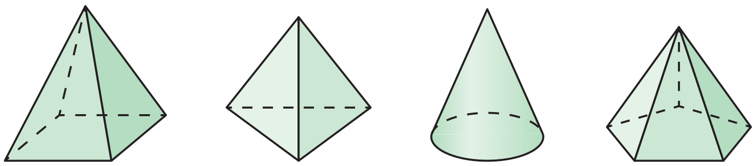
11.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

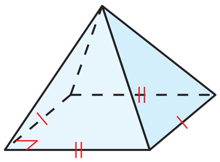
1. **VOCABULARY** A(n) ___________ is a solid that is bounded by polygons.

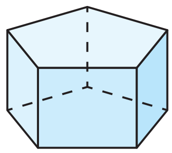
2. **WHICH ONE DOESN'T BELONG?** Which solid does *not* belong with the other three? Explain your reasoning.

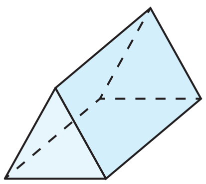


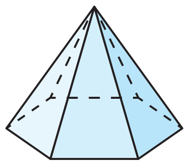
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, match the polyhedron with its name.

3. 

4. 

5. 

6. 

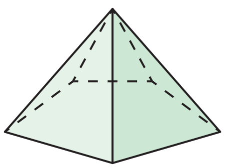
A. triangular prism

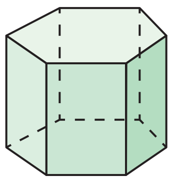
B. rectangular pyramid

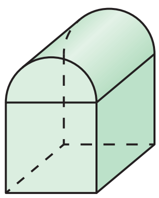
C. hexagonal pyramid

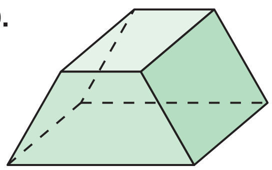
D. pentagonal prism

In Exercises 7–10, tell whether the solid is a polyhedron. If it is, name the polyhedron. *(See Example 1.)*

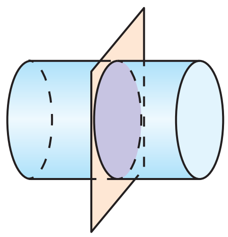
7. 

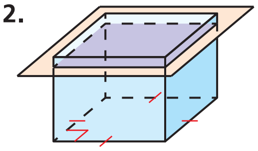
8. 

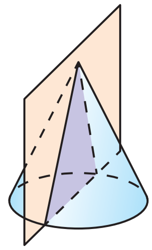
9. 

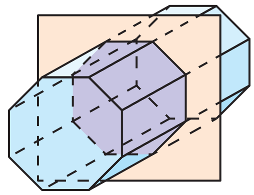
10. 

In Exercises 11–14, describe the cross section formed by the intersection of the plane and the solid. *(See Example 2.)*

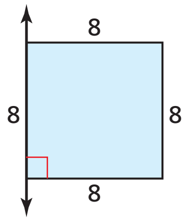
11. 

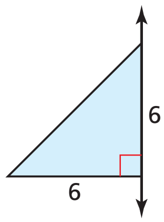
12. 

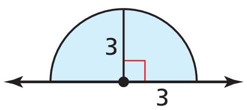
13. 

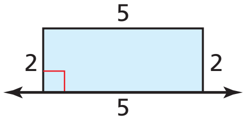
14. 

In Exercises 15–18, sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid. *(See Example 3.)*

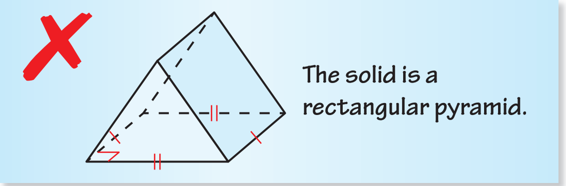
15. 

16. 

17. 

18. 

19. **ERROR ANALYSIS** Describe and correct the error in identifying the solid.

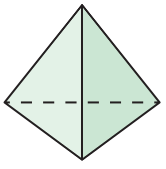


20. **HOW DO YOU SEE IT?** Is the swimming pool shown a polyhedron? If it is, name the polyhedron. If not, explain why not.

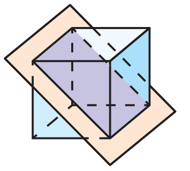
In Exercises 21–26, sketch the polyhedron.

21. triangular prism
22. rectangular prism
23. pentagonal prism
24. hexagonal prism
25. square pyramid
26. pentagonal pyramid

27. **MAKING AN ARGUMENT** Your friend says that the polyhedron shown is a triangular prism. Your cousin says that it is a triangular pyramid. Who is correct? Explain your reasoning.



28. **ATTENDING TO PRECISION** The figure shows a plane intersecting a cube through four of its vertices. The edge length of the cube is 6 inches.



a. Describe the shape of the cross section.

b. What is the perimeter of the cross section?

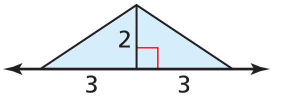
c. What is the area of the cross section?

REASONING **In Exercises 29–34, tell whether it is possible for a cross section of a cube to have the given shape. If it is, describe or sketch how the plane could intersect the cube.**

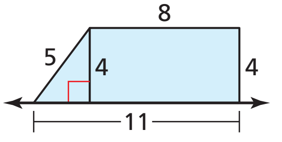
29. circle
30. pentagon
31. rhombus
32. isosceles triangle
33. hexagon
34. scalene triangle

35. **REASONING** Sketch the composite solid produced by rotating the figure around the given axis. Then identify and describe the composite solid.

a.



b.



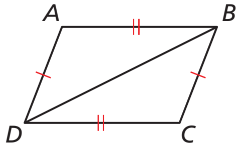
36. **THOUGHT PROVOKING** Describe how Plato might have argued that there are precisely five *Platonic Solids* (see page 617). (*Hint*: Consider the angles that meet at a vertex.)

Maintaining Mathematical Proficiency

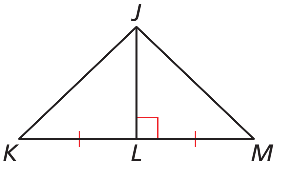
Reviewing what you learned in previous grades and lessons

Decide whether enough information is given to prove that the triangles are congruent. If so, state the theorem you would use. *(Sections 5.3, 5.5, and 5.6)*

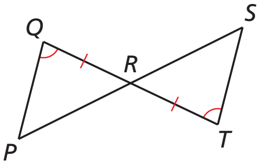
37. $\triangle ABD$, $\triangle CDB$



38. $\triangle JLK$, $\triangle JLM$



39. $\triangle RQP$, $\triangle RTS$



11.1–11.4 What Did You Learn?

Core Vocabulary

circumference, *p. 594*
arc length, *p. 595*
radian, *p. 597*
population density, *p. 603*
sector of a circle, *p. 604*
center of a regular polygon, *p. 611*
radius of a regular polygon, *p. 611*
apothem of a regular polygon, *p. 611*
central angle of a regular polygon, *p. 611*
polyhedron, *p. 618*
face, *p. 618*
edge, *p. 618*
vertex, *p. 618*
cross section, *p. 619*
solid of revolution, *p. 620*
axis of revolution, *p. 620*

Core Concepts

Section 11.1

Circumference of a Circle, *p. 594*
Arc Length, *p. 595*
Converting between Degrees and Radians, *p. 597*

Section 11.2

Area of a Circle, *p. 602*
Population Density, *p. 603*
Area of a Sector, *p. 604*

Section 11.3

Area of a Rhombus or Kite, *p. 610*
Area of a Regular Polygon, *p. 612*

Section 11.4

Types of Solids, *p. 618*
Cross Section of a Solid, *p. 619*
Solids of Revolution, *p. 620*

Mathematical Practices

1. In Exercise 13 on page 598, why does it matter how many revolutions the wheel makes?
2. Your friend is confused with Exercise 19 on page 606. What question(s) could you ask your friend to help them figure it out?
3. In Exercise 38 on page 615, write a proof to support your answer.

Study Skills

Kinesthetic Learners

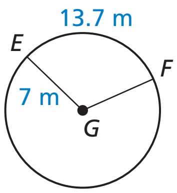
Incorporate physical activity.

- Act out a word problem as much as possible. Use props when you can.
- Solve a word problem on a large whiteboard. The physical action of writing is more kinesthetic when the writing is larger and you can move around while doing it.
- Make a review card.

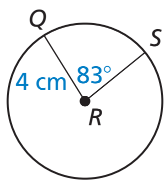
11.1–11.4 Quiz

Find the indicated measure. *(Section 11.1)*

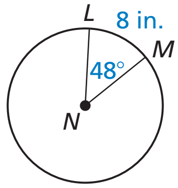
1. $m\widehat{EF}$



2. arc length of $\widehat{QS}$



3. circumference of $\odot N$

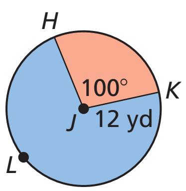


4. Convert 26° to radians and $\frac{5\pi}{9}$ radians to degrees. *(Section 11.1)*

Use the figure to find the indicated measure. *(Section 11.2)*

5. area of red sector

6. area of blue sector

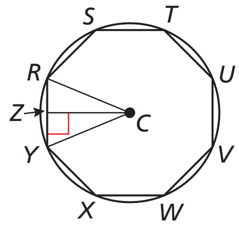


In the diagram, *RSTUVWXY* is a regular octagon inscribed in $\odot C$. *(Section 11.3)*

7. Identify the center, a radius, an apothem, and a central angle of the polygon.

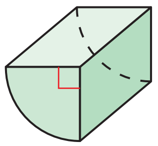
8. Find $m\angle RCY$, $m\angle RCZ$, and $m\angle ZRC$.

9. The radius of the circle is 8 units. Find the area of the octagon.

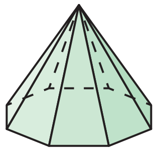


Tell whether the solid is a polyhedron. If it is, name the polyhedron. *(Section 11.4)*

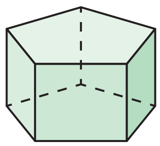
10.

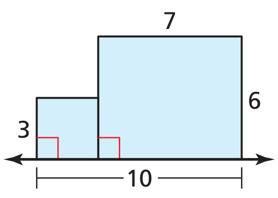


11.



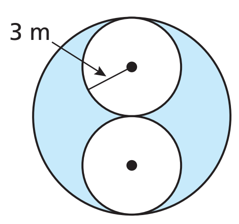
12.



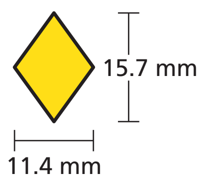


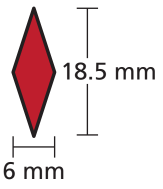
13. Sketch the composite solid produced by rotating the figure around the given axis. Then identify and describe the composite solid. *(Section 11.4)*

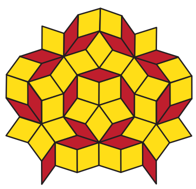
14. The two white congruent circles just fit into the blue circle. What is the area of the blue region? *(Section 11.2)*



15. Find the area of each rhombus tile. Then find the area of the pattern. *(Section 11.3)*





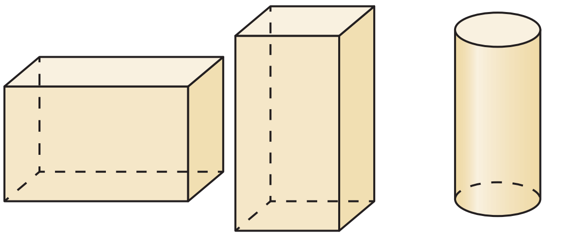


11.5 Volumes of Prisms and Cylinders

Essential Question How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

Recall that the volume V of a right prism or a right cylinder is equal to the product of the area of a base B and the height h.

$$V = Bh$$



EXPLORATION 1 **Finding Volume**

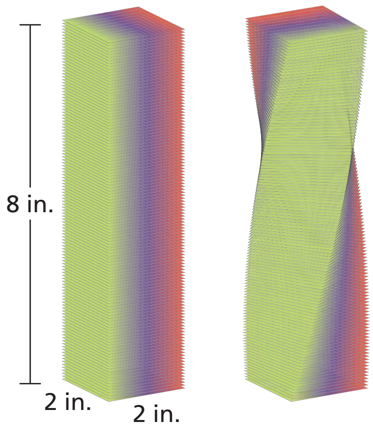
Work with a partner. Consider a stack of square papers that is in the form of a right prism.

a. What is the volume of the prism?

b. When you twist the stack of papers, as shown at the right, do you change the volume? Explain your reasoning.

c. Write a carefully worded conjecture that describes the conclusion you reached in part (b).

d. Use your conjecture to find the volume of the twisted stack of papers.



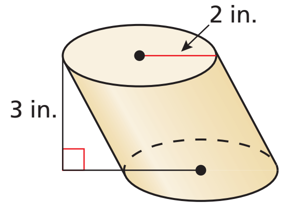
ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely to others.

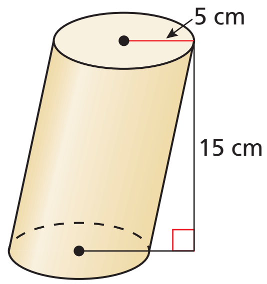
EXPLORATION 2 **Finding Volume**

Work with a partner. Use the conjecture you wrote in Exploration 1 to find the volume of the cylinder.

a.



b.



Communicate Your Answer

3. How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

4. In Exploration 1, would the conjecture you wrote change if the papers in each stack were not squares? Explain your reasoning.

11.5 Lesson

Core Vocabulary

volume, *p. 626*
Cavalieri's Principle, *p. 626*
density, *p. 628*
similar solids, *p. 630*

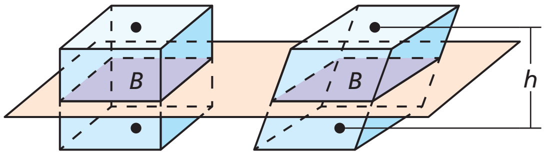
Previous
prism
cylinder
composite solid

What You Will Learn

- Find volumes of prisms and cylinders.
- Use the formula for density.
- Use volumes of prisms and cylinders.

Finding Volumes of Prisms and Cylinders

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3). **Cavalieri's Principle**, named after Bonaventura Cavalieri (1598–1647), states that if two solids have the same height and the same cross-sectional area at every level, then they have the same volume. The prisms below have equal heights h and equal cross-sectional areas B at every level. By Cavalieri's Principle, the prisms have the same volume.



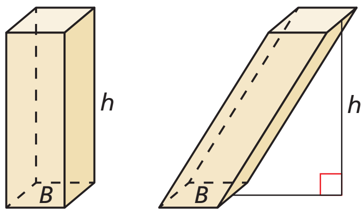
Core Concept

Volume of a Prism

The volume V of a prism is

$V = Bh$

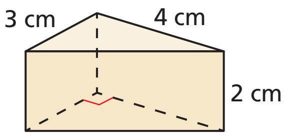
where B is the area of a base and h is the height.



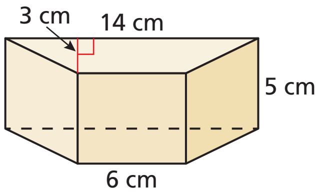
EXAMPLE 1 Finding Volumes of Prisms

Find the volume of each prism.

a.



b.



SOLUTION

a. The area of a base is $B = \frac{1}{2}(3)(4) = 6\ cm^2$ and the height is $h = 2$ cm.

$V = Bh$	Formula for volume of a prism
$= 6(2)$	Substitute.
$= 12$	Simplify.

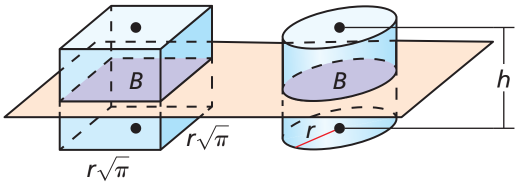
The volume is 12 cubic centimeters.

b. The area of a base is $B = \frac{1}{2}(3)(6 + 14) = 30\ cm^2$ and the height is $h = 5$ cm.

$V = Bh$	Formula for volume of a prism
$= 30(5)$	Substitute.
$= 150$	Simplify.

The volume is 150 cubic centimeters.

Consider a cylinder with height h and base radius r and a rectangular prism with the same height that has a square base with sides of length $r\sqrt{\pi}$.



The cylinder and the prism have the same cross-sectional area, πr^2, at every level and the same height. By Cavalieri's Principle, the prism and the cylinder have the same volume. The volume of the prism is $V = Bh = \pi r^2 h$, so the volume of the cylinder is also $V = Bh = \pi r^2 h$.

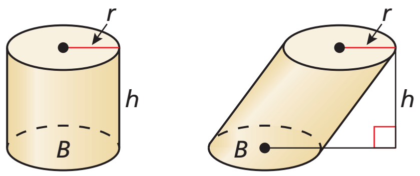
Core Concept

Volume of a Cylinder

The volume V of a cylinder is

$$V = Bh = \pi r^2 h$$

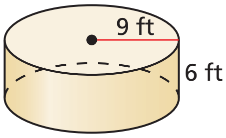
where B is the area of a base, h is the height, and r is the radius of a base.



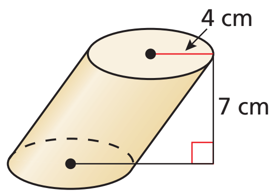
EXAMPLE 2 Finding Volumes of Cylinders

Find the volume of each cylinder.

a.



b.



SOLUTION

a. The dimensions of the cylinder are $r = 9$ ft and $h = 6$ ft.

$$V = \pi r^2 h = \pi(9)^2(6) = 486\pi \approx 1526.81$$

▶ The volume is 486π, or about 1526.81 cubic feet.

b. The dimensions of the cylinder are $r = 4$ cm and $h = 7$ cm.

$$V = \pi r^2 h = \pi(4)^2(7) = 112\pi \approx 351.86$$

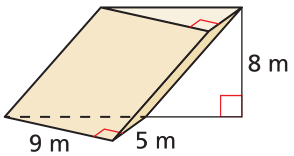
▶ The volume is 112π, or about 351.86 cubic centimeters.

Monitoring Progress

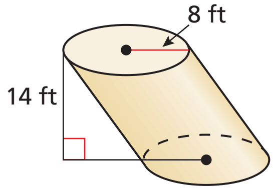
Help in English and Spanish at *BigIdeasMath.com*

Find the volume of the solid.

1.



2.



Using the Formula for Density

Density is the amount of matter that an object has in a given unit of volume. The density of an object is calculated by dividing its mass by its volume.

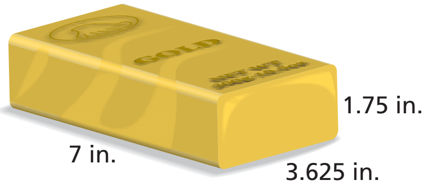
$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Different materials have different densities, so density can be used to distinguish between materials that look similar. For example, table salt and sugar look alike. However, table salt has a density of 2.16 grams per cubic centimeter, while sugar has a density of 1.58 grams per cubic centimeter.

According to the U.S. Mint, Fort Knox houses about 9.2 million pounds of gold.

EXAMPLE 3 Using the Formula for Density

The diagram shows the dimensions of a standard gold bar at Fort Knox. Gold has a density of 19.3 grams per cubic centimeter. Find the mass of a standard gold bar to the nearest gram.



SOLUTION

Step 1 Convert the dimensions to centimeters using 1 inch = 2.54 centimeters.

Length $7 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 17.78 \text{ cm}$

Width $3.625 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 9.2075 \text{ cm}$

Height $1.75 \text{ in.} \cdot \frac{2.54 \text{ cm}}{1 \text{ in.}} = 4.445 \text{ cm}$

Step 2 Find the volume.

The area of a base is $B = 17.78(9.2075) = 163.70935 \text{ cm}^2$ and the height is $h = 4.445$ cm.

$$V = Bh = 163.70935(4.445) \approx 727.69 \text{ cm}^3$$

Step 3 Let x represent the mass in grams. Substitute the values for the volume and the density in the formula for density and solve for x.

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$ Formula for density

$19.3 \approx \frac{x}{727.69}$ Substitute.

$14{,}044 \approx x$ Multiply each side by 727.69.

▶ The mass of a standard gold bar is about 14,044 grams.

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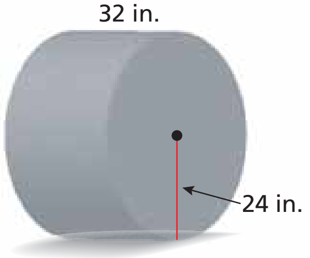
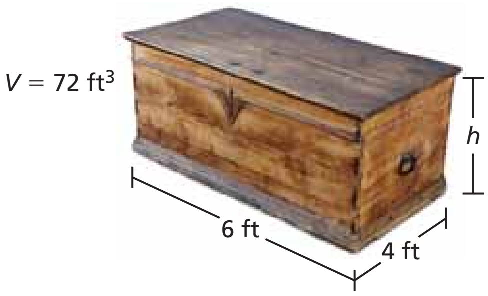
3. The diagram shows the dimensions of a concrete cylinder. Concrete has a density of 2.3 grams per cubic centimeter. Find the mass of the concrete cylinder to the nearest gram.



Using Volumes of Prisms and Cylinders

EXAMPLE 4 Modeling with Mathematics

You are building a rectangular chest. You want the length to be 6 feet, the width to be 4 feet, and the volume to be 72 cubic feet. What should the height be?



SOLUTION

1. **Understand the Problem** You know the dimensions of the base of a rectangular prism and the volume. You are asked to find the height.

2. **Make a Plan** Write the formula for the volume of a rectangular prism, substitute known values, and solve for the height h.

3. **Solve the Problem** The area of a base is $B = 6(4) = 24$ ft^2 and the volume is $V = 72$ ft^3.

$V = Bh$ — Formula for volume of a prism

$72 = 24h$ — Substitute.

$3 = h$ — Divide each side by 24.

▶ The height of the chest should be 3 feet.

4. **Look Back** Check your answer.

$V = Bh = 24(3) = 72$ ✓

EXAMPLE 5 Solving a Real-Life Problem

You are building a 6-foot-tall dresser. You want the volume to be 36 cubic feet. What should the area of the base be? Give a possible length and width.

SOLUTION

$V = Bh$ — Formula for volume of a prism

$36 = B \cdot 6$ — Substitute.

$6 = B$ — Divide each side by 6.

▶ The area of the base should be 6 square feet. The length could be 3 feet and the width could be 2 feet.

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4. **WHAT IF?** In Example 4, you want the length to be 5 meters, the width to be 3 meters, and the volume to be 60 cubic meters. What should the height be?

5. **WHAT IF?** In Example 5, you want the height to be 5 meters and the volume to be 75 cubic meters. What should the area of the base be? Give a possible length and width.

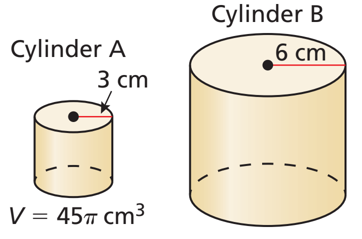
Core Concept

Similar Solids

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The ratio of the corresponding linear measures of two similar solids is called the *scale factor*. If two similar solids have a scale factor of k, then the ratio of their volumes is equal to k^3.

EXAMPLE 6 Finding the Volume of a Similar Solid

Cylinder A and cylinder B are similar. Find the volume of cylinder B.



SOLUTION

The scale factor is $k = \dfrac{\text{Radius of cylinder B}}{\text{Radius of cylinder A}}$

$= \dfrac{6}{3} = 2.$

Use the scale factor to find the volume of cylinder B.

$$\frac{\text{Volume of cylinder B}}{\text{Volume of cylinder A}} = k^3 \qquad \text{The ratio of the volumes is } k^3.$$

$$\frac{\text{Volume of cylinder B}}{45\pi} = 2^3 \qquad \text{Substitute.}$$

$$\text{Volume of cylinder B} = 360\pi \qquad \text{Solve for volume of cylinder B.}$$

▶ The volume of cylinder B is 360π cubic centimeters.

COMMON ERROR

Be sure to write the ratio of the volumes in the same order you wrote the ratio of the radii.

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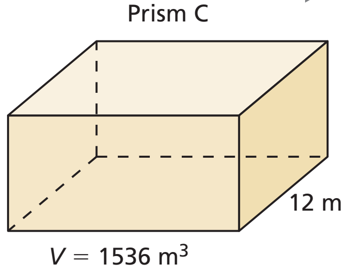
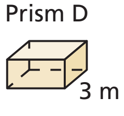
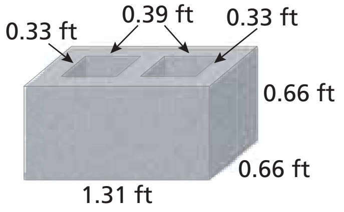




6. Prism C and prism D are similar. Find the volume of prism D.

EXAMPLE 7 Finding the Volume of a Composite Solid

Find the volume of the concrete block.



SOLUTION

To find the area of the base, subtract two times the area of the small rectangle from the large rectangle.

$B = $ Area of large rectangle $- 2 \cdot$ Area of small rectangle

$= 1.31(0.66) - 2(0.33)(0.39)$

$= 0.6072$

Using the formula for the volume of a prism, the volume is

$V = Bh = 0.6072(0.66) \approx 0.40.$

▶ The volume is about 0.40 cubic foot.

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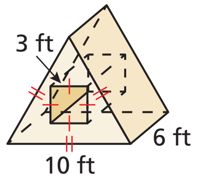


7. Find the volume of the composite solid.

11.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

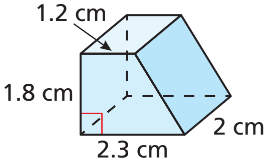
Vocabulary and Core Concept Check

1. **VOCABULARY** In what type of units is the volume of a solid measured?

2. **COMPLETE THE SENTENCE** Density is the amount of ______ that an object has in a given unit of _________.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the volume of the prism. *(See Example 1.)*

3.


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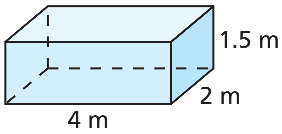

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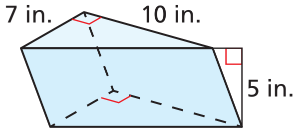

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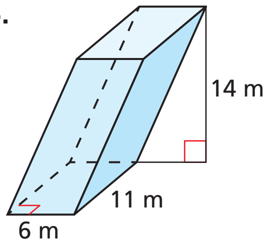
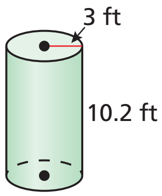


In Exercises 7–10, find the volume of the cylinder. *(See Example 2.)*

7.


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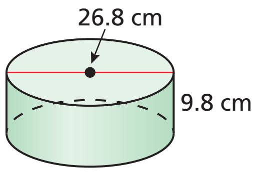

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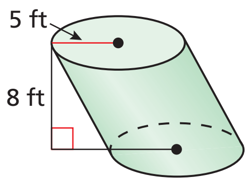

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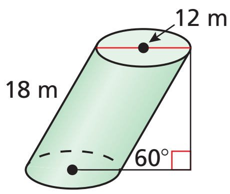


In Exercises 11 and 12, make a sketch of the solid and find its volume. Round your answer to the nearest hundredth.

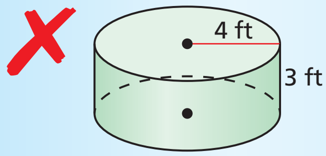
11. A prism has a height of 11.2 centimeters and an equilateral triangle for a base, where each base edge is 8 centimeters.

12. A pentagonal prism has a height of 9 feet and each base edge is 3 feet.

13. **PROBLEM SOLVING** A piece of copper with a volume of 8.25 cubic centimeters has a mass of 73.92 grams. A piece of iron with a volume of 5 cubic centimeters has a mass of 39.35 grams. Which metal has the greater density?

14. **PROBLEM SOLVING** The United States has minted one-dollar silver coins called the American Eagle Silver Bullion Coin since 1986. Each coin has a diameter of 40.6 millimeters and is 2.98 millimeters thick. The density of silver is 10.5 grams per cubic centimeter. What is the mass of an American Eagle Silver Bullion Coin to the nearest gram? *(See Example 3.)*

15. **ERROR ANALYSIS** Describe and correct the error in finding the volume of the cylinder.



$V = 2\pi rh$

$= 2\pi(4)(3)$

$= 24\pi$

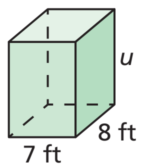
So, the volume of the cylinder is 24π cubic feet.

16. **ERROR ANALYSIS** Describe and correct the error in finding the density of an object that has a mass of 24 grams and a volume of 28.3 cubic centimeters.

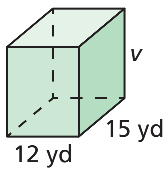
> ✗ density $= \frac{28.3}{24} \approx 1.18$
>
> So, the density is about 1.18 cubic centimeters per gram.

In Exercises 17–22, find the missing dimension of the prism or cylinder. *(See Example 4.)*

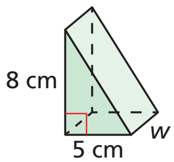
17. Volume $= 560\text{ ft}^3$



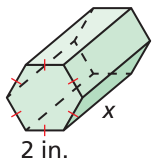
18. Volume $= 2700\text{ yd}^3$



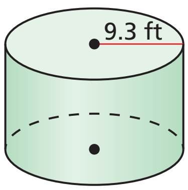
19. Volume $= 80\text{ cm}^3$



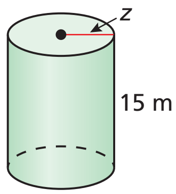
20. Volume $= 72.66\text{ in.}^3$



21. Volume $= 3000\text{ ft}^3$



22. Volume $= 1696.5\text{ m}^3$



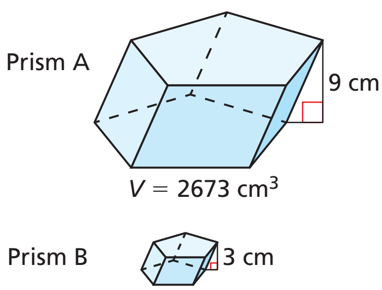
In Exercises 23 and 24, find the area of the base of the rectangular prism with the given volume and height. Then give a possible length and width. *(See Example 5.)*

23. $V = 154\text{ in.}^3$, $h = 11$ in.

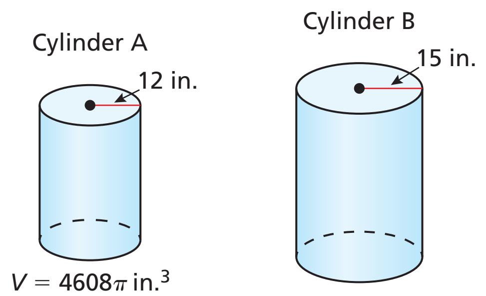
24. $V = 27\text{ m}^3$, $h = 3$ m

In Exercises 25 and 26, the solids are similar. Find the volume of solid B. *(See Example 6.)*

25.

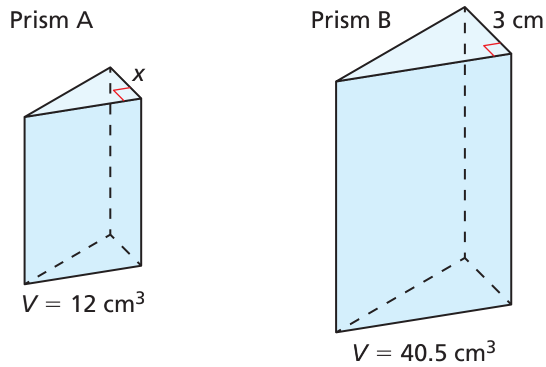


26.

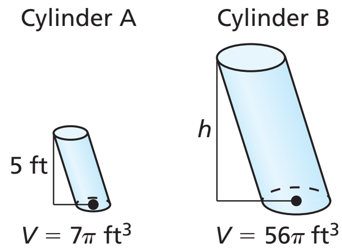


In Exercises 27 and 28, the solids are similar. Find the indicated measure.

27. height x of the base of prism A

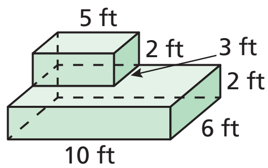


28. height h of cylinder B

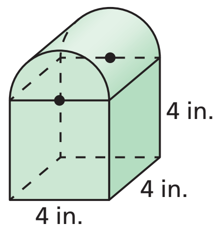


In Exercises 29–32, find the volume of the composite solid. *(See Example 7.)*

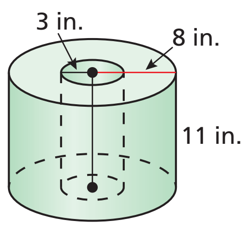
29.



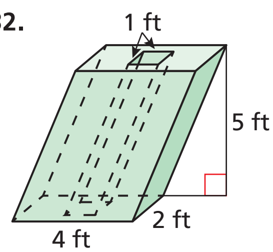
30.



31.



32.

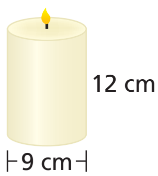


33. **MODELING WITH MATHEMATICS** The Great Blue Hole is a cylindrical trench located off the coast of Belize. It is approximately 1000 feet wide and 400 feet deep. About how many gallons of water does the Great Blue Hole contain? (1 $ft^3 \approx 7.48$ gallons)

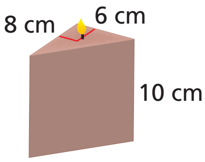
34. **COMPARING METHODS** The *Volume Addition Postulate* states that the volume of a solid is the sum of the volumes of all its nonoverlapping parts. Use this postulate to find the volume of the block of concrete in Example 7 by subtracting the volume of each hole from the volume of the large rectangular prism. Which method do you prefer? Explain your reasoning.

REASONING **In Exercises 35 and 36, you are melting a rectangular block of wax to make candles. How many candles of the given shape can be made using a block that measures 10 centimeters by 9 centimeters by 20 centimeters?**

35.



36.



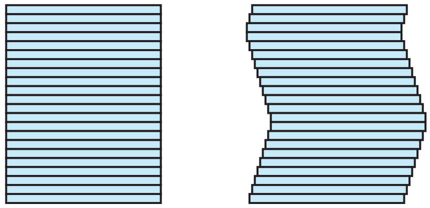
37. **PROBLEM SOLVING** An aquarium shaped like a rectangular prism has a length of 30 inches, a width of 10 inches, and a height of 20 inches. You fill the aquarium $\frac{3}{4}$ full with water. When you submerge a rock in the aquarium, the water level rises 0.25 inch.

a. Find the volume of the rock.

b. How many rocks of this size can you place in the aquarium before water spills out?

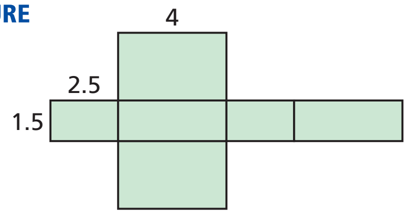
38. **PROBLEM SOLVING** You drop an irregular piece of metal into a container partially filled with water and measure that the water level rises 4.8 centimeters. The square base of the container has a side length of 8 centimeters. You measure the mass of the metal to be 450 grams. What is the density of the metal?

39. **WRITING** Both of the figures shown are made up of the same number of congruent rectangles. Explain how Cavalieri's Principle can be adapted to compare the areas of these figures.

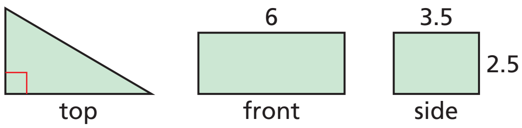


40. **HOW DO YOU SEE IT?** Each stack of memo papers contains 500 equally-sized sheets of paper. Compare their volumes. Explain your reasoning.

41. **USING STRUCTURE** Sketch the solid formed by the net. Then find the volume of the solid.



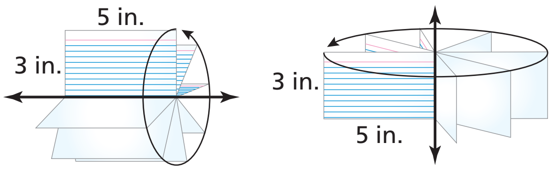
42. **USING STRUCTURE** Sketch the solid with the given views. Then find the volume of the solid.



43. **OPEN-ENDED** Sketch two rectangular prisms that have volumes of 100 cubic inches but different surface areas. Include dimensions in your sketches.

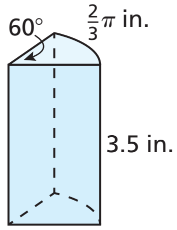
44. **MODELING WITH MATHEMATICS** Which box gives you more cereal for your money? Explain.

45. **CRITICAL THINKING** A 3-inch by 5-inch index card is rotated around a horizontal line and a vertical line to produce two different solids. Which solid has a greater volume? Explain your reasoning.



46. **CRITICAL THINKING** The height of cylinder X is twice the height of cylinder Y. The radius of cylinder X is half the radius of cylinder Y. Compare the volumes of cylinder X and cylinder Y. Justify your answer.

47. **USING STRUCTURE** Find the volume of the solid shown. The bases of the solid are sectors of circles.



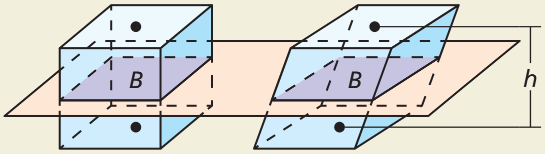
48. **MATHEMATICAL CONNECTIONS** You drill a circular hole of radius r through the base of a cylinder of radius R. Assume the hole is drilled completely through to the other base. You want the volume of the hole to be half the volume of the cylinder. Express r as a function of R.

49. **ANALYZING RELATIONSHIPS** How can you change the height of a cylinder so that the volume is increased by 25% but the radius remains the same?

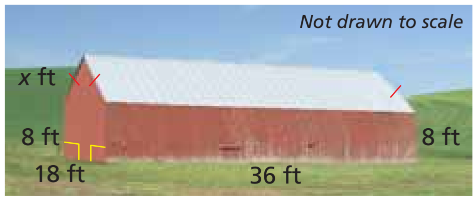
50. **ANALYZING RELATIONSHIPS** How can you change the edge length of a cube so that the volume is reduced by 40%?

51. **MAKING AN ARGUMENT** You have two objects of equal volume. Your friend says you can compare the densities of the objects by comparing their mass, because the heavier object will have a greater density. Is your friend correct? Explain your reasoning.

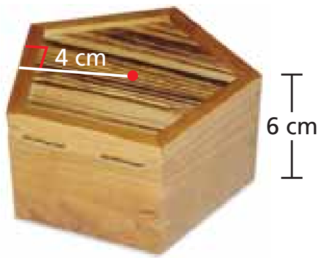
52. **THOUGHT PROVOKING** Cavalieri's Principle states that the two solids shown below have the same volume. Do they also have the same surface area? Explain your reasoning.



53. **PROBLEM SOLVING** A barn is in the shape of a pentagonal prism with the dimensions shown. The volume of the barn is 9072 cubic feet. Find the dimensions of each half of the roof.



54. **PROBLEM SOLVING** A wooden box is in the shape of a regular pentagonal prism. The sides, top, and bottom of the box are 1 centimeter thick. Approximate the volume of wood used to construct the box. Round your answer to the nearest tenth.

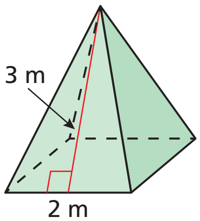


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the surface area of the regular pyramid. *(Skills Review Handbook)*

55.



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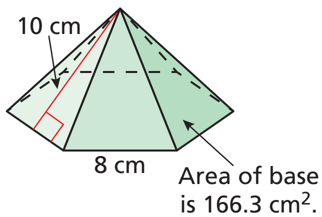

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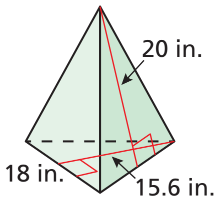


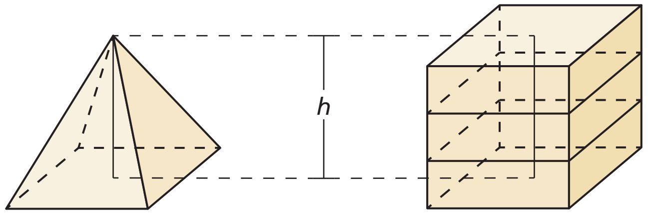
11.6 Volumes of Pyramids

Essential Question

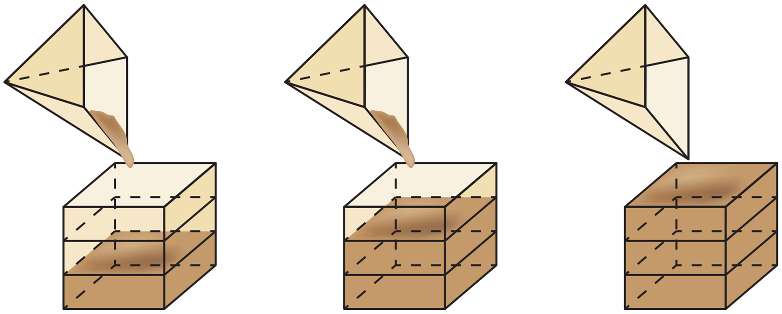
How can you find the volume of a pyramid?

EXPLORATION 1 Finding the Volume of a Pyramid

Work with a partner. The pyramid and the prism have the same height and the same square base.



When the pyramid is filled with sand and poured into the prism, it takes three pyramids to fill the prism.



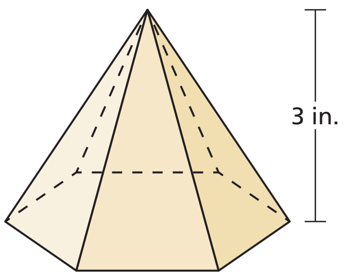
LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

Use this information to write a formula for the volume V of a pyramid.

EXPLORATION 2 Finding the Volume of a Pyramid

Work with a partner. Use the formula you wrote in Exploration 1 to find the volume of the hexagonal pyramid.



Communicate Your Answer

3. How can you find the volume of a pyramid?

4. In Section 11.7, you will study volumes of cones. How do you think you could use a method similar to the one presented in Exploration 1 to write a formula for the volume of a cone? Explain your reasoning.

11.6 Lesson

Core Vocabulary

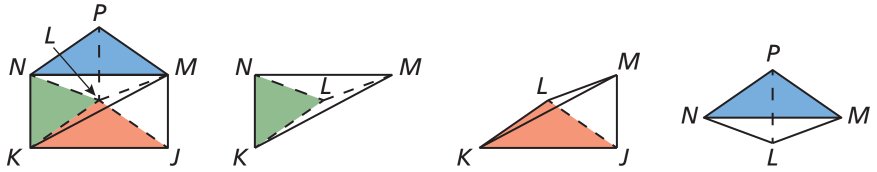
Previous
pyramid
composite solid

What You Will Learn

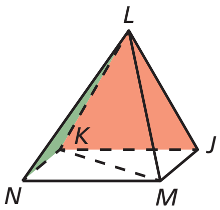
- Find volumes of pyramids.
- Use volumes of pyramids.

Finding Volumes of Pyramids

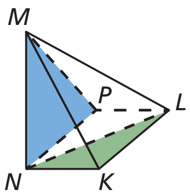
Consider a triangular prism with parallel, congruent bases $\triangle JKL$ and $\triangle MNP$. You can divide this triangular prism into three triangular pyramids.



Triangular prism   Triangular pyramid 1   Triangular pyramid 2   Triangular pyramid 3



Pyramid *Q*



Pyramid *R*

You can combine triangular pyramids 1 and 2 to form a pyramid with a base that is a parallelogram, as shown at the left. Name this pyramid *Q*. Similarly, you can combine triangular pyramids 1 and 3 to form pyramid *R* with a base that is a parallelogram.

In pyramid *Q*, diagonal $\overline{KM}$ divides $\square JKNM$ into two congruent triangles, so the bases of triangular pyramids 1 and 2 are congruent. Similarly, you can divide any cross section parallel to $\square JKNM$ into two congruent triangles that are the cross sections of triangular pyramids 1 and 2.

By Cavalieri's Principle, triangular pyramids 1 and 2 have the same volume. Similarly, using pyramid *R*, you can show that triangular pyramids 1 and 3 have the same volume. By the Transitive Property of Equality, triangular pyramids 2 and 3 have the same volume.

The volume of each pyramid must be one-third the volume of the prism, or $V = \frac{1}{3}Bh$. You can generalize this formula to say that the volume of any pyramid with any base is equal to $\frac{1}{3}$ the volume of a prism with the same base and height because you can divide any polygon into triangles and any pyramid into triangular pyramids.

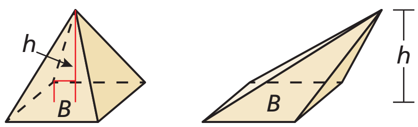
Core Concept

Volume of a Pyramid

The volume V of a pyramid is

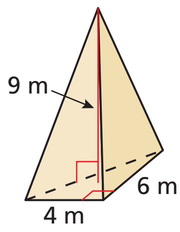
$$V = \frac{1}{3}Bh$$

where B is the area of the base and h is the height.



EXAMPLE 1 Finding the Volume of a Pyramid

Find the volume of the pyramid.



SOLUTION

$V = \frac{1}{3}Bh$ Formula for volume of a pyramid

$= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 6\right)(9)$ Substitute.

$= 36$ Simplify.

- The volume is 36 cubic meters.

Monitoring Progress

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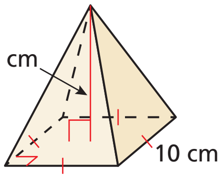
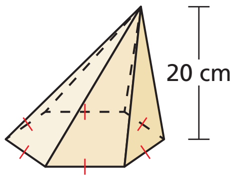
Find the volume of the pyramid.

1.



2.



Using Volumes of Pyramids

EXAMPLE 2 Using the Volume of a Pyramid

Khafre's Pyramid, Egypt

Originally, Khafre's Pyramid had a height of about 144 meters and a volume of about 2,218,800 cubic meters. Find the side length of the square base.

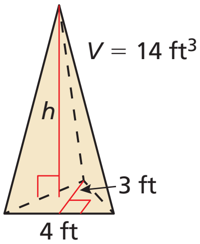
SOLUTION

$V = \frac{1}{3}Bh$	Formula for volume of a pyramid
$2{,}218{,}800 \approx \frac{1}{3}x^2(144)$	Substitute.
$6{,}656{,}400 \approx 144x^2$	Multiply each side by 3.
$46{,}225 \approx x^2$	Divide each side by 144.
$215 \approx x$	Find the positive square root.

▶ Originally, the side length of the square base was about 215 meters.

EXAMPLE 3 Using the Volume of a Pyramid

Find the height of the triangular pyramid.



SOLUTION

The area of the base is $B = \frac{1}{2}(3)(4) = 6\text{ ft}^2$ and the volume is $V = 14\text{ ft}^3$.

$V = \frac{1}{3}Bh$	Formula for volume of a pyramid
$14 = \frac{1}{3}(6)h$	Substitute.
$7 = h$	Solve for h.

▶ The height is 7 feet.

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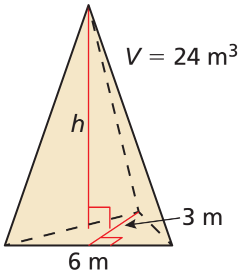
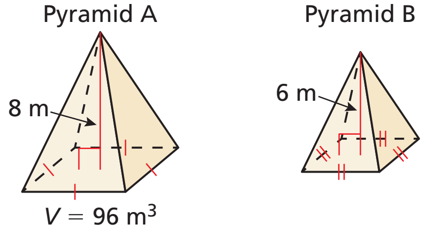


3. The volume of a square pyramid is 75 cubic meters and the height is 9 meters. Find the side length of the square base.

4. Find the height of the triangular pyramid at the left.

EXAMPLE 4 Finding the Volume of a Similar Solid

Pyramid A and pyramid B are similar. Find the volume of pyramid B.



SOLUTION

The scale factor is $k = \frac{\text{Height of pyramid B}}{\text{Height of pyramid A}} = \frac{6}{8} = \frac{3}{4}$.

Use the scale factor to find the volume of pyramid B.

$\frac{\text{Volume of pyramid B}}{\text{Volume of pyramid A}} = k^3$ — The ratio of the volumes is k^3.

$\frac{\text{Volume of pyramid B}}{96} = \left(\frac{3}{4}\right)^3$ — Substitute.

Volume of pyramid B $= 40.5$ — Solve for volume of pyramid B.

▶ The volume of pyramid B is 40.5 cubic meters.

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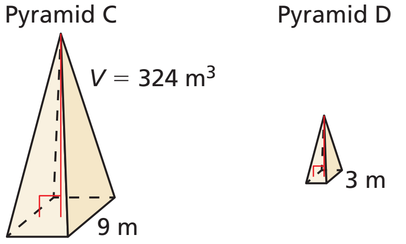
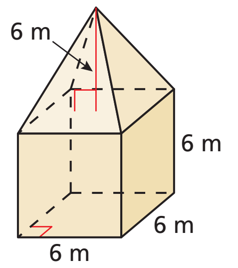
5. Pyramid C and pyramid D are similar. Find the volume of pyramid D.



EXAMPLE 5 Finding the Volume of a Composite Solid

Find the volume of the composite solid.



SOLUTION

Volume of solid	=	Volume of cube	+	Volume of pyramid

$= s^3 + \frac{1}{3}Bh$ — Write formulas.

$= 6^3 + \frac{1}{3}(6)^2 \cdot 6$ — Substitute.

$= 216 + 72$ — Simplify.

$= 288$ — Add.

▶ The volume is 288 cubic meters.

Monitoring Progress

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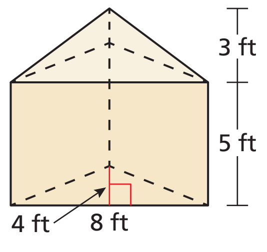
6. Find the volume of the composite solid.



11.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

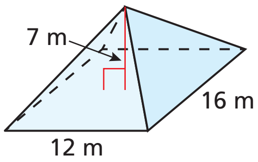
Vocabulary and Core Concept Check

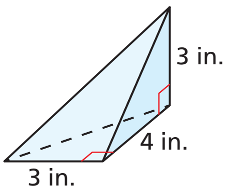
1. **VOCABULARY** Explain the difference between a triangular prism and a triangular pyramid.

2. **REASONING** A square pyramid and a cube have the same base and height. Compare the volume of the square pyramid to the volume of the cube.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, find the volume of the pyramid. *(See Example 1.)*

3.


4.


In Exercises 5–8, find the indicated measure. *(See Example 2.)*

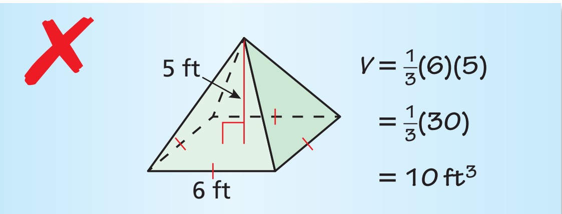
5. A pyramid with a square base has a volume of 120 cubic meters and a height of 10 meters. Find the side length of the square base.

6. A pyramid with a square base has a volume of 912 cubic feet and a height of 19 feet. Find the side length of the square base.

7. A pyramid with a rectangular base has a volume of 480 cubic inches and a height of 10 inches. The width of the rectangular base is 9 inches. Find the length of the rectangular base.

8. A pyramid with a rectangular base has a volume of 105 cubic centimeters and a height of 15 centimeters. The length of the rectangular base is 7 centimeters. Find the width of the rectangular base.

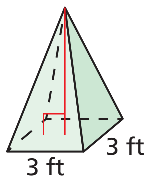
9. **ERROR ANALYSIS** Describe and correct the error in finding the volume of the pyramid.



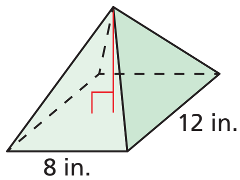
10. **OPEN-ENDED** Give an example of a pyramid and a prism that have the same base and the same volume. Explain your reasoning.

In Exercises 11–14, find the height of the pyramid. *(See Example 3.)*

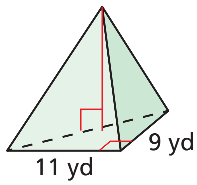
11. Volume $= 15 \text{ ft}^3$



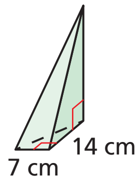
12. Volume $= 224 \text{ in.}^3$



13. Volume $= 198 \text{ yd}^3$

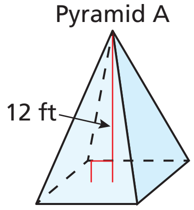


14. Volume $= 392 \text{ cm}^3$



In Exercises 15 and 16, the pyramids are similar. Find the volume of pyramid B. *(See Example 4.)*

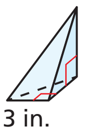
15. Pyramid A



$V = 256 \text{ ft}^3$

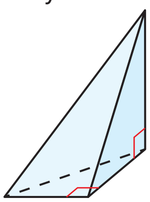
Pyramid B

16. Pyramid A



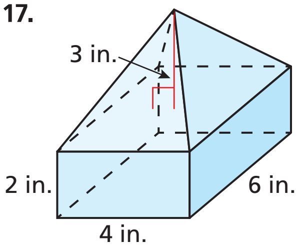
$V = 10 \text{ in.}^3$

Pyramid B



In Exercises 17–20, find the volume of the composite solid. *(See Example 5.)*

17.



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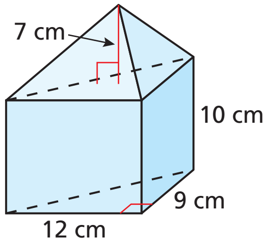

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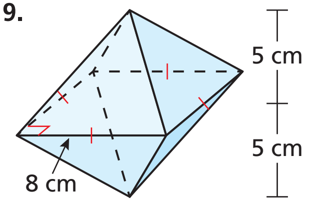

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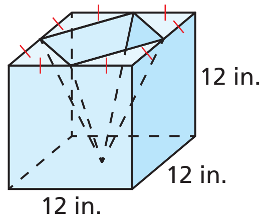


21. **ABSTRACT REASONING** A pyramid has a height of 8 feet and a square base with a side length of 6 feet.

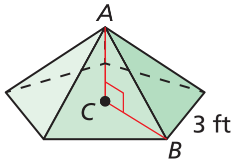
a. How does the volume of the pyramid change when the base stays the same and the height is doubled?

b. How does the volume of the pyramid change when the height stays the same and the side length of the base is doubled?

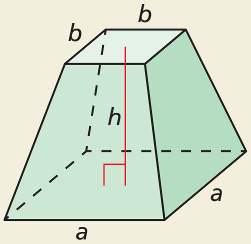
c. Are your answers to parts (a) and (b) true for any square pyramid? Explain your reasoning.

22. **HOW DO YOU SEE IT?** The cube shown is formed by three pyramids, each with the same square base and the same height. How could you use this to verify the formula for the volume of a pyramid?

23. **CRITICAL THINKING** Find the volume of the regular pentagonal pyramid. Round your answer to the nearest hundredth. In the diagram, $m\angle ABC = 35°$.



24. **THOUGHT PROVOKING** A *frustum* of a pyramid is the part of the pyramid that lies between the base and a plane parallel to the base, as shown. Write a formula for the volume of the frustum of a square pyramid in terms of a, b, and h. (*Hint*: Consider the "missing" top of the pyramid and use similar triangles.)



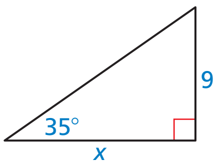
25. **MODELING WITH MATHEMATICS** Nautical deck prisms were used as a safe way to illuminate decks on ships. The deck prism shown here is composed of the following three solids: a regular hexagonal prism with an edge length of 3.5 inches and a height of 1.5 inches, a regular hexagonal prism with an edge length of 3.25 inches and a height of 0.25 inch, and a regular hexagonal pyramid with an edge length of 3 inches and a height of 3 inches. Find the volume of the deck prism.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of x. Round your answer to the nearest tenth. *(Section 9.4 and Section 9.5)*

26.



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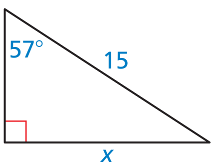

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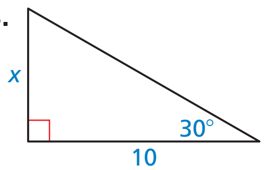

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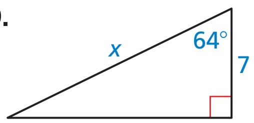
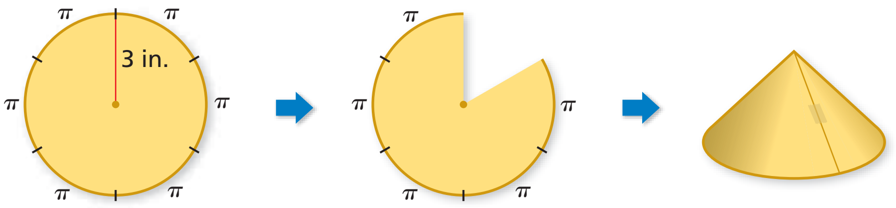


11.7 Surface Areas and Volumes of Cones

Essential Question How can you find the surface area and the volume of a cone?

EXPLORATION 1 Finding the Surface Area of a Cone

Work with a partner. Construct a circle with a radius of 3 inches. Mark the circumference of the circle into six equal parts, and label the length of each part. Then cut out one sector of the circle and make a cone.



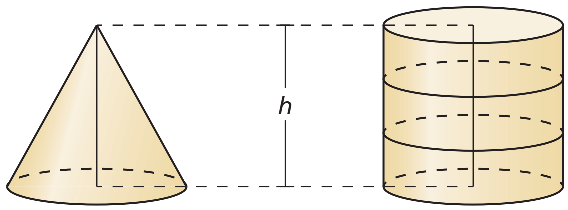
a. Explain why the base of the cone is a circle. What are the circumference and radius of the base?

b. What is the area of the original circle? What is the area with one sector missing?

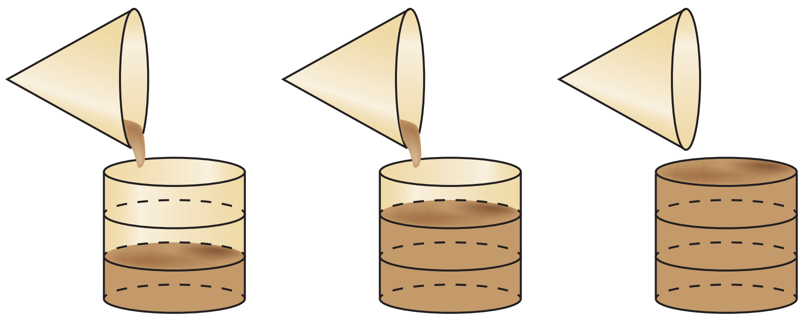
c. Describe the surface area of the cone, including the base. Use your description to find the surface area.

EXPLORATION 2 Finding the Volume of a Cone

Work with a partner. The cone and the cylinder have the same height and the same circular base.



When the cone is filled with sand and poured into the cylinder, it takes three cones to fill the cylinder.



Use this information to write a formula for the volume V of a cone.

CONSTRUCTING VIABLE ARGUMENTS
To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results in constructing arguments.

Communicate Your Answer

3. How can you find the surface area and the volume of a cone?

4. In Exploration 1, cut another sector from the circle and make a cone. Find the radius of the base and the surface area of the cone. Repeat this three times, recording your results in a table. Describe the pattern.

11.7 Lesson

Core Vocabulary

lateral surface of a cone, *p. 642*

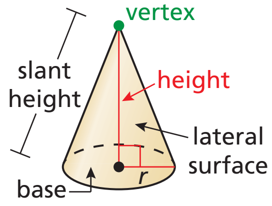
Previous
cone
net
composite solid

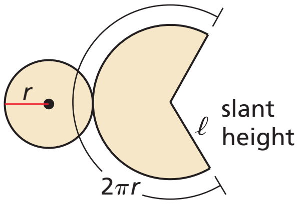
What You Will Learn

- Find surface areas of right cones.
- Find volumes of cones.
- Use volumes of cones.

Finding Surface Areas of Right Cones

Recall that a *circular cone*, or *cone*, has a circular *base* and a *vertex* that is not in the same plane as the base. The *altitude*, or *height*, is the perpendicular distance between the vertex and the base. In a *right cone*, the height meets the base at its center and the *slant height* is the distance between the vertex and a point on the base edge.





The **lateral surface of a cone** consists of all segments that connect the vertex with points on the base edge. When you cut along the slant height and lay the right cone flat, you get the net shown at the left. In the net, the circular base has an area of πr^2 and the lateral surface is a sector of a circle. You can find the area of this sector by using a proportion, as shown below.

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}}$$ Set up proportion.

$$\frac{\text{Area of sector}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell}$$ Substitute.

$$\text{Area of sector} = \pi \ell^2 \cdot \frac{2\pi r}{2\pi \ell}$$ Multiply each side by $\pi\ell^2$.

$$\text{Area of sector} = \pi r \ell$$ Simplify.

The surface area of a right cone is the sum of the base area and the lateral area, $\pi r \ell$.

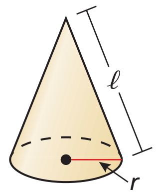
Core Concept

Surface Area of a Right Cone

The surface area S of a right cone is

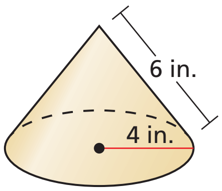
$$S = \pi r^2 + \pi r \ell$$

where r is the radius of the base and ℓ is the slant height.



EXAMPLE 1 Finding Surface Areas of Right Cones

Find the surface area of the right cone.



SOLUTION

$$S = \pi r^2 + \pi r \ell = \pi \cdot 4^2 + \pi(4)(6) = 40\pi \approx 125.66$$

► The surface area is 40π, or about 125.66 square inches.

Monitoring Progress

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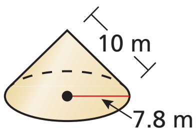
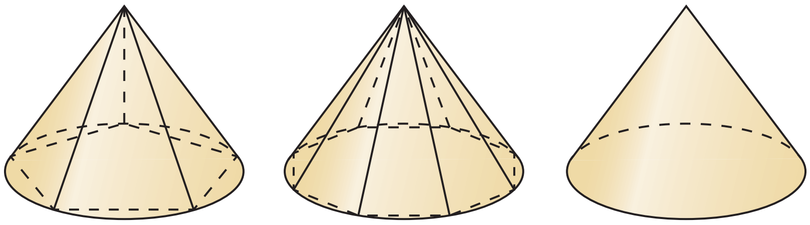
1. Find the surface area of the right cone.



Finding Volumes of Cones

Consider a cone with a regular polygon inscribed in the base. The pyramid with the same vertex as the cone has volume $V = \frac{1}{3}Bh$. As you increase the number of sides of the polygon, it approaches the base of the cone and the pyramid approaches the cone. The volume approaches $\frac{1}{3}\pi r^2 h$ as the base area B approaches πr^2.



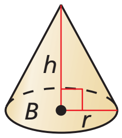
Core Concept

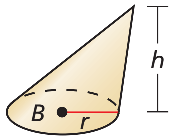
Volume of a Cone

The volume V of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$

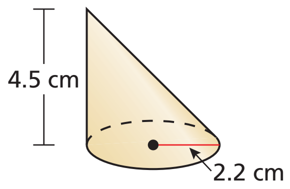
where B is the area of the base, h is the height, and r is the radius of the base.





EXAMPLE 2 **Finding the Volume of a Cone**

Find the volume of the cone.



SOLUTION

$V = \frac{1}{3}\pi r^2 h$	Formula for volume of a cone
$= \frac{1}{3}\pi \cdot (2.2)^2 \cdot 4.5$	Substitute.
$= 7.26\pi$	Simplify.
≈ 22.81	Use a calculator.

▶ The volume is 7.26π, or about 22.81 cubic centimeters.

Monitoring Progress

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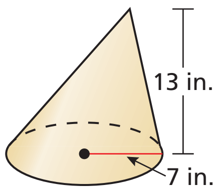
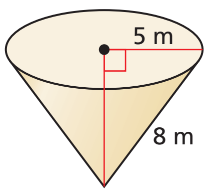
Find the volume of the cone.

2.



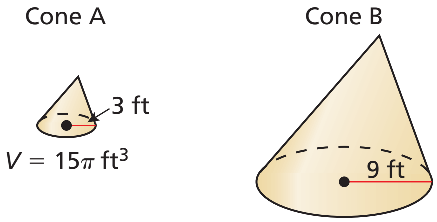
3.



Using Volumes of Cones

EXAMPLE 3 Finding the Volume of a Similar Solid

Cone A and cone B are similar. Find the volume of cone B.



SOLUTION

The scale factor is $k = \dfrac{\text{Radius of cone B}}{\text{Radius of cone A}} = \dfrac{9}{3} = 3.$

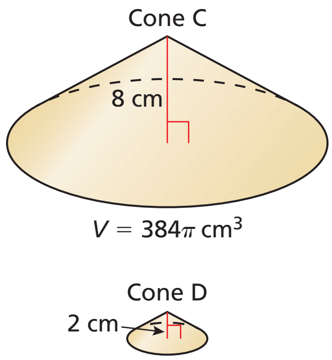
Use the scale factor to find the volume of cone B.

$$\frac{\text{Volume of cone B}}{\text{Volume of cone A}} = k^3 \qquad \text{The ratio of the volumes is } k^3.$$

$$\frac{\text{Volume of cone B}}{15\pi} = 3^3 \qquad \text{Substitute.}$$

$$\text{Volume of cone B} = 405\pi \qquad \text{Solve for volume of cone B.}$$

▶ The volume of cone B is 405π cubic feet.



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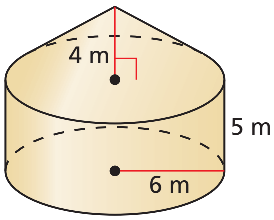
4. Cone C and cone D are similar. Find the volume of cone D.

EXAMPLE 4 Finding the Volume of a Composite Solid

Find the volume of the composite solid.



SOLUTION

Let h_1 be the height of the cylinder and let h_2 be the height of the cone.

Volume of solid	=	Volume of cylinder	+	Volume of cone

$$= \pi r^2 h_1 + \tfrac{1}{3}\pi r^2 h_2 \qquad \text{Write formulas.}$$

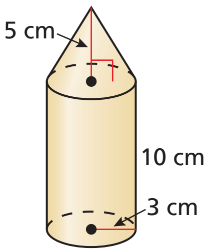
$$= \pi \cdot 6^2 \cdot 5 + \tfrac{1}{3}\pi \cdot 6^2 \cdot 4 \qquad \text{Substitute.}$$

$$= 180\pi + 48\pi \qquad \text{Simplify.}$$

$$= 228\pi \qquad \text{Add.}$$

$$\approx 716.28 \qquad \text{Use a calculator.}$$

▶ The volume is 228π, or about 716.28 cubic meters.



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5. Find the volume of the composite solid.

11.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

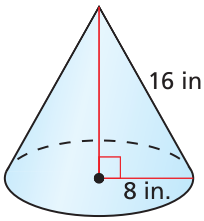
1. WRITING Describe the differences between pyramids and cones. Describe their similarities.

2. COMPLETE THE SENTENCE The volume of a cone with radius r and height h is $\frac{1}{3}$ the volume of a(n) _________ with radius r and height h.

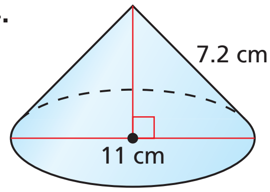
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the surface area of the right cone. *(See Example 1.)*

3.



4.

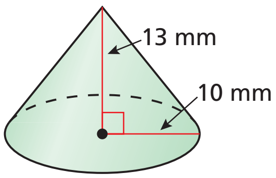


5. A right cone has a radius of 9 inches and a height of 12 inches.

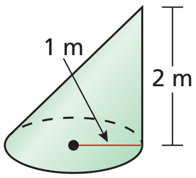
6. A right cone has a diameter of 11.2 feet and a height of 9.2 feet.

In Exercises 7–10, find the volume of the cone. *(See Example 2.)*

7.



8.

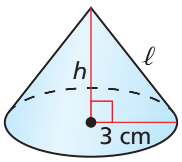


9. A cone has a diameter of 11.5 inches and a height of 15.2 inches.

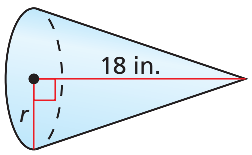
10. A right cone has a radius of 3 feet and a slant height of 6 feet.

In Exercises 11 and 12, find the missing dimension(s).

11. Surface area $= 75.4$ cm^2

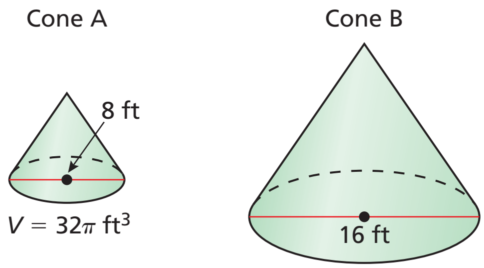


12. Volume $= 216\pi$ in.^{3}

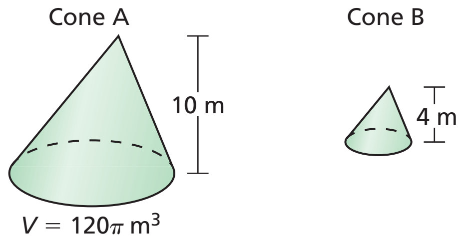


In Exercises 13 and 14, the cones are similar. Find the volume of cone B. *(See Example 3.)*

13.

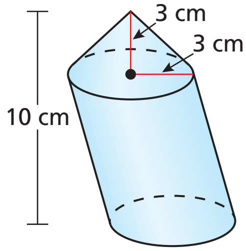


14.

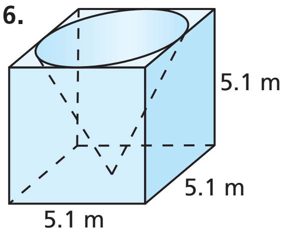


In Exercises 15 and 16, find the volume of the composite solid. *(See Example 4.)*

15.

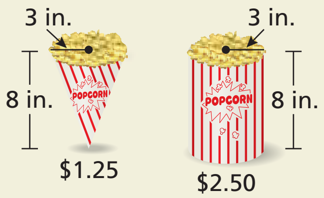


16.



17. ANALYZING RELATIONSHIPS A cone has height h and a base with radius r. You want to change the cone so its volume is doubled. What is the new height if you change only the height? What is the new radius if you change only the radius? Explain.

18. **HOW DO YOU SEE IT** A snack stand serves a small order of popcorn in a cone-shaped container and a large order of popcorn in a cylindrical container. Do not perform any calculations.

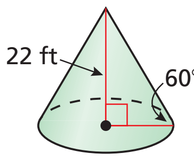


a. How many small containers of popcorn do you have to buy to equal the amount of popcorn in a large container? Explain.

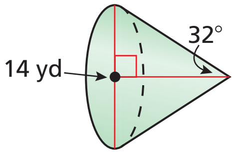
b. Which container gives you more popcorn for your money? Explain.

In Exercises 19 and 20, find the volume of the right cone.

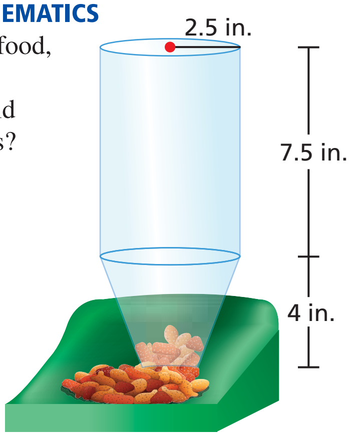
19.



20.

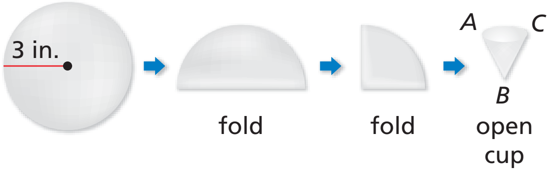


21. **MODELING WITH MATHEMATICS** A cat eats half a cup of food, twice per day. Will the automatic pet feeder hold enough food for 10 days? Explain your reasoning. (1 cup ≈ 14.4 in.^{3})

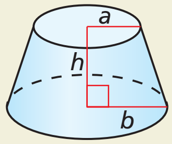


22. **MODELING WITH MATHEMATICS** During a chemistry lab, you use a funnel to pour a solvent into a flask. The radius of the funnel is 5 centimeters and its height is 10 centimeters. You pour the solvent into the funnel at a rate of 80 milliliters per second and the solvent flows out of the funnel at a rate of 65 milliliters per second. How long will it be before the funnel overflows? (1 mL = 1 cm^3)

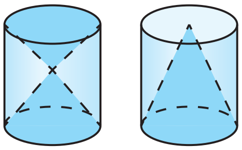
23. **REASONING** To make a paper drinking cup, start with a circular piece of paper that has a 3-inch radius, then follow the given steps. How does the surface area of the cup compare to the original paper circle? Find $m\angle ABC$.



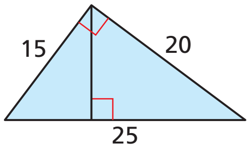
24. **THOUGHT PROVOKING** A *frustum* of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Write a formula for the volume of the frustum of a cone in terms of a, b, and h. (*Hint*: Consider the "missing" top of the cone and use similar triangles.)



25. **MAKING AN ARGUMENT** In the figure, the two cylinders are congruent. The combined height of the two smaller cones equals the height of the larger cone. Your friend claims that this means the total volume of the two smaller cones is equal to the volume of the larger cone. Is your friend correct? Justify your answer.



26. **CRITICAL THINKING** When the given triangle is rotated around each of its sides, solids of revolution are formed. Describe the three solids and find their volumes. Give your answers in terms of π.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the indicated measure. *(Section 11.2)*

27. area of a circle with a radius of 7 feet

28. area of a circle with a diameter of 22 centimeters

29. diameter of a circle with an area of 256π square meters

30. radius of a circle with an area of 529π square inches

11.8 Surface Areas and Volumes of Spheres

Essential Question

How can you find the surface area and the volume of a sphere?

EXPLORATION 1 Finding the Surface Area of a Sphere

Work with a partner. Remove the covering from a baseball or softball.

You will end up with two "figure 8" pieces of material, as shown above. From the amount of material it takes to cover the ball, what would you estimate the surface area S of the ball to be? Express your answer in terms of the radius r of the ball.

$S =$ ________ Surface area of a sphere

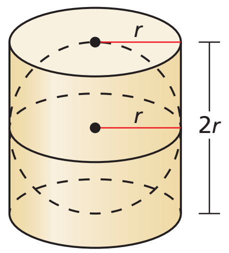
Use the Internet or some other resource to confirm that the formula you wrote for the surface area of a sphere is correct.

USING TOOLS STRATEGICALLY

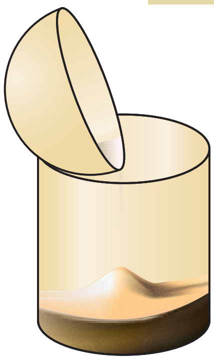
To be proficient in math, you need to identify relevant external mathematical resources, such as content located on a website.

EXPLORATION 2 Finding the Volume of a Sphere

Work with a partner. A cylinder is circumscribed about a sphere, as shown. Write a formula for the volume V of the cylinder in terms of the radius r.



$V =$ ________ Volume of cylinder



When half of the sphere (a *hemisphere*) is filled with sand and poured into the cylinder, it takes three hemispheres to fill the cylinder. Use this information to write a formula for the volume V of a sphere in terms of the radius r.

$V =$ ________ Volume of a sphere

Communicate Your Answer

3. How can you find the surface area and the volume of a sphere?
4. Use the results of Explorations 1 and 2 to find the surface area and the volume of a sphere with a radius of (a) 3 inches and (b) 2 centimeters.

11.8 Lesson

Core Vocabulary

chord of a sphere, *p. 648*
great circle, *p. 648*

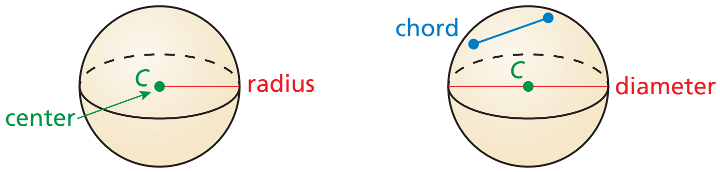
Previous
sphere
center of a sphere
radius of a sphere
diameter of a sphere
hemisphere

What You Will Learn

- Find surface areas of spheres.
- Find volumes of spheres.

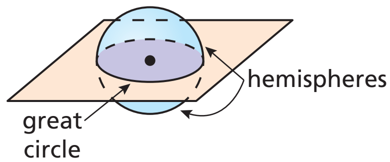
Finding Surface Areas of Spheres

A *sphere* is the set of all points in space equidistant from a given point. This point is called the *center* of the sphere. A *radius* of a sphere is a segment from the center to a point on the sphere. A **chord of a sphere** is a segment whose endpoints are on the sphere. A *diameter* of a sphere is a chord that contains the center.



As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

If a plane intersects a sphere, then the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. The circumference of a great circle is the circumference of the sphere. Every great circle of a sphere separates the sphere into two congruent halves called *hemispheres*.



Core Concept

Surface Area of a Sphere

The surface area S of a sphere is

$$S = 4\pi r^2$$

where r is the radius of the sphere.

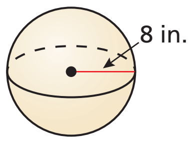
To understand the formula for the surface area of a sphere, think of a baseball. The surface area of a baseball is sewn from two congruent shapes, each of which resembles two joined circles.

So, the entire covering of the baseball consists of four circles, each with radius r. The area A of a circle with radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the formula for the surface area of a sphere.

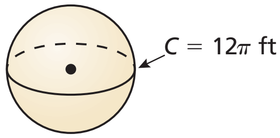
EXAMPLE 1 Finding the Surface Areas of Spheres

Find the surface area of each sphere.

a.



b.



SOLUTION

a. $S = 4\pi r^2$ Formula for surface area of a sphere

$= 4\pi(8)^2$ Substitute 8 for *r*.

$= 256\pi$ Simplify.

≈ 804.25 Use a calculator.

▶ The surface area is 256π, or about 804.25 square inches.

b. The circumference of the sphere is 12π, so the radius of the sphere is $\frac{12\pi}{2\pi} = 6$ feet.

$S = 4\pi r^2$ Formula for surface area of a sphere

$= 4\pi(6)^2$ Substitute 6 for *r*.

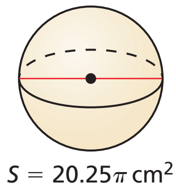
$= 144\pi$ Simplify.

≈ 452.39 Use a calculator.

▶ The surface area is 144π, or about 452.39 square feet.

EXAMPLE 2 Finding the Diameter of a Sphere

Find the diameter of the sphere.



SOLUTION

$S = 4\pi r^2$ Formula for surface area of a sphere

$20.25\pi = 4\pi r^2$ Substitute 20.25π for *S*.

$5.0625 = r^2$ Divide each side by 4π.

$2.25 = r$ Find the positive square root.

▶ The diameter is $2r = 2 \bullet 2.25 = 4.5$ centimeters.

COMMON ERROR

Be sure to multiply the value of *r* by 2 to find the diameter.

Monitoring Progress

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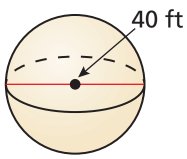
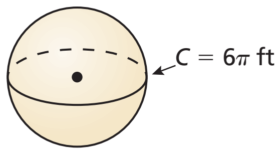
Find the surface area of the sphere.

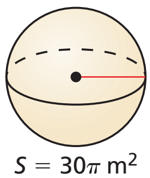
1.



2.

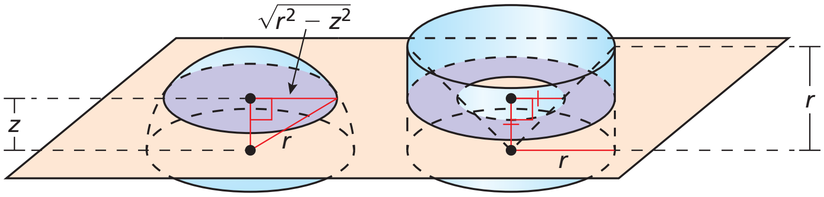


3. Find the radius of the sphere.



Finding Volumes of Spheres

The figure shows a hemisphere and a cylinder with a cone removed. A plane parallel to their bases intersects the solids z units above their bases.



Using the AA Similarity Theorem (Theorem 8.3), you can show that the radius of the cross section of the cone at height z is z. The area of the cross section formed by the plane is $\pi(r^2 - z^2)$ for both solids. Because the solids have the same height and the same cross-sectional area at every level, they have the same volume by Cavalieri's Principle.

$$V_{\text{hemisphere}} = V_{\text{cylinder}} - V_{\text{cone}}$$

$$= \pi r^2(r) - \frac{1}{3}\pi r^2(r)$$

$$= \frac{2}{3}\pi r^3$$

So, the volume of a sphere of radius r is

$$2 \cdot V_{\text{hemisphere}} = 2 \cdot \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3.$$

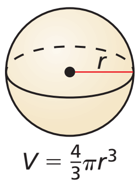
Core Concept

Volume of a Sphere

The volume V of a sphere is

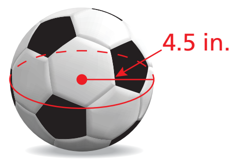
$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.



EXAMPLE 3 Finding the Volume of a Sphere

Find the volume of the soccer ball.



SOLUTION

$V = \frac{4}{3}\pi r^3$	Formula for volume of a sphere
$= \frac{4}{3}\pi(4.5)^3$	Substitute 4.5 for r.
$= 121.5\pi$	Simplify.
≈ 381.70	Use a calculator.

▶ The volume of the soccer ball is 121.5π, or about 381.70 cubic inches.

EXAMPLE 4 Finding the Volume of a Sphere

The surface area of a sphere is 324π square centimeters. Find the volume of the sphere.

SOLUTION

Step 1 Use the surface area to find the radius.

$S = 4\pi r^2$	Formula for surface area of a sphere
$324\pi = 4\pi r^2$	Substitute 324π for S.
$81 = r^2$	Divide each side by 4π.
$9 = r$	Find the positive square root.

The radius is 9 centimeters.

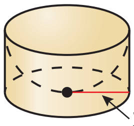
Step 2 Use the radius to find the volume.

$V = \frac{4}{3}\pi r^3$	Formula for volume of a sphere
$= \frac{4}{3}\pi(9)^3$	Substitute 9 for r.
$= 972\pi$	Simplify.
≈ 3053.63	Use a calculator.

▶ The volume is 972π, or about 3053.63 cubic centimeters.

EXAMPLE 5 Finding the Volume of a Composite Solid

Find the volume of the composite solid.



SOLUTION

Volume of solid = Volume of cylinder − Volume of hemisphere

$= \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	Write formulas.
$= \pi(2)^2(2) - \frac{2}{3}\pi(2)^3$	Substitute.
$= 8\pi - \frac{16}{3}\pi$	Multiply.
$= \frac{24}{3}\pi - \frac{16}{3}\pi$	Rewrite fractions using least common denominator.
$= \frac{8}{3}\pi$	Subtract.
≈ 8.38	Use a calculator.

▶ The volume is $\frac{8}{3}\pi$, or about 8.38 cubic inches.

Monitoring Progress

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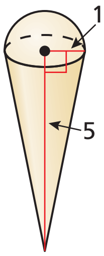


4. The radius of a sphere is 5 yards. Find the volume of the sphere.
5. The diameter of a sphere is 36 inches. Find the volume of the sphere.
6. The surface area of a sphere is 576π square centimeters. Find the volume of the sphere.
7. Find the volume of the composite solid at the left.

Dynamic Solutions available at *BigIdeasMath.com*

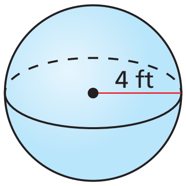
Vocabulary and Core Concept Check

1. **VOCABULARY** When a plane intersects a sphere, what must be true for the intersection to be a great circle?

2. **WRITING** Explain the difference between a sphere and a hemisphere.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the surface area of the sphere. *(See Example 1.)*

3.


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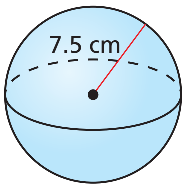

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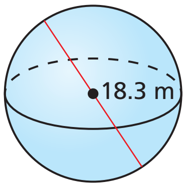

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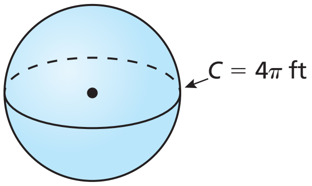


In Exercises 7–10, find the indicated measure. *(See Example 2.)*

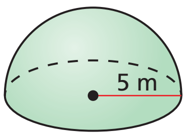
7. Find the radius of a sphere with a surface area of 4π square feet.

8. Find the radius of a sphere with a surface area of 1024π square inches.

9. Find the diameter of a sphere with a surface area of 900π square meters.

10. Find the diameter of a sphere with a surface area of 196π square centimeters.

In Exercises 11 and 12, find the surface area of the hemisphere.

11.


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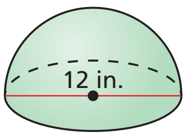
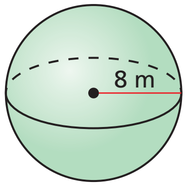


In Exercises 13–18, find the volume of the sphere. *(See Example 3.)*

13.


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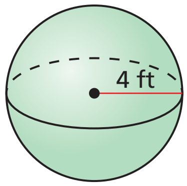

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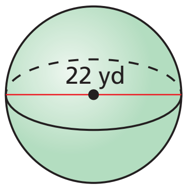

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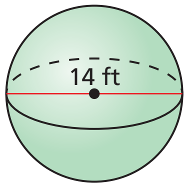

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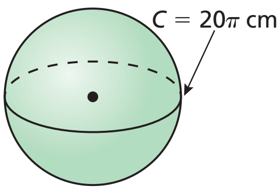

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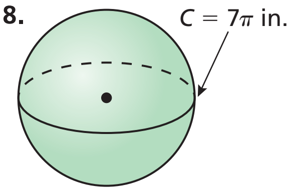
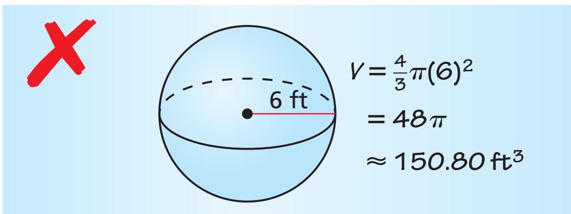


In Exercises 19 and 20, find the volume of the sphere with the given surface area. *(See Example 4.)*

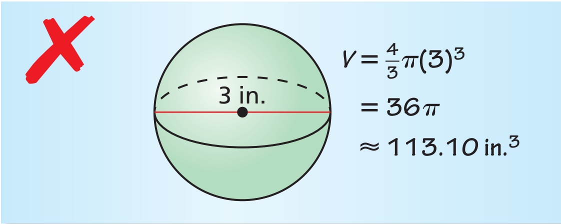
19. Surface area $= 16\pi$ ft^2

20. Surface area $= 484\pi$ cm^2

21. **ERROR ANALYSIS** Describe and correct the error in finding the volume of the sphere.

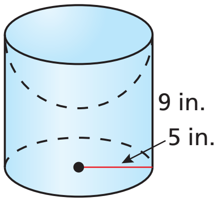


22. **ERROR ANALYSIS** Describe and correct the error in finding the volume of the sphere.



In Exercises 23–26, find the volume of the composite solid. *(See Example 5.)*

23.



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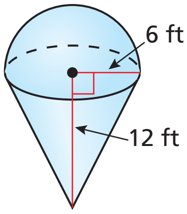

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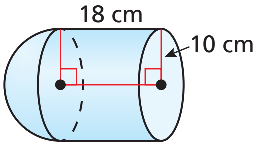

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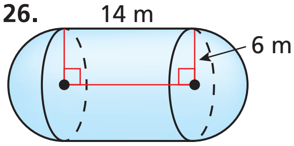


In Exercises 27–32, find the surface area and volume of the ball.

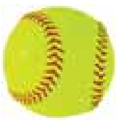
27. bowling ball

$d = 8.5$ in.

28. basketball

$C = 29.5$ in.

29. softball



$C = 12$ in.

30. golf ball

$d = 1.7$ in.

31. volleyball

$C = 26$ in.

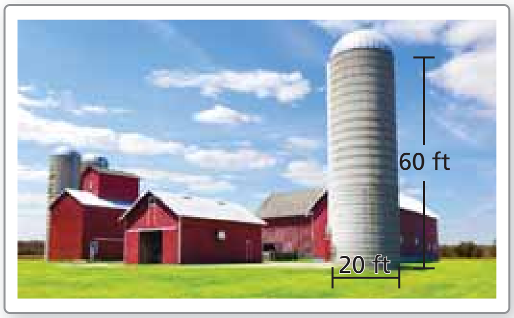
32. baseball

$C = 9$ in.

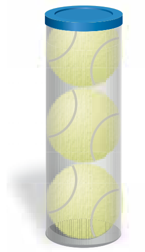
33. **MAKING AN ARGUMENT** You friend claims that if the radius of a sphere is doubled, then the surface area of the sphere will also be doubled. Is your friend correct? Explain your reasoning.

34. **REASONING** A semicircle with a diameter of 18 inches is rotated about its diameter. Find the surface area and the volume of the solid formed.

35. **MODELING WITH MATHEMATICS** A silo has the dimensions shown. The top of the silo is a hemispherical shape. Find the volume of the silo.



36. **MODELING WITH MATHEMATICS** Three tennis balls are stored in a cylindrical container with a height of 8 inches and a radius of 1.43 inches. The circumference of a tennis ball is 8 inches.



a. Find the volume of a tennis ball.

b. Find the amount of space within the cylinder not taken up by the tennis balls.

37. **ANALYZING RELATIONSHIPS** Use the table shown for a sphere.

Radius	Surface area	Volume
3 in.	36π in.^{2}	36π in.^{3}
6 in.		
9 in.		
12 in.		

a. Copy and complete the table. Leave your answers in terms of π.

b. What happens to the surface area of the sphere when the radius is doubled? tripled? quadrupled?

c. What happens to the volume of the sphere when the radius is doubled? tripled? quadrupled?

38. **MATHEMATICAL CONNECTIONS** A sphere has a diameter of $4(x + 3)$ centimeters and a surface area of 784π square centimeters. Find the value of x.

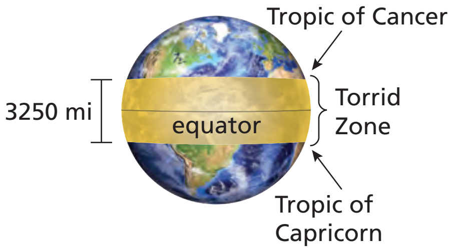
39. **MODELING WITH MATHEMATICS** The radius of Earth is about 3960 miles. The radius of the moon is about 1080 miles.

a. Find the surface area of Earth and the moon.

b. Compare the surface areas of Earth and the moon.

c. About 70% of the surface of Earth is water. How many square miles of water are on Earth's surface?

40. **MODELING WITH MATHEMATICS** The Torrid Zone on Earth is the area between the Tropic of Cancer and the Tropic of Capricorn. The distance between these two tropics is about 3250 miles. You can estimate the distance as the height of a cylindrical belt around the Earth at the equator.

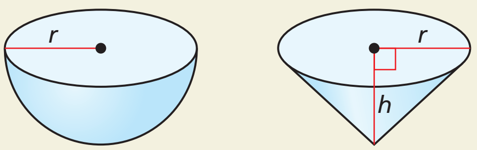


a. Estimate the surface area of the Torrid Zone. (The radius of Earth is about 3960 miles.)

b. A meteorite is equally likely to hit anywhere on Earth. Estimate the probability that a meteorite will land in the Torrid Zone.

41. **ABSTRACT REASONING** A sphere is inscribed in a cube with a volume of 64 cubic inches. What is the surface area of the sphere? Explain your reasoning.

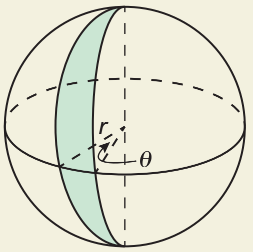
42. **HOW DO YOU SEE IT?** The formula for the volume of a hemisphere and a cone are shown. If each solid has the same radius and $r = h$, which solid will have a greater volume? Explain your reasoning.



$V = \frac{2}{3}\pi r^3$ $V = \frac{1}{3}\pi r^2 h$

43. **CRITICAL THINKING** Let V be the volume of a sphere, S be the surface area of the sphere, and r be the radius of the sphere. Write an equation for V in terms of r and S. $\left(\textit{Hint}\text{: Start with the ratio } \frac{V}{S}.\right)$

44. **THOUGHT PROVOKING** A *spherical lune* is the region between two great circles of a sphere. Find the formula for the area of a lune.



45. **CRITICAL THINKING** The volume of a right cylinder is the same as the volume of a sphere. The radius of the sphere is 1 inch. Give three possibilities for the dimensions of the cylinder.

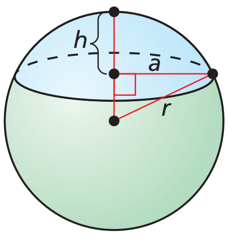
46. **PROBLEM SOLVING** A *spherical cap* is a portion of a sphere cut off by a plane. The formula for the volume of a spherical cap is $V = \frac{\pi h}{6}(3a^2 + h^2)$, where a is the radius of the base of the cap and h is the height of the cap. Use the diagram and given information to find the volume of each spherical cap.

a. $r = 5$ ft, $a = 4$ ft

b. $r = 34$ cm, $a = 30$ cm

c. $r = 13$ m, $h = 8$ m

d. $r = 75$ in., $h = 54$ in.



47. **CRITICAL THINKING** A sphere with a radius of 2 inches is inscribed in a right cone with a height of 6 inches. Find the surface area and the volume of the cone.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the triangle. Round decimal answers to the nearest tenth. *(Section 9.7)*

48. $A = 26°, C = 35°, b = 13$

49. $B = 102°, C = 43°, b = 21$

50. $a = 23, b = 24, c = 20$

51. $A = 103°, b = 15, c = 24$

11.5–11.8 What Did You Learn?

Core Vocabulary

volume, *p. 626*
Cavalieri's Principle, *p. 626*
density, *p. 628*
similar solids, *p. 630*
lateral surface of a cone, *p. 642*
chord of a sphere, *p. 648*
great circle, *p. 648*

Core Concepts

Section 11.5

Cavalieri's Principle, *p. 626*
Volume of a Prism, *p. 626*
Volume of a Cylinder, *p. 627*
Density, *p. 628*
Similar Solids, *p. 630*

Section 11.6

Volume of a Pyramid, *p. 636*

Section 11.7

Surface Area of a Right Cone, *p. 642*
Volume of a Cone, *p. 643*

Section 11.8

Surface Area of a Sphere, *p. 648*
Volume of a Sphere, *p. 650*

Mathematical Practices

1. Search online for advertisements for products that come in different sizes. Then compare the unit prices, as done in Exercise 44 on page 633. Do you get results similar to Exercise 44? Explain.
2. In Exercise 15 on page 639, explain why the volume changed by a factor of $\frac{1}{64}$.
3. In Exercise 38 on page 653, explain the steps you used to find the value of x.

Performance Task

Water Park Renovation

The city council will consider reopening the closed water park if your team can come up with a cost analysis for painting some of the structures, filling the pool water reservoirs, and resurfacing some of the surfaces. What is your plan to convince the city council to open the water park?

To explore the answers to these questions and more, go to ***BigIdeasMath.com***.

Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool
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ANSWERS

1. *Sample answer:* yes; The larger container usually has a lesser unit cost.
2. *Sample answer:* The scale factor is $\frac{1}{4}$ and the ratio of the volumes is the scale factor cubed.
3. *Sample answer:* Substitute $r = 2x + 6$ into the surface area formula and set equal to 784π, then solve for x.

ANSWERS

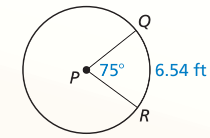
1. about 30.00 ft
2. about 56.57 cm
3. about 26.09 in.
4. 218 ft
5. about 169.65 in.^{2}
6. about 17.72 in.^{2}
7. 173.166 ft^2

11 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

11.1 Circumference and Arc Length *(pp. 593–600)*

The arc length of $\widehat{QR}$ is 6.54 feet. Find the radius of ⊙*P*.



$$\frac{\text{Arc length of } \widehat{QR}}{2\pi r} = \frac{m\widehat{QR}}{360^\circ}$$ Formula for arc length

$$\frac{6.54}{2\pi r} = \frac{75^\circ}{360^\circ}$$ Substitute.

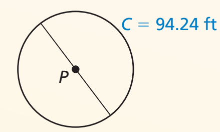
$$6.54(360) = 75(2\pi r)$$ Cross Products Property

$$5.00 \approx r$$ Solve for *r*.

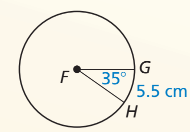
▶ The radius of ⊙*P* is about 5 feet.

Find the indicated measure.

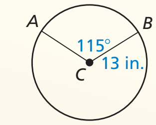
1. diameter of ⊙*P*



2. circumference of ⊙*F*



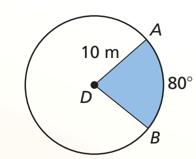
3. arc length of $\widehat{AB}$



4. A mountain bike tire has a diameter of 26 inches. To the nearest foot, how far does the tire travel when it makes 32 revolutions?

11.2 Areas of Circles and Sectors *(pp. 601–608)*

Find the area of sector *ADB*.



$$\text{Area of sector } ADB = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$$ Formula for area of a sector

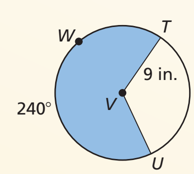
$$= \frac{80^\circ}{360^\circ} \cdot \pi \cdot 10^2$$ Substitute.

$$\approx 69.81$$ Use a calculator.

▶ The area of sector *ADB* is about 69.81 square meters.

Find the area of the blue shaded region.

5.



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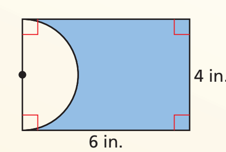

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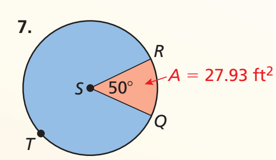
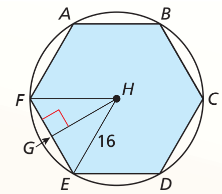


11.3 Areas of Polygons (pp. 609–616)

A regular hexagon is inscribed in $\odot H$. Find (a) $m\angle EHG$, and (b) the area of the hexagon.



a. $\angle FHE$ is a central angle, so $m\angle FHE = \frac{360°}{6} = 60°$.

Apothem $\overline{GH}$ bisects $\angle FHE$.

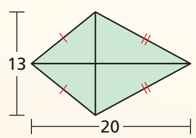
▶ So, $m\angle EHG = 30°$.

b. Because $\triangle EHG$ is a 30°-60°-90° triangle, $GE = \frac{1}{2} \cdot HE = 8$ and $GH = \sqrt{3} \cdot GE = 8\sqrt{3}$. So, $s = 2(GE) = 16$ and $a = GH = 8\sqrt{3}$.

▶ The area is $A = \frac{1}{2}a \cdot ns = \frac{1}{2}(8\sqrt{3})(6)(16) \approx 665.1$ square units.

Find the area of the kite or rhombus.

8.



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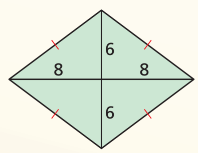

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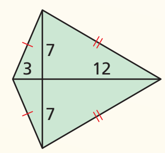
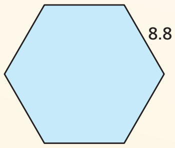


Find the area of the regular polygon.

11.



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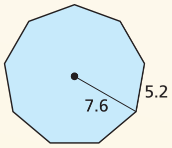
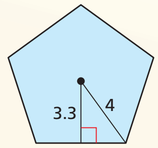


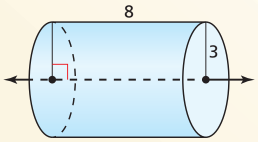
13.



14. A platter is in the shape of a regular octagon with an apothem of 6 inches. Find the area of the platter.

11.4 Three-Dimensional Figures (pp. 617–622)

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.



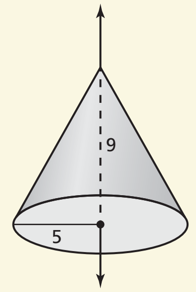
▶ The solid is a cylinder with a height of 8 and a radius of 3.

ANSWERS

8. 130 square units
9. 96 square units
10. 105 square units
11. about 201.20 square units
12. about 167.11 square units
13. about 37.30 square units
14. about 119.29 in.^{2}

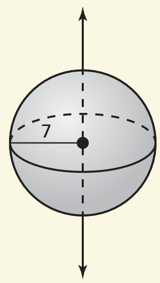
ANSWERS

15.



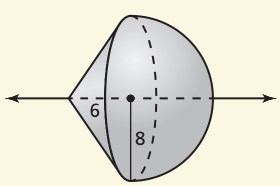
cone with height 9 and base radius 5

16.



sphere with radius 7

17.



cone with height 6 and base radius 8 and hemisphere with radius 8

18. rectangle

19. square

20. triangle

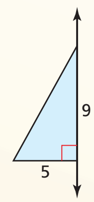
21. 11.34 m^3

22. about 100.53 mm^3

23. about 27.53 yd^3

Sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

15.



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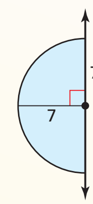

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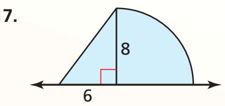
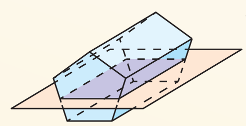
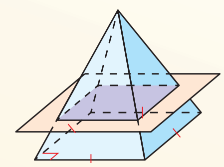


Describe the cross section formed by the intersection of the plane and the solid.

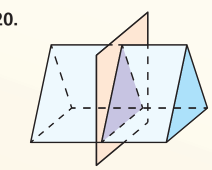
18.



19.



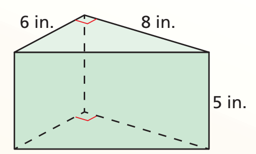
20.



11.5 Volumes of Prisms and Cylinders (pp. 625–634)

Find the volume of the triangular prism.

The area of a base is $B = \frac{1}{2}(6)(8) = 24 \text{ in.}^2$ and the height is $h = 5$ in.

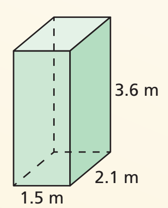


$V = Bh$	Formula for volume of a prism
$= 24(5)$	Substitute.
$= 120$	Simplify.

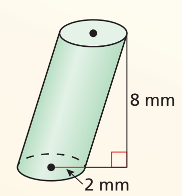
▶ The volume is 120 cubic inches.

Find the volume of the solid.

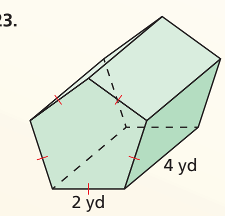
21.



22.

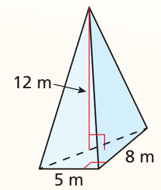


23.



11.6 Volumes of Pyramids (pp. 635–640)

Find the volume of the pyramid.



$V = \frac{1}{3}Bh$ Formula for volume of a pyramid

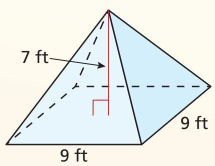
$= \frac{1}{3}\left(\frac{1}{2} \cdot 5 \cdot 8\right)(12)$ Substitute.

$= 80$ Simplify.

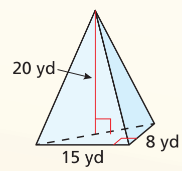
▶ The volume is 80 cubic meters.

Find the volume of the pyramid.

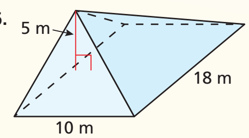
24.



25.



26.

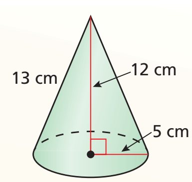


27. The volume of a square pyramid is 60 cubic inches and the height is 15 inches. Find the side length of the square base.

28. The volume of a square pyramid is 1024 cubic inches. The base has a side length of 16 inches. Find the height of the pyramid.

11.7 Surface Areas and Volumes of Cones (pp. 641–646)

Find the (a) surface area and (b) volume of the cone.



a. $S = \pi r^2 + \pi r\ell$ Formula for surface area of a cone

$= \pi \cdot 5^2 + \pi(5)(13)$ Substitute.

$= 90\pi$ Simplify.

≈ 282.74 Use a calculator.

▶ The surface area is 90π, or about 282.74 square centimeters.

b. $V = \frac{1}{3}\pi r^2 h$ Formula for volume of a cone

$= \frac{1}{3}\pi \cdot 5^2 \cdot 12$ Substitute.

$= 100\pi$ Simplify.

≈ 314.16 Use a calculator.

▶ The volume is 100π, or about 314.16 cubic centimeters.

ANSWERS

24. 189 ft^3
25. 400 yd^3
26. 300 m^3
27. about 3.46 in.
28. 12 in.

ANSWERS

29. $S \approx 678.58\text{ cm}^2$; $V \approx 1017.88\text{ cm}^3$

30. $S \approx 2513.27\text{ cm}^2$; $V \approx 8042.48\text{ cm}^3$

31. $S \approx 439.82\text{ m}^2$; $V \approx 562.10\text{ m}^3$

32. 15 cm

33. $S \approx 615.75\text{ in.}^2$; $V \approx 1436.76\text{ in.}^3$

34. $S \approx 907.92\text{ ft}^2$; $V \approx 2572.44\text{ ft}^3$

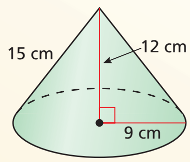
35. $S \approx 2827.43\text{ ft}^2$; $V \approx 14{,}137.17\text{ ft}^3$

36. $S \approx 74.8$ million km^2; $V \approx 60.8$ billion km^3

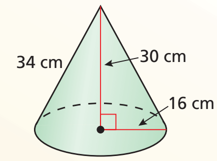
37. about 272.55 m^3

Find the surface area and the volume of the cone.

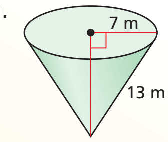
29.



30.



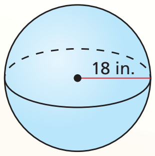
31.



32. A cone with a diameter of 16 centimeters has a volume of 320π cubic centimeters. Find the height of the cone.

11.8 Surface Areas and Volumes of Spheres *(pp. 647–654)*

Find the (a) surface area and (b) volume of the sphere.



a.

$S = 4\pi r^2$	Formula for surface area of a sphere
$= 4\pi(18)^2$	Substitute 18 for *r*.
$= 1296\pi$	Simplify.
≈ 4071.50	Use a calculator.

▶ The surface area is 1296π, or about 4071.50 square inches.

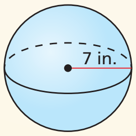
b.

$V = \frac{4}{3}\pi r^3$	Formula for volume of a sphere
$= \frac{4}{3}\pi(18)^3$	Substitute 18 for *r*.
$= 7776\pi$	Simplify.
$\approx 24{,}429.02$	Use a calculator.

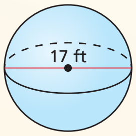
▶ The volume is 7776π, or about 24,429.02 cubic inches.

Find the surface area and the volume of the sphere.

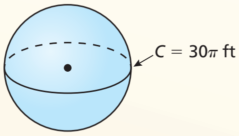
33.



34.



35.

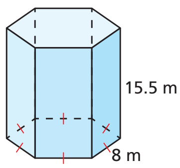


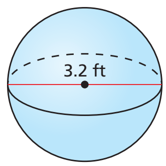
36. The shape of Mercury can be approximated by a sphere with a diameter of 4880 kilometers. Find the surface area and the volume of Mercury.

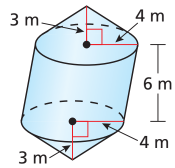
37. A solid is composed of a cube with a side length of 6 meters and a hemisphere with a diameter of 6 meters. Find the volume of the composite solid.

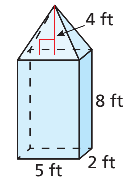
11 Chapter Test

Find the volume of the solid.

1. 

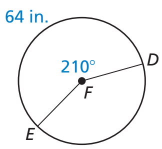
2. 

3. 

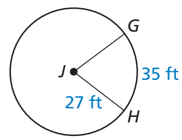
4. 

Find the indicated measure.

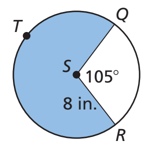
5. circumference of $\odot F$



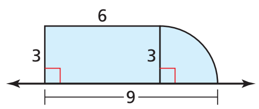
6. $m\widehat{GH}$



7. area of shaded sector

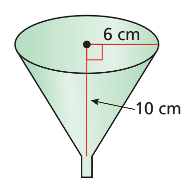


8. Sketch the composite solid produced by rotating the figure around the given axis. Then identify and describe the composite solid.



9. Find the surface area of a right cone with a diameter of 10 feet and a height of 12 feet.

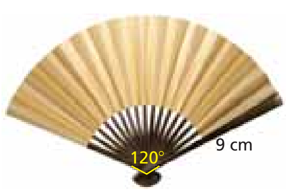
10. You have a funnel with the dimensions shown.
    a. Find the approximate volume of the funnel.
    b. You use the funnel to put oil in a car. Oil flows out of the funnel at a rate of 45 milliliters per second. How long will it take to empty the funnel when it is full of oil? (1 mL = 1 cm^3)
    c. How long would it take to empty a funnel with a radius of 10 centimeters and a height of 6 centimeters if oil flows out of the funnel at a rate of 45 milliliters per second?
    d. Explain why you can claim that the time calculated in part (c) is greater than the time calculated in part (b) without doing any calculations.



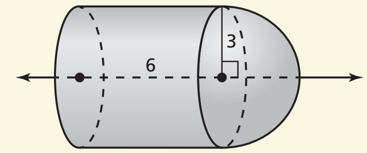
11. A water bottle in the shape of a cylinder has a volume of 500 cubic centimeters. The diameter of a base is 7.5 centimeters. What is the height of the bottle? Justify your answer.

12. Find the area of a dodecagon (12 sides) with a side length of 9 inches.

13. In general, a cardboard fan with a greater area does a better job of moving air and cooling you. The fan shown is a sector of a cardboard circle. Another fan has a radius of 6 centimeters and an intercepted arc of 150°. Which fan does a better job of cooling you?



ANSWERS

1. about 2577.29 m^3
2. about 17.16 ft^3
3. about 402.12 m^3
4. $93\frac{1}{3}$ ft^3
5. about 109.71 in.
6. about 74.27°
7. about 142.42 in.^{2}
8. 

   cylinder with height 6 and base radius 3, and hemisphere with radius 3
9. 90π ft^2 or about 282.74 ft^2
10. a. about 376.99 cm^3
    b. about 8.38 sec
    c. about 13.96 sec
    d. *Sample answer:* Changing the radius has a greater effect than changing the height.
11. about 11.32 cm; *Sample answer:* $500 = \pi(3.75)^3h$, so $h \approx 11.32$.
12. about 906.89 in.^{2}
13. the fan shown

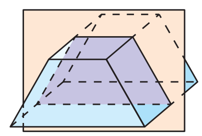
If students need help...	*If students got it...*
Lesson Tutorials	Resources by Chapter • Enrichment and Extension • Cumulative Review
Skills Review Handbook	Performance Task
BigIdeasMath.com	Start the *next* Section

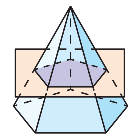
ANSWERS

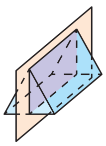
1. **a.** trapezoid
   b. pentagon
   c. rectangle
2. $\overline{PQ} \perp \overleftrightarrow{RS}$
3. **a.** about 4650 mm^3
   b. about 75,267 mm^3
4. A

11 Cumulative Assessment

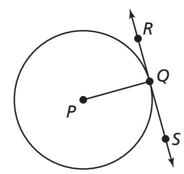
1. Identify the shape of the cross section formed by the intersection of the plane and the solid.

a. 

b. 

c. 

2. In the diagram, $\overleftrightarrow{RS}$ is tangent to $\odot P$ at Q and $\overline{PQ}$ is a radius of $\odot P$. What must be true about $\overleftrightarrow{RS}$ and $\overline{PQ}$? Select all that apply.

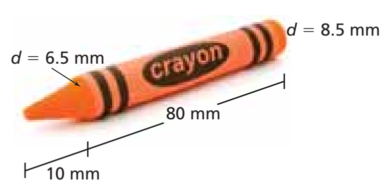


$PQ = \frac{1}{2}RS$	$PQ = RS$	$\overline{PQ}$ is tangent to $\odot P$.	$\overline{PQ} \perp \overleftrightarrow{RS}$

3. A crayon can be approximated by a composite solid made from a cylinder and a cone. A crayon box is a rectangular prism. The dimensions of a crayon and a crayon box containing 24 crayons are shown.

a. Find the volume of a crayon.

b. Find the amount of space within the crayon box not taken up by the crayons.



4. What is the equation of the line passing through the point (2, 5) that is parallel to the line $x + \frac{1}{2}y = -1$?

Ⓐ $y = -2x + 9$

Ⓑ $y = 2x + 1$

Ⓒ $y = \frac{1}{2}x + 4$

Ⓓ $y = -\frac{1}{2}x + 6$

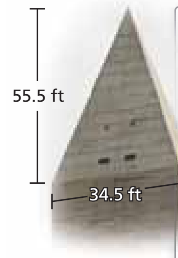
5. The top of the Washington Monument in Washington, D.C., is a square pyramid, called a *pyramidion*. What is the volume of the pyramidion?

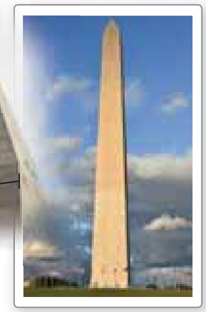
Ⓐ $22{,}019.63\text{ ft}^3$

Ⓑ $172{,}006.91\text{ ft}^3$

Ⓒ $66{,}058.88\text{ ft}^3$

Ⓓ $207{,}530.08\text{ ft}^3$

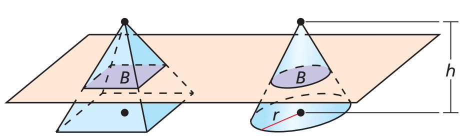




6. Prove or disprove that the point $\left(1, \sqrt{3}\right)$ lies on the circle centered at the origin and containing the point (0, 2).

7. Your friend claims that the house shown can be described as a composite solid made from a rectangular prism and a triangular prism. Do you support your friend's claim? Explain your reasoning.

8. The diagram shows a square pyramid and a cone. Both solids have the same height, h, and the base of the cone has radius r. According to Cavalieri's Principle, the solids will have the same volume if the square base has sides of length ____.



9. About 19,400 people live in a region with a 5-mile radius. Find the population density in people per square mile.

ANSWERS

5. A

6. *Sample answer:* The radius of the circle is 2.

$d = \sqrt{(0-1)^2 + \left(0 - \sqrt{3}\right)^2} = 2,$

so $\left(1, \sqrt{3}\right)$ is on the circle.

7. yes; *Sample answer:* The bottom part of the house has parallel rectangular bases at the bottom and top, and the top part of the house has parallel triangular bases on two of the sides.

8. $r\sqrt{\pi}$

9. about 247 people/mi^2