

5 Ratios and Rates

5.1 Ratios

5.2 Ratio Tables

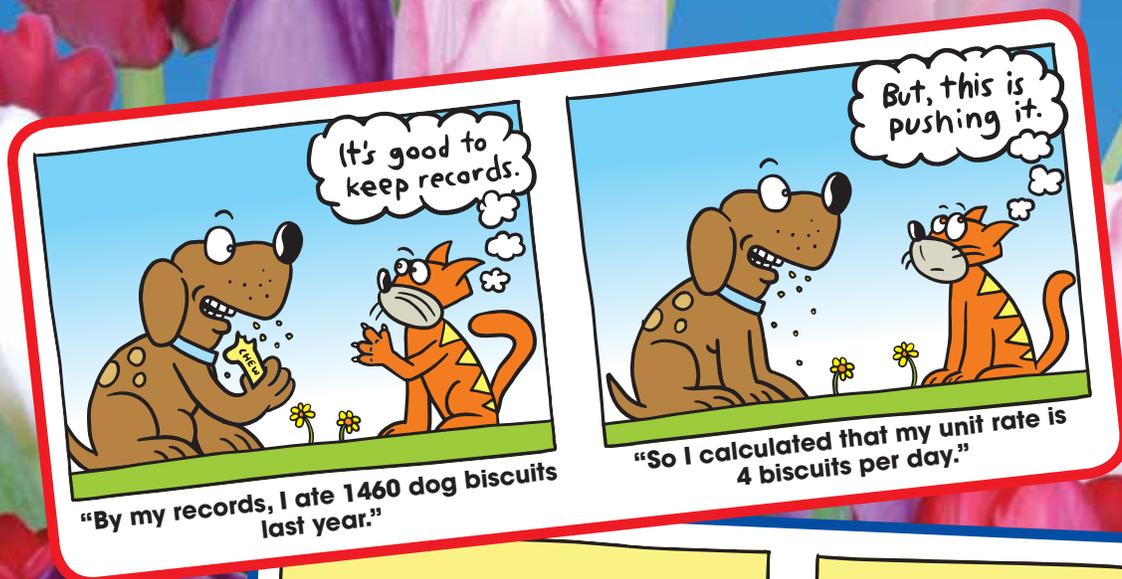
5.3 Rates

5.4 Comparing and Graphing Ratios

5.5 Percents

5.6 Solving Percent Problems

5.7 Converting Measures



What You Learned Before

Identifying Patterns (MACC.5.OA.2.3)

Example 1 Using the numbers from the table, find and state the rule in words. Then find the missing value.

x	y
1	6
2	12
3	18
4	

Each y-value is 6 times the x-value.

∴ The x-value times 6 equals the y-value.
The missing value is $6(4) = 24$.

Try It Yourself

Using the numbers from the table, find and state the rule in words. Then find the missing value.

1.

x	y
1	2
3	6
5	10
7	

2.

x	y
2	8
4	16
6	24
8	

3.

x	y
1	5
2	10
3	15
4	

Multiplying and Dividing by Fractions (MACC.5.NF.2.4a, MACC.6.NS.1.1)

Example 2 Find $\frac{5}{6} \cdot \frac{3}{4}$.

$$\begin{aligned} \frac{5}{6} \cdot \frac{3}{4} &= \frac{5 \cdot \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \cdot 4} \\ &= \frac{5}{8} \end{aligned}$$

Example 3 Find $2 \div \frac{9}{10}$.

$$\begin{aligned} 2 \div \frac{9}{10} &= 2 \cdot \frac{10}{9} \\ &= \frac{2 \cdot 10}{9} \\ &= \frac{20}{9} \end{aligned}$$

Multiply by the reciprocal of the divisor.

Try It Yourself

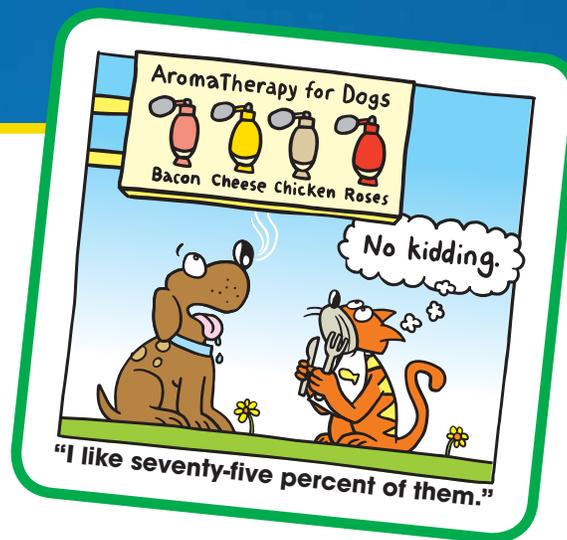
Evaluate the expression. Write the answer in simplest form.

4. $\frac{1}{5} \cdot \frac{13}{20}$

5. $\frac{3}{4} \cdot \frac{13}{25}$

6. $7 \div \frac{9}{10}$

7. $4 \div \frac{16}{17}$

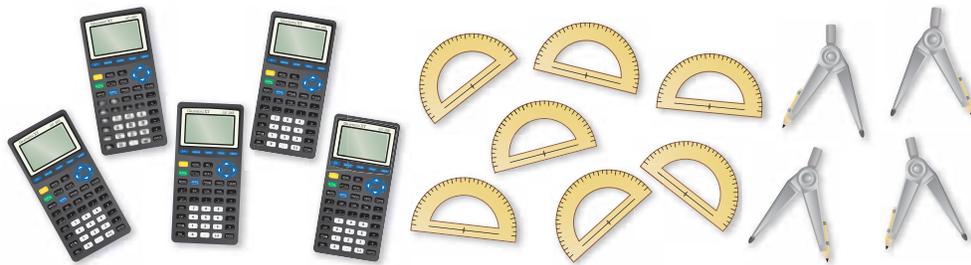


5.1 Ratios

Essential Question How can you represent a relationship between two quantities?

1 ACTIVITY: Comparing Quantities

Work with a partner. Use the collection of objects to complete each statement.



There are graphing calculators to protractors.

There are protractors to graphing calculators.

There are compasses to protractors.

There are graphing calculators to compasses.

There are protractors to total objects.

The number of graphing calculators is $\frac{\text{input}}{\text{input}}$ of the total number of objects.

2 ACTIVITY: Playing Garbage Basketball

Work with a partner.

- Take turns shooting a ball or other object into a wastebasket from a reasonable distance.

- Organize the numbers of shots you made and shots you missed in a chart.

- Write a statement similar to those in Activity 1 that describes the relationship between the number of shots you made and the number of shots you missed.
- Write a statement similar to those in Activity 1 that describes the relationship between the number of shots you made and the total number of shots.
- What fraction of your shots did you make? What fraction did you miss?



Ratios

In this lesson, you will

- understand the concept of a ratio.
- use ratios to describe the relationship between two quantities.

Learning Standard
MACC.6.RP.1.1

3 ACTIVITY: Reading a Diagram

Work with a partner. You mix different amounts of paint to create new colors. Write a statement that describes the relationship between the amounts of paint shown in each diagram.



There are  parts blue for every  parts green.



There are  for every .



Math Practice 4

Use a Table or Diagram

What are the quantities in this problem? How does a table or diagram represent the relationship between the quantities?

4 ACTIVITY: Describing Relationships

Work with a partner. Use a table or a diagram to represent the relationship between the two quantities.

- For every 3 boys standing in a line, there are 4 girls.
- For each vote Brian received, Sasha received 6 votes.
- A class counts the number of vehicles that pass by its school from 1:00 to 2:00 P.M. There are 3 times as many cars as trucks.
- A hand sanitizer contains 5 parts aloe for every 2 parts distilled water.

What Is Your Answer?

- IN YOUR OWN WORDS** How can you represent a relationship between two quantities? Give examples to support your explanation.
- MODELING** You make 48 pints of pink paint by using 5 pints of red paint for every 3 pints of white paint. Use a diagram to find the number of pints of red paint and white paint in your mixture. Explain.

Practice

Use what you learned about comparing two quantities to complete Exercises 4 and 5 on page 194.

Key Vocabulary

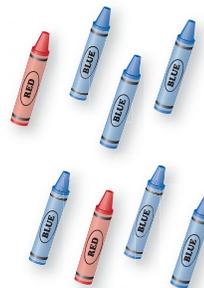
ratio, p. 192

Key Idea

Ratio

Words A **ratio** is a comparison of two quantities. Ratios can be part-to-part, part-to-whole, or whole-to-part comparisons.

Examples 2 red crayons *to* 6 blue crayons
1 red crayon *for every* 3 blue crayons
3 blue crayons *per* 1 red crayon
3 blue crayons *for each* red crayon
3 blue crayons *out of every* 4 crayons
2 red crayons *out of* 8 crayons



Algebra The ratio of a to b can be written as $a : b$.

EXAMPLE 1 Writing Ratios

Remember

Part-to-whole relationships compare a part of a whole to the whole. Fractions represent part-to-whole relationships. Part-to-part relationships compare a part of a whole to another part of the whole.

You have the coins shown.

a. Write the ratio of pennies to quarters.

6 pennies → 6 to 7 ← 7 quarters

So, the ratio of pennies to quarters is 6 to 7, or 6 : 7.

b. Write the ratio of quarters to dimes.

7 quarters → 7 to 3 ← 3 dimes

So, the ratio of quarters to dimes is 7 to 3, or 7 : 3.

c. Write the ratio of dimes to the total number of coins.

3 dimes → 3 to 16 ← 16 coins

So, the ratio of dimes to the total number of coins is 3 to 16, or 3 : 16.

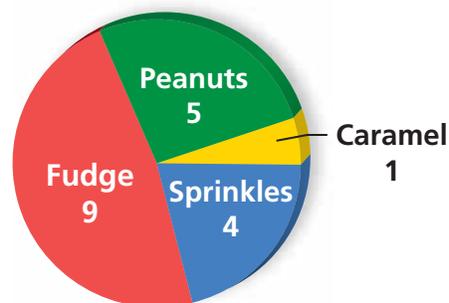


On Your Own

Now You're Ready
Exercises 6–13

- In Example 1, write the ratio of dimes to pennies.
- The circle graph shows the favorite ice-cream toppings of several students. Use ratio language to compare the number of students who favor peanuts to the total number of students.

Favorite Toppings

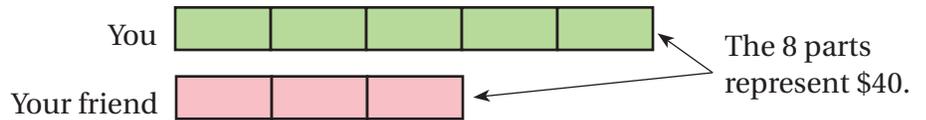


A *tape diagram* is a diagram that looks like a segment of tape. It shows the relationship between two quantities.

EXAMPLE 2 Using a Tape Diagram

The ratio of your monthly allowance to your friend's monthly allowance is 5 : 3. The monthly allowances total \$40. How much is each allowance?

To help visualize the problem, express the ratio 5 : 3 using a tape diagram.



Because there are 8 parts, you know that 1 part represents $\$40 \div 8 = \5 .

$$5 \text{ parts represent } \$5 \cdot 5 = \$25.$$

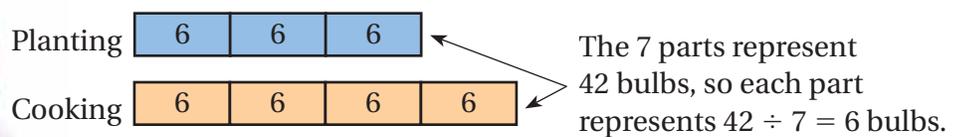
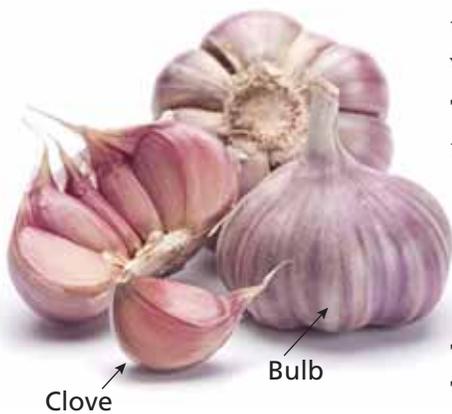
$$3 \text{ parts represent } \$5 \cdot 3 = \$15.$$

∴ So, your monthly allowance is \$25, and your friend's monthly allowance is \$15.

EXAMPLE 3 Using a Tape Diagram

You separate 42 bulbs of garlic into two groups: one for planting and one for cooking. You will plant 3 bulbs for every 4 bulbs that you will use for cooking. Each bulb has about 8 cloves. About how many cloves will you plant?

To help visualize the problem, express the ratio 3 for every 4 using a tape diagram.



There are $3 \cdot 6 = 18$ bulbs for planting and $4 \cdot 6 = 24$ bulbs for cooking. The group of 18 bulbs has about $18 \cdot 8 = 144$ cloves.

∴ So, you will plant about 144 cloves.

On Your Own

Now You're Ready
Exercises 15
and 16

3. **WHAT IF?** In Example 2, the ratio is 2 to 3. How much is each allowance?

4. **WHAT IF?** In Example 3, you will plant 1 bulb for every 2 bulbs that you will use for cooking. Will you plant more or fewer cloves than originally planned? Explain your reasoning.

Vocabulary and Concept Check

- VOCABULARY** The ratio of vowels to consonants in a word is 5 to 7. Are there more vowels or consonants in the word? Explain.
- NUMBER SENSE** You are comparing apples to oranges in a fruit bowl. Is the ratio 2 : 3 the same as the ratio 3 : 2? Explain.
- WHICH ONE DOESN'T BELONG?** Which ratio does *not* belong with the other three? Explain your reasoning.

2 parts to 5 parts

2 out of every 5

2 for each 5

2 for every 5

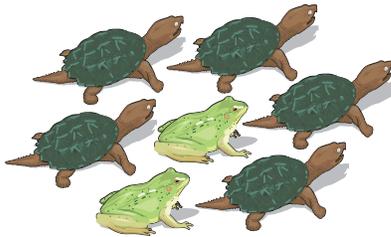
Practice and Problem Solving

Use a table or a diagram to represent the relationship between the two quantities.

- For each lion, there are 7 giraffes.
- For every 5 seats, there are 4 fans.

Write the ratio. Explain what the ratio means.

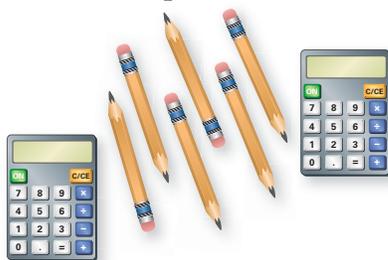
- 1 6. frogs to turtles



7. basketballs to soccer balls



8. calculators : pencils



9. shirts : pants



Use the table to write the ratio. Explain what the ratio means.

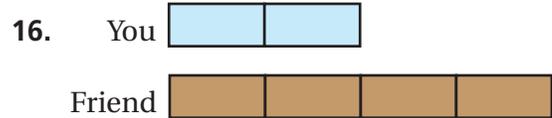
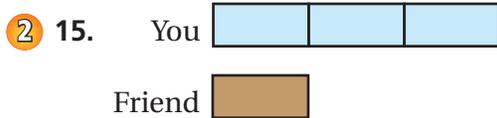
- dramas to movies
- comedies to movies
- movies : action
- movies : dramas

Topic	Stamps
Birds	7
Celebrity	14
Horses	5
Ships	9

Movie	Number
Drama	3
Comedy	8
Action	4

14. **STAMP COLLECTING** The table shows the numbers of stamps in a new stamp collection. Use ratio language to compare the number of celebrity stamps to the total number of stamps.

You and a friend tutor for a total of 12 hours. Use the tape diagram to find how many hours you tutor.



17. **REASONING** Twelve of the 28 students in a class have a dog. What is the ratio of students who have a dog to students who do not?

18. **GEOGRAPHY** In the continental United States, the ratio of states that border an ocean to states that do not border an ocean is 7 : 9. How many of the states border an ocean?

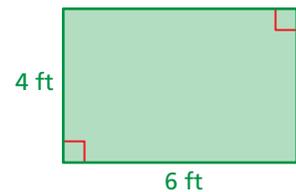
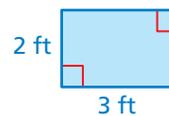


19. **CHECKERS** During a checkers game, there are 16 pieces left. The ratio of black to red is 3 : 5. How many black pieces are on the board? Explain how you found your answer.

20. **SCHOOL PLAY** There are 48 students in a school play. The ratio of boys to girls is 5 : 7. How many more girls than boys are in the play? Explain how you found your answer.

21. **GEOMETRY** Use the blue and green rectangles.

a. Find the ratio of the length of the blue rectangle to the length of the green rectangle. Repeat this for width, perimeter, and area.



b. Compare and contrast your ratios in part (a).

22. **PERIMETER** The ratio of the side lengths of a triangle is 2 : 3 : 4. The shortest side is 15 inches. What is the perimeter? Explain.

23. **PRECISION** You mix soda water, fruit punch concentrate, and ginger ale in the ratio of 1 : 2 : 5 to make fruit punch. How many pints of each ingredient should you use to make 4 gallons of fruit punch? Is your answer reasonable? Explain.

24. **Reasoning** There are 12 boys and 10 girls in your gym class. If 6 boys joined the class, how many girls would need to join for the ratio of boys to girls to remain the same? Justify your answer.



Fair Game Review What you learned in previous grades & lessons

Divide. (Section 2.6)

25. $13.8 \div 3$

26. $16.45 \div 5$

27. $53.13 \div 21$

28. $19.214 \div 13$

29. **MULTIPLE CHOICE** What is the value of the expression $x \div y$ when $x = 30$ and $y = 18$? (Section 3.1)

(A) $\frac{3}{5}$

(B) $1\frac{2}{3}$

(C) 12

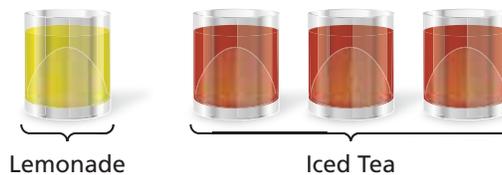
(D) 48

5.2 Ratio Tables

Essential Question How can you find two ratios that describe the same relationship?

1 ACTIVITY: Making a Mixture

Work with a partner. A mixture calls for 1 cup of lemonade and 3 cups of iced tea.



- a. How many total cups does the mixture contain? cups

For every cup of lemonade, there are cups of iced tea.

- b. How do you make a larger batch of this mixture? Describe your procedure and use the table below to organize your results. Add more columns to the table if needed.

Cups of Lemonade						
Cups of Iced Tea						
Total Cups						

- c. Which operations did you use to complete your table? Do you think there is more than one way to complete the table? Explain.
- d. How many total cups are in your final mixture? How many of those cups are lemonade? How many are iced tea? Compare your results with those of other groups in your class.
- e. Suppose you take a sip from every group's final mixture. Do you think all the mixtures should taste the same? Do you think the color of all the mixtures should be the same? Explain your reasoning.
- f. Why do you think it is useful to use a table when organizing your results in this activity? Explain.



Ratios

In this lesson, you will

- use ratio tables to find equivalent ratios.
- solve real-life problems.

Learning Standards

MACC.6.RP.1.1

MACC.6.RP.1.3a

2 ACTIVITY: Using a Multiplication Table

Math Practice 2

Use Operations

For each part of this problem, how do you know which operation to use?

Work with a partner. Use the information in Activity 1 and the multiplication table below.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48

- A mixture contains 8 cups of lemonade. How many cups of iced tea are in the mixture?
- A mixture contains 21 cups of iced tea. How many cups of lemonade are in the mixture?
- A mixture has a total of 40 cups. How many cups are lemonade? How many are iced tea?
- REPEATED REASONING** Explain how a multiplication table may have helped you in Activity 1.

3 ACTIVITY: Using More than One Ratio to Describe a Quantity

Work with a partner.

- Find the ratio of pitchers of lemonade to pitchers of iced tea.



- How can you divide the pitchers into equal groups? Is there more than one way? Use your results to describe the entire collection of pitchers.
- Three more pitchers of lemonade are added. Is there more than one way to divide the pitchers into equal groups? Explain.
- The number of pitchers of lemonade and iced tea are doubled. Can you use the ratio in part (b) to describe the entire collection of pitchers? Explain.

What Is Your Answer?

- IN YOUR OWN WORDS** How can you find two ratios that describe the same relationship? Give examples to support your explanation.

Practice

Use what you learned about ratios to complete Exercises 4 and 5 on page 201.

Key Vocabulary

equivalent ratios,
p. 198
ratio table, p. 198

Two ratios that describe the same relationship are **equivalent ratios**. You can find equivalent ratios by:

- adding or subtracting quantities in equivalent ratios.
- multiplying or dividing each quantity in a ratio by the same number.

You can find and organize equivalent ratios in a **ratio table**.

EXAMPLE 1 Completing Ratio Tables

Find the missing value(s) in each ratio table. Then write the equivalent ratios.

a.

Pens	1	2	
Pencils	3		9

b.

Dogs	4		24
Cats	6	12	

- a. You can use repeated addition with the original ratio to find the missing values.

Pens	1	2	3
Pencils	3	6	9

∴ The equivalent ratios are 1 : 3, 2 : 6, and 3 : 9.

- b. You can use multiplication to find the missing values.

Dogs	4	8	24
Cats	6	12	36

∴ The equivalent ratios are 4 : 6, 8 : 12, and 24 : 36.

On Your Own

 Now You're Ready
Exercises 6–11

Find the missing value(s) in the ratio table. Then write the equivalent ratios.

1.

Plantains	4		12
Bananas	3	6	

2.

Euros	5	10	
Dollars	4		32

EXAMPLE 2 Making a Ratio Table

You are making sugar water for your hummingbird feeder. A website indicates to use 4 parts of water for every 1 part of sugar. You use 20 cups of water. How much sugar do you need?

You can solve this problem by using equivalent ratios. The ratio of water to sugar is 4 parts to 1 part. So, for every 4 cups of water, you need 1 cup of sugar. Find an equivalent ratio with 20 parts water.

Method 1: Use a ratio table and addition.

You can think of making a larger batch of sugar water as combining several batches of 4 to 1 mixtures. Use addition to obtain 20 in the water column.



Water (cups)	4	8	12	16	20
Sugar (cups)	1	2	3	4	5

Arrows above the table show additions of 4 to the water column: 4 to 8, 8 to 12, 12 to 16, and 16 to 20. Arrows below the table show additions of 1 to the sugar column: 1 to 2, 2 to 3, 3 to 4, and 4 to 5.

The ratio 20 to 5 is equivalent to 4 to 1.

∴ So, you need 5 cups of sugar.

Method 2: Use a ratio table and multiplication.

You multiplied the amount of water in the recipe by 5 because $20 \div 4 = 5$. So, you need to multiply the amount of sugar by 5. Multiply each part of the ratio in the original recipe by 5.

Water (cups)	4	20
Sugar (cups)	1	5

Arrows above and below the table show multiplication by 5: 4 to 20 for water and 1 to 5 for sugar.

The ratio 20 to 5 is equivalent to 4 to 1.

∴ So, you need 5 cups of sugar.

Study Tip

In Example 2, Method 1, notice that you can eliminate a step by adding columns 2 and 3 to obtain $8 + 12 = 20$ cups of water for $2 + 3 = 5$ cups of sugar.

On Your Own

Now You're Ready
Exercises 13
and 14

- WHAT IF?** You use 24 cups of water. How much sugar do you need?
- You make a sweeter mixture of sugar water for your hummingbird feeder using 3 parts of water for every 1 part of sugar. You use 9 quarts of water. How much sugar do you need?

EXAMPLE 3 Using a Ratio Table

The nutrition facts label on a box of crackers shows that there are 240 milligrams of sodium in every 36 crackers.

a. You eat 15 crackers. How much sodium do you consume?

The ratio of sodium to crackers is 240 to 36. Use a ratio table to find an equivalent ratio with 15 crackers.

Sodium (milligrams)	240	120	20	100
Crackers	36	18	3	15

$\div 2$ $\div 6$ $\times 5$
 $\div 2$ $\div 6$ $\times 5$



The ratio 100 to 15 is equivalent to 240 to 36.

So, you consume 100 milligrams of sodium.

b. You eat 21 crackers. How much sodium do you consume?

Notice that you can add the two middle columns in the table above.

So, you consume $120 + 20 = 140$ milligrams of sodium in $18 + 3 = 21$ crackers.

Study Tip

In Example 3, notice that you could use one step in the ratio table: multiply by

$$\frac{1}{2} \cdot \frac{1}{6} \cdot 5 = \frac{5}{12}$$

EXAMPLE 4 Using a Ratio Table

You mix 3 pints of yellow paint for every 4 pints of blue paint to make green paint. You use 10 pints of blue paint. How much green paint do you make?

The ratio of yellow paint to blue paint is 3 to 4. Use a ratio table to find an equivalent ratio with 10 parts blue paint.

Yellow (pints)	3	$\frac{3}{2}$	$7\frac{1}{2}$
Blue (pints)	4	2	10

$\div 2$ $\times 5$
 $\div 2$ $\times 5$

You use $7\frac{1}{2}$ pints of yellow paint and 10 pints of blue paint.

So, you make $7\frac{1}{2} + 10 = 17\frac{1}{2}$ pints of green paint.

Study Tip

In Example 4, notice that you could use one step in the ratio table: multiply by

$$\frac{1}{2} \cdot 5 = \frac{5}{2}$$

On Your Own

- WHAT IF?** In Example 3, you eat 24 crackers. How much sodium do you consume?
- WHAT IF?** In Example 4, you mix 2 pints of yellow paint for every 3 pints of blue paint. You use 5 pints of yellow paint. How much green paint do you make?

Now You're Ready
Exercises 15
and 16

5.2 Exercises

Vocabulary and Concept Check

- VOCABULARY** How can you tell whether two ratios are equivalent?
- NUMBER SENSE** Consider the ratio 3 : 5. Can you create an equivalent ratio by adding the same number to each quantity in the ratio? Explain.
- WHICH ONE DOESN'T BELONG?** Which ratio does *not* belong with the other three? Explain your reasoning.

3 : 4

9 : 12

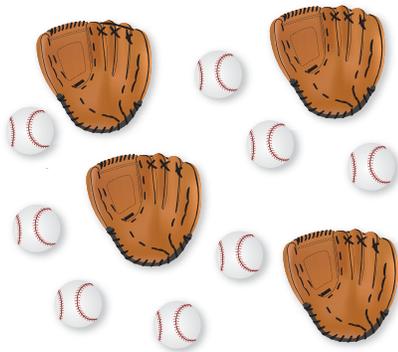
12 : 15

12 : 16

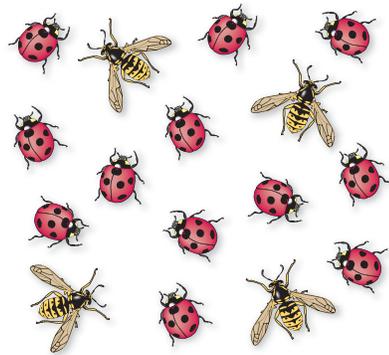
Practice and Problem Solving

Write several ratios that describe the collection.

4. baseballs to gloves



5. ladybugs to bees



Find the missing value(s) in the ratio table. Then write the equivalent ratios.

1 6.

Boys	1	
Girls	5	10

7.

Violins	8	24
Cellos	3	

8.

Taxis	6		36
Buses	5	15	

9.

Burgers	3		9
Hot Dogs	5	10	

10.

Towels	14	7	
Blankets	8		16

11.

Forks	16	8	
Spoons	10		30

12. **WORK** Your neighbor pays you \$17 for every 2 hours you work. You work for 8 hours on Saturday. How much does your neighbor owe you?

Complete the ratio table to solve the problem.

- 2 13. For every 3 tickets you sell, your friend sells 4. You sell a total of 12 tickets. How many does your friend sell?

You	3			12
Friend	4			

14. A store sells 2 printers for every 5 computers. The store sells 40 computers. How many printers does the store sell?

Printers	2		8	
Computers	5	10		40

- 3 15. First and second place in a contest use a ratio to share a cash prize. When first place pays \$100, second place pays \$60. How much does first place pay when second place pays \$36?

First	100		
Second	60		36

16. A grade has 81 girls and 72 boys. The grade is split into groups that have the same ratio of girls to boys as the whole grade. How many girls are in a group that has 16 boys?

Girls	81		
Boys	72		16

ERROR ANALYSIS Describe and correct the error in making the ratio table.

17. 

A	3	8	13
B	7	12	17

18. 

A	5	25	125
B	3	9	27

19. **DONATION** A sports store donates basketballs and soccer balls to the boys and girls club. The ratio of basketballs to soccer balls is 7 : 6. The store donates 24 soccer balls. How many basketballs does the store donate?

20. **DOWNLOAD** You are downloading songs to your MP3 player. The ratio of pop songs to rock songs is 5 : 4. You download 40 pop songs. How many rock songs do you download?



SCRAMBLED EGGS In Exercises 21–25, use the ratio table showing different batches of the same recipe for scrambled eggs.

Recipe	A	B	C	D	E	F
Servings	4	2	6	3	5	9
Eggs	8	4	12	6	10	18
Milk (cups)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{5}{8}$	$1\frac{1}{8}$

21. How can you use Recipes B and D to create Recipe E?
 22. How can you use Recipes C and D to create Recipe F?
 23. How can you use Recipes B and C to create Recipe A?

24. How can you use Recipes C and F to create Recipe D?

25. Describe one way to use the recipes to create a batch with 11 servings.

Two whole numbers A and B satisfy the following conditions. Find A and B .

26. $A + B = 30$
 $A : B$ is equivalent to $2 : 3$.

27. $A + B = 44$
 $A : B$ is equivalent to $4 : 7$.

28. $A - B = 18$
 $A : B$ is equivalent to $11 : 5$.

29. $A - B = 25$
 $A : B$ is equivalent to $13 : 8$.

Nutrition Facts	
Serving Size: 1 ounce (28g)	
Amount Per Serving	
Calories 161	Calories from Fat 109
% Daily Value*	
Total Fat 13g	20%
Saturated Fat 3g	13%
Trans Fat	
Cholesterol 0mg	0%
Sodium 4mg	0%
Total Carbohydrate 9g	3%
Dietary Fiber 1g	3%
Sugars 1g	
Protein 4g	
Vitamin A 0%	Vitamin C 0%
Calcium 1%	Iron 9%

*Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.

30. **CASHEWS** The nutrition facts label on a container of dry roasted cashews indicates there are 161 calories in 28 grams. You eat 9 cashews totaling 12 grams.

- How many calories do you consume?
- How many cashews are in one serving?

31. **REASONING** The ratio of three numbers is $4 : 3 : 1$. The sum of the numbers is 64. What is the greatest number?

32. **SURVEY** Seven out of every 8 students surveyed owns a bike. The difference between the number of students who own a bike and those who do not is 72. How many students were surveyed?

33. **BUG COLLECTION** You and a classmate have a bug collection for science class. You find 5 out of every 9 bugs in the collection. You find 4 more bugs than your classmate. How many bugs are in the collection?

34. **Problem Solving** You and a friend each have a collection of tokens. Initially, for every 8 tokens you had, your friend had 3. After you give half of your tokens to your friend, your friend now has 18 more tokens than you. Initially, how many more tokens did you have than your friend?



Fair Game Review What you learned in previous grades & lessons

Factor the expression using the GCF. (Section 3.4)

35. $54 + 27$

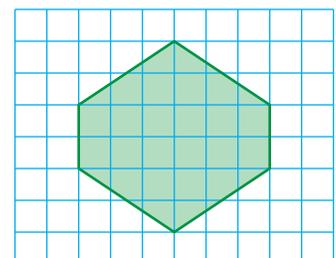
36. $60x - 84$

37. $42x + 28y$

38. **MULTIPLE CHOICE** Which expression does *not* give the area of the shaded figure? (Section 4.3)

(A) $2(6) + 2\left(\frac{1}{2}(6)(2)\right)$ (B) $2\left(\frac{1}{2}(3)(2 + 6)\right)$

(C) $6(6) - 4\left(\frac{1}{2}(3)(2)\right)$ (D) $6(6) - \frac{1}{2}(6)(2)$



5.3 Rates

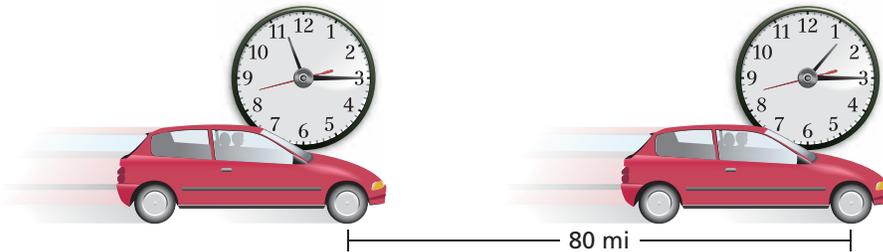
Essential Question How can you use rates to describe changes in real-life problems?

1 ACTIVITY: Stories Without Words

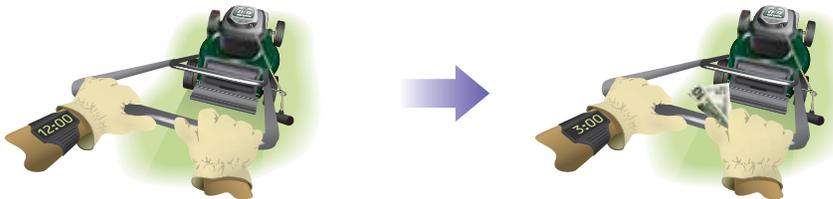
Work with a partner. Each diagram shows a story problem.

- Describe the story problem in your own words.
- Write the rate indicated by the diagram. What are the units?

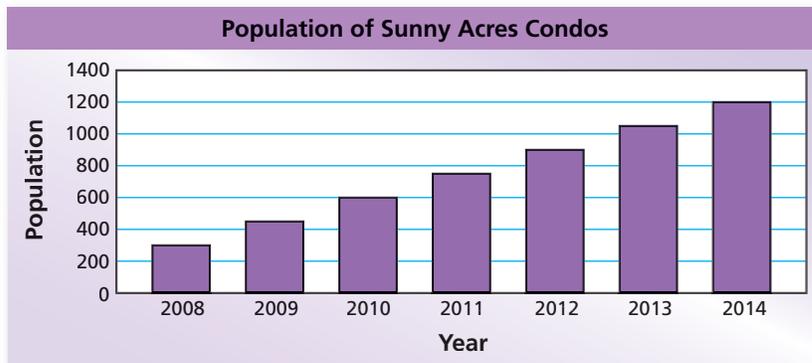
a.



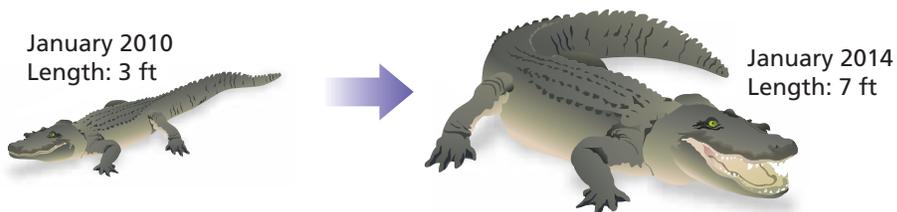
b.



c.



d.



Rates

In this lesson, you will

- understand the concepts of rates and unit rates.
- write unit rates.
- solve real-life problems.

Learning Standards

MACC.6.RP.1.2

MACC.6.RP.1.3a

MACC.6.RP.1.3b

2 ACTIVITY: Finding Equivalent Rates

Math Practice 6

Specify Units

How do the given units help you find the units for your answer?

Work with a partner. Use the diagrams in Activity 1. Explain how you found each answer.

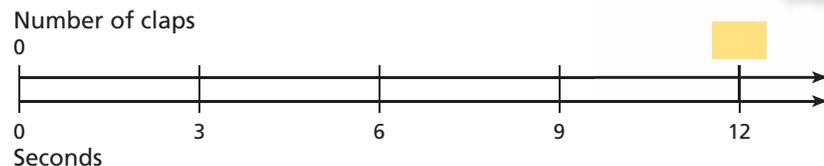
- How many miles does the car travel in 1 hour?
- How much money does the person earn every hour?
- How much does the population of Sunny Acres Condos increase each year?
- How many feet does the alligator grow per year?

3 ACTIVITY: Using a Double Number Line

Work with a partner. Count the number of times you can clap your hands in 12 seconds. Have your partner keep track of the time and record your results.



- Use the results to complete the double number line.



- Explain how to use the double number line to find the number of times you clap your hands in 6 seconds and in 4 seconds.
- Find the number of times you can clap your hands in 1 minute. Explain how you found your answer.
- How can you find the number of times you can clap your hands in 2 minutes? 3 minutes? Explain.

What Is Your Answer?

- IN YOUR OWN WORDS** How can you use rates to describe changes in real-life problems? Give examples to support your explanation.
- MODELING** Use a double number line to model each story in Activity 1. Show how to use the double number line to answer each question in Activity 2. Why is a double number line a good problem-solving tool for these types of problems?

Practice

Use what you learned about rates to complete Exercises 3 and 4 on page 208.

Key Vocabulary

rate, p. 206
unit rate, p. 206
equivalent rates,
p. 206

Key Idea

Rate and Unit Rate

Words A **rate** is a ratio of two quantities using different units. A **unit rate** compares a quantity to one unit of another quantity. **Equivalent rates** have the same unit rate.

Numbers You pay \$27 for 3 pizzas.

Rate: \$27 : 3 pizzas



Unit rate: \$9 : 1 pizza

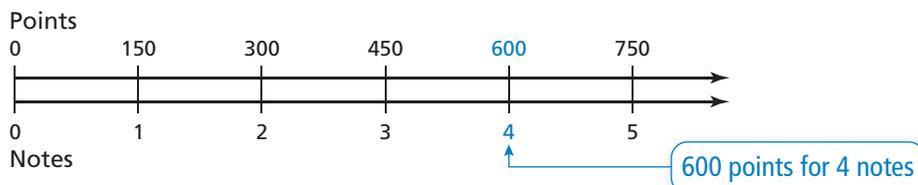
Algebra Rate: a units : b units Unit rate: $\frac{a}{b}$ units : 1 unit

Study Tip

In a rate $a : b$, you can divide both a and b by b to find the unit rate.

EXAMPLE 1 Writing a Rate

The double number line shows the rate at which you earn points for successfully hitting notes in a music video game. Write a rate that represents this situation.



One possible rate is 600 points for every 4 notes.

EXAMPLE 2 Finding a Unit Rate



A piece of space junk travels 5 miles in 8 seconds. How far does it travel per second?

Use a ratio table and divide by 8 to write an equivalent rate in which the time is 1 second.

Distance (miles)	5	$\frac{5}{8}$
Time (seconds)	8	1

Red arrows indicate dividing both 5 and 8 by 8 to get $\frac{5}{8}$ and 1.

The rate 5 miles : 8 seconds is equivalent to $\frac{5}{8}$ mile : 1 second.

So, the space junk travels $\frac{5}{8}$ mile per second.

On Your Own

Now You're Ready
Exercises 3–14

- Write another rate that represents the situation in Example 1.
- A Japanese bullet train travels 558 miles in 3 hours. How far does it travel every hour?
- You pay \$8 for 16 ounces of sliced turkey. Write a rate that gives the price for each ounce of turkey.

EXAMPLE 3 Finding Equivalent Rates

- a. A chef buys 6 pounds of salmon fillets for \$51. How much will the chef pay for 9 more pounds of salmon fillets?**

Using a ratio table, divide to find the unit rate and then multiply to find the cost for 9 pounds of salmon fillets.



Cost (dollars)	51	8.5	76.5
Salmon (pounds)	6	1	9

Diagram showing the process of finding the unit rate and then the cost for 9 pounds. Red arrows indicate the operations: $\div 6$ (from 51 to 8.5) and $\times 9$ (from 8.5 to 76.5) in the top row; $\div 6$ (from 6 to 1) and $\times 9$ (from 1 to 9) in the bottom row.

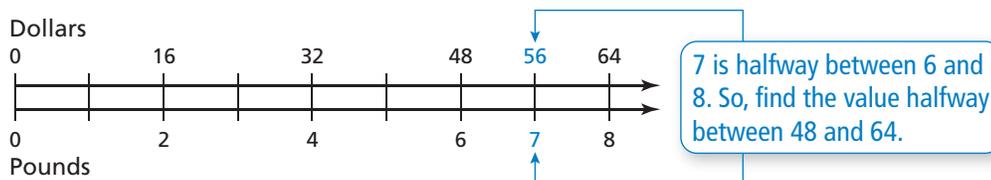
∴ So, the chef will pay \$76.50 for 9 more pounds of salmon fillets.

Study Tip

The unit rate of cost per unit is called *unit cost*. The unit cost of the salmon in Example 3(a) is \$8.50 per pound.

- b. You buy 2 pounds of tilapia fillets for \$16. What is the cost for 7 pounds of tilapia fillets?**

Because \$16 is easily divided into halves, fourths, and eighths, it is appropriate to model the rate using a double number line.



∴ So, the cost for 7 pounds of tilapia fillets is \$56.

On Your Own

Now You're Ready
Exercises 17
and 18

- Your download speed is 3 megabytes every 4 seconds.
 - How many megabytes can you download in 1 minute?
 - Construct a double number line that represents the situation. How many megabytes can you download in 10 seconds?

Vocabulary and Concept Check

- WRITING** Describe a unit rate that you use in real life.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

What is the cost per bagel?

What is the cost per dozen bagels?

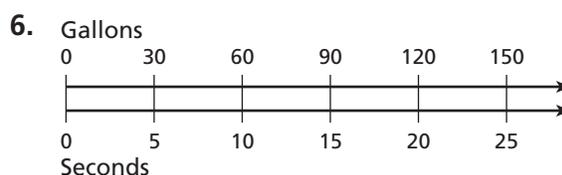
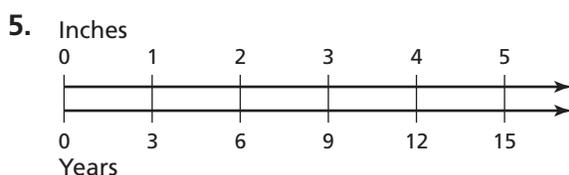
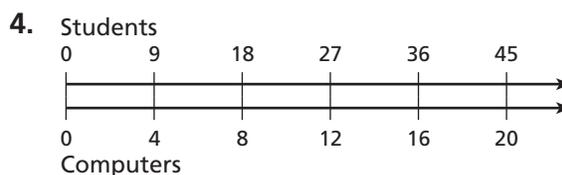
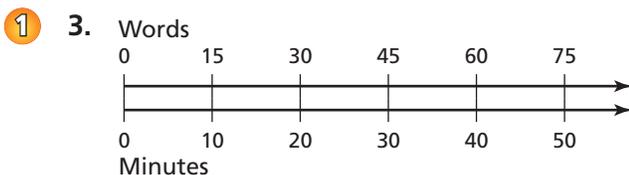
What is the unit cost of a bagel?

How much does each bagel cost?



Practice and Problem Solving

Write a rate that represents the situation.



Write a unit rate for the situation.

7. \$28 saved in 4 weeks
8. 18 necklaces made in 3 hours
9. 270 miles in 6 hours
10. 228 students in 12 classes
11. 2520 kilobytes in 18 seconds
12. 880 calories in 8 servings
13. 1080 miles on 15 gallons
14. \$12.50 for 5 ounces
15. **LIGHTNING** Lightning strikes Earth 1000 times in 10 seconds. How many times does lightning strike per second?
16. **HEART RATE** Your heart beats 240 times in 4 minutes. How many times does your heart beat each minute?
17. **CAR WASH** You earn \$35 for washing 7 cars. How much do you earn for washing 4 cars?
18. **5K RACE** You jog 2 kilometers in 12 minutes. At this rate, how long will it take you to complete a 5-kilometer race?



Decide whether the rates are equivalent.

19. 24 laps in 6 minutes
72 laps in 18 minutes
20. 126 points every 3 games
210 points every 5 games
21. 15 breaths every 36 seconds
90 breaths every 3 minutes
22. \$16 for 4 pounds
\$1 for 4 ounces
23. **PRINTER** A printer prints 28 photos in 8 minutes.
- How many minutes does it take to print 21 more photos?
 - Construct a double number line diagram that represents the situation. How many minutes does it take to print 35 more photos?

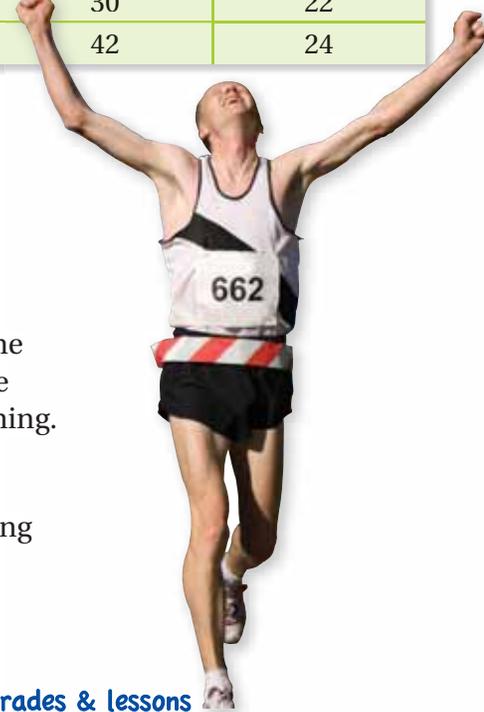


24. **SUN VISOR** An athletic director pays \$90 for 12 sun visors for the softball team.
- How much will the athletic director pay to buy 15 more sun visors?
 - Construct a double number line diagram that represents the situation. What is the cost of 16 sun visors?

25. **FOOD DRIVE** The table shows the amounts of food collected by two homerooms. Homeroom A collects 21 additional items of food. How many more items does Homeroom B need to collect to have more items per student?

	Homeroom A	Homeroom B
Students	24	16
Canned Food	30	22
Dry Food	42	24

26. **MARATHON** A runner completed a 26.2-mile marathon in 210 minutes.
- Estimate the unit rate, in miles per minute.
 - Estimate the unit rate, in minutes per mile.
 - Another runner says, "I averaged 10-minute miles in the marathon." Is this runner talking about the kind of rate described in part (a) or in part (b)? Explain your reasoning.
27. **Logic** You can do one-half of a job in an hour. Your friend can do one-third of the same job in an hour. How long will it take to do the job if you work together?



Fair Game Review what you learned in previous grades & lessons

Write two fractions that are equivalent to the given fraction. (*Skills Review Handbook*)

28. $\frac{1}{3}$

29. $\frac{5}{6}$

30. $\frac{2}{5}$

31. $\frac{4}{9}$

32. **MULTIPLE CHOICE** Which expression is equivalent to $6(x) - 6(2)$? (*Section 3.4*)

(A) $2(x - 6)$

(B) $6(x - 2)$

(C) $12(x - 1)$

(D) $36(x - 2)$

5.4 Comparing and Graphing Ratios

Essential Question How can you compare two ratios?

1 ACTIVITY: Comparing Ratio Tables

Work with a partner.

- You make purple frosting by adding 1 drop of red food coloring for every 3 drops of blue food coloring.
 - Your teacher makes purple frosting by adding 3 drops of red food coloring for every 5 drops of blue food coloring.
- a. Copy and complete the ratio table for each frosting mixture.

Your Frosting	
Drops of Red	Drops of Blue
1	
2	
3	
4	
5	



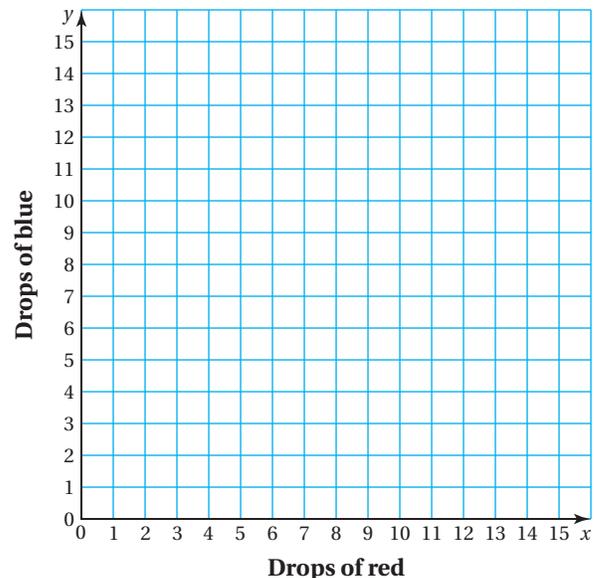
Your Teacher's Frosting	
Drops of Red	Drops of Blue
3	
6	
9	
12	
15	

- b. Whose frosting is bluer? Whose frosting is redder? Justify your answers.
- c. **STRUCTURE** Insert and complete a new column for each ratio table above that shows the total number of drops. How can you use this column to answer part (b)?

2 ACTIVITY: Graphing from a Ratio Table

Work with a partner.

- a. Explain how you can use the values from the ratio table for your frosting to create a graph in the coordinate plane.
- b. Use the values in the table to plot the points. Then connect the points and describe the graph. What do you notice?
- c. What does the line represent?



Ratios and Rates

In this lesson, you will

- compare ratios.
- compare unit rates.
- graph ordered pairs to compare ratios and rates.

Learning Standards

MACC.6.RP.1.2

MACC.6.RP.1.3a

3 ACTIVITY: Comparing Graphs from Ratio Tables

Math Practice 7

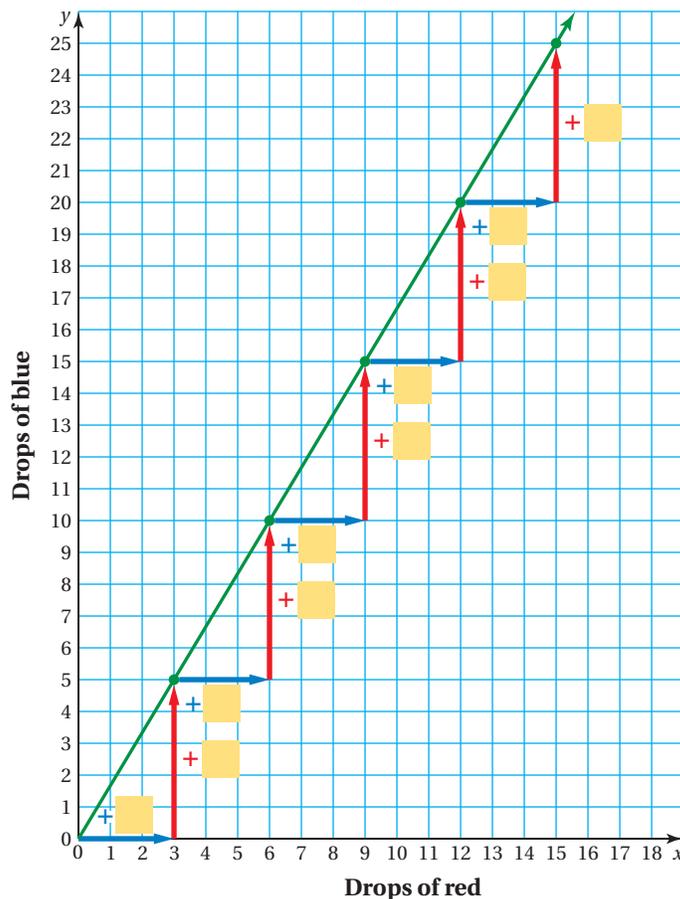
Look for Patterns

What patterns do you notice in the graph? What does this tell you about the problem?

Work with a partner. The graph shows the values from the ratio table for your teacher's frosting.

- a. Complete the table and the graph.

Your Teacher's Frosting	
Drops of Red	Drops of Blue
3	
6	
9	
12	
15	



- b. Explain the relationship between the entries in the ratio table and the points on the graph.
- c. How is this graph similar to the graph in Activity 2? How is it different?
- d. How can you use the graphs to determine whose frosting has more red or blue in it? Explain.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you compare two ratios?
5. **PRECISION** Your teacher's frosting mixture has 7 drops of red in it. How can you use the graph to find how many drops of blue are needed to make the purple frosting? Is your answer exact? Explain.

Practice

Use what you learned about comparing ratios to complete Exercises 3 and 4 on page 214.

One way to compare ratios is by using ratio tables.

EXAMPLE 1 Comparing Ratios



You mix 8 tablespoons of hot sauce and 3 cups of salsa in a green bowl. You mix 12 tablespoons of hot sauce and 4 cups of salsa in an orange bowl. Which mixture is hotter?

Use ratio tables to compare the mixtures. Find a larger batch of each mixture in which the amount of hot sauce or salsa is the same.

Green Bowl

		$\times 4$
Hot Sauce (tablespoons)	8	32
Salsa (cups)	3	12
		$\times 4$

Orange Bowl

		$\times 3$
Hot Sauce (tablespoons)	12	36
Salsa (cups)	4	12
		$\times 3$



The tables show that for a larger batch of each mixture using 12 cups of salsa, the orange bowl would have $36 - 32 = 4$ more tablespoons of hot sauce.

So, the mixture in the orange bowl is hotter.

EXAMPLE 2 Comparing Unit Rates



Which bag of dog food is the better buy?

Use ratio tables to find and compare the unit costs.

20-Pound Bag

		$\div 20$
Cost (dollars)	17.20	0.86
Food (pounds)	20	1
		$\div 20$

30-Pound Bag

		$\div 30$
Cost (dollars)	25.20	0.84
Food (pounds)	30	1
		$\div 30$

The 20-pound bag costs \$0.86 per pound, and the 30-pound bag costs \$0.84 per pound.

Because \$0.84 is less than \$0.86, the 30-pound bag is the better buy.

On Your Own

- In Example 1, you mix 10 tablespoons of hot sauce and 3 cups of salsa in a red bowl. Which mixture is the mildest? Explain.
- A 30-pack of paper towels costs \$48.30. A 32-pack costs \$49.60. Which is the better buy? Explain.

Now You're Ready
Exercises 3–10

EXAMPLE 3 Graphing Values from Ratio Tables

A hot-air balloon rises 9 meters every 3 seconds. A blimp rises 7 meters every 2 seconds.



Rises 9 meters every 3 seconds.

a. Complete the ratio table for each aircraft. Which rises faster?

Balloon	
Time (seconds)	Height (meters)
3	9
6	18
9	27
12	36

$\times 2$
 $\times 3$
 $\times 4$

Blimp	
Time (seconds)	Height (meters)
2	7
4	14
6	21
8	28

$\times 2$
 $\times 3$
 $\times 4$

Every 6 seconds, the balloon rises 18 meters and the blimp rises 21 meters.

∴ So, the blimp rises faster.

b. Graph the ordered pairs (time, height) from the tables in part (a). What can you conclude?

Write the ordered pairs.

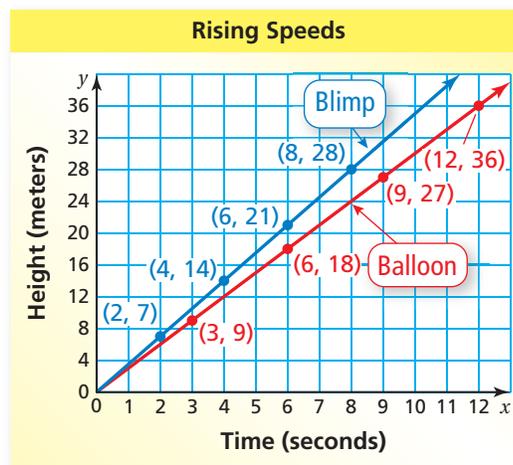
Balloon: (3, 9), (6, 18), (9, 27), (12, 36)

Blimp: (2, 7), (4, 14), (6, 21), (8, 28)

Study Tip

When graphing speed, you often place time on the horizontal axis and distance on the vertical axis.

Plot and label each set of ordered pairs. Then draw a line through each set of points.



∴ Both graphs begin at (0, 0). The graph for the blimp is steeper, so the blimp rises faster than the hot-air balloon.

On Your Own

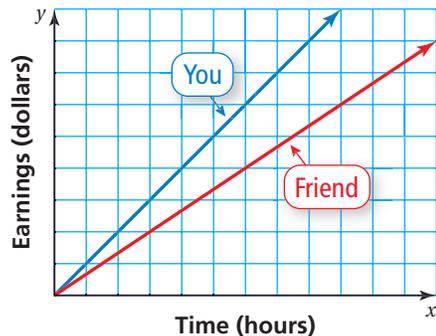
3. **WHAT IF?** The blimp rises 6 meters every 2 seconds. How does this affect your conclusion?

Now You're Ready
Exercises 12 and 13

5.4 Exercises

Vocabulary and Concept Check

- WRITING** Explain how to use tables to compare ratios.
- NUMBER SENSE** Just by looking at the graph, determine who earns a greater hourly wage. Explain.



Practice and Problem Solving

Determine which car gets the better gas mileage.

1 3.

Car	A	B
Distance (miles)	125	120
Gallons Used	5	6

4.

Car	A	B
Distance (miles)	300	320
Gallons Used	8	10

5.

Car	A	B
Distance (miles)	450	405
Gallons Used	15	12

6.

Car	A	B
Distance (miles)	360	270
Gallons Used	20	18

Determine which is the better buy.

2 7.

Air Freshener	A	B
Cost (dollars)	6	12
Refills	2	3

8.

Kitten Food	A	B
Cost (dollars)	15	9
Cans	18	12

9.

Ham	A	B
Cost (dollars)	5.70	8.75
Pounds	3	5

10.

Cheese	A	B
Cost (dollars)	3.59	5.12
Slices	10	16

11. **SALT WATER GARGLE** Salt water gargle can temporarily relieve a sore throat. One recipe calls for $\frac{3}{4}$ teaspoon of salt in 1 cup of water. A second recipe calls for 1 teaspoon of salt in 2 cups of water. Which recipe will taste saltier?



Complete the ratio tables and graph the ordered pairs from the tables.
What can you conclude?

12.

Water Tank		Swimming Pool	
Time (min)	Liters Leaked	Time (min)	Liters Leaked
2	4	3	2
4		6	
6		9	
8		12	

13.

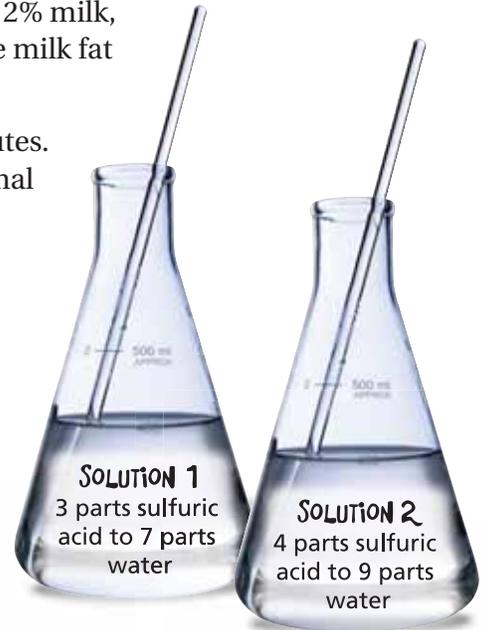
Zoo		Museum	
People	Cost (dollars)	People	Cost (dollars)
4	60	5	95
8		10	
12		15	
16		20	

14. **MILK** In whole milk, 13 parts out of 400 are milk fat. In 2% milk, 1 part out of 50 is milk fat. Which type of milk has more milk fat per cup?

15. **HEART RATE** A horse's heart beats 440 times in 10 minutes. A cow's heart beats 390 times in 6 minutes. Which animal has a greater heart rate?

16. **CHOOSE TOOLS** A chemist prepares two acid solutions.

- Use a ratio table to determine which solution is more acidic.
- Use a graph to determine which solution is more acidic.
- Which method do you prefer? Explain.



17. **NUT MIXTURE** A company offers a nut mixture with 7 peanuts for every 4 almonds. The company changes the mixture to have 8 peanuts for every 5 almonds, but the number of nuts per container does not change.

- Create a ratio table for each mixture. How many nuts are in the smallest possible container?
- Graph the ordered pairs from the tables. What can you conclude?
- Almonds cost more than peanuts. Should the company charge more or less for the new mixture? Explain your reasoning.

18. **Structure** The point (p, q) is on the graph of values from a ratio table. What is another point on the graph?



Fair Game Review what you learned in previous grades & lessons

Divide. (Section 1.1)

19. $544 \div 34$

20. $1520 \div 83$

21. $8439 \div 245$

22. **MULTIPLE CHOICE** Which of the following numbers is equal to 9.32 when you increase it by 4.65? (Section 2.4)

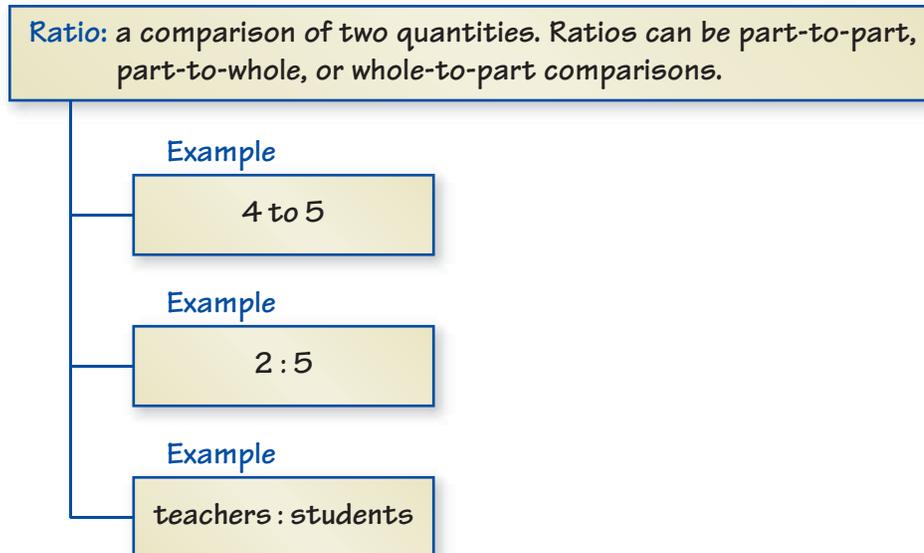
(A) 4.33

(B) 4.67

(C) 5.67

(D) 13.97

You can use a **definition and example chart** to organize information about a concept. Here is an example of a definition and example chart for ratio.



On Your Own

Make definition and example charts to help you study these topics.

1. equivalent ratios
2. ratio table
3. rate
4. unit rate
5. equivalent rates

After you complete this chapter, make definition and example charts for the following topics.

6. percent
7. U.S. customary system
8. metric system
9. conversion factor
10. unit analysis



"My math teacher taught us how to make a **definition and example chart.**"

5.1–5.4 Quiz

Write the ratio. Explain what the ratio means. (Section 5.1)

1. tulips to lilies



2. crayons to markers



Find the missing values in the ratio table. Then write the equivalent ratios. (Section 5.2)

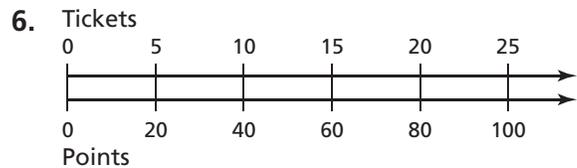
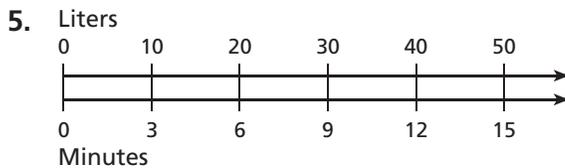
3.

Shoes	7		49
Boots	2	8	

4.

Trains	3	12	
Airplanes	8		48

Write a rate that represents the situation. (Section 5.3)



Write a unit rate for the situation. (Section 5.3)

7. 12 touchdowns in 6 games

8. 15 text messages in 5 minutes

9. 80 entries in 4 contests

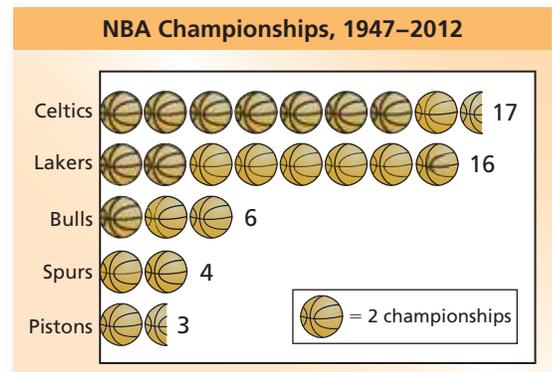
10. 75 questions in 25 minutes

11. **DOWNLOADS** Three album downloads cost \$36. How much do 5 album downloads cost? (Section 5.3)

12. **SHAMPOO** You can buy 20 fluid ounces of shampoo for \$4.40 or 24 fluid ounces for \$4.80. Which is the better buy? Explain. (Section 5.4)

13. **NBA CHAMPIONSHIPS** Write each ratio. Explain what the ratio means. (Section 5.1)

- Celtics championships to Lakers championships
- Pistons championships to Spurs championships
- Bulls championships to Lakers championships



5.5 Percents

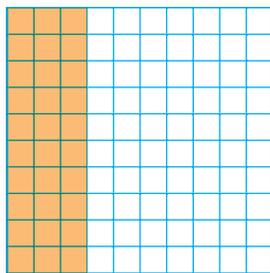
Essential Question What is the connection between ratios, fractions, and percents?

1 ACTIVITY: Writing Ratios

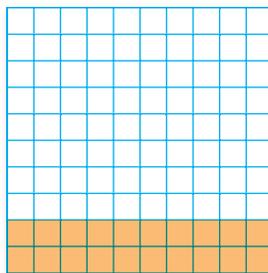
Work with a partner.

- Write the fraction of the squares that are shaded.
- Write the ratio of the number of shaded squares to the total number of squares.
- How are the ratios and the fractions related?
- When can you write ratios as fractions?

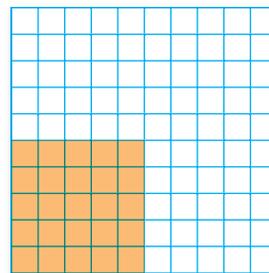
a.



b.



c.



The Meaning of a Word ● Percent

A century is 100 years.

A cent is one hundredth of a dollar.

In Mexico, a centavo is one hundredth of a peso.



Percents

In this lesson, you will

- write percents as fractions with denominators of 100.
- write fractions as percents.

Learning Standard
MACC.6.RP.1.3c



Cent means *one hundred*, so **percent** means *per one hundred*. The symbol for percent is %.

Key Vocabulary

percent, p. 220

Key Idea

Writing Percents as Fractions

Words A **percent** is a part-to-whole ratio where the whole is 100. So, you can write a percent as a fraction with a denominator of 100.

Numbers $60\% = 60 \text{ out of } 100 = \frac{60}{100}$

Annotations:
 - "part" points to 60
 - "per" points to the fraction bar
 - "one hundred (whole)" points to 100

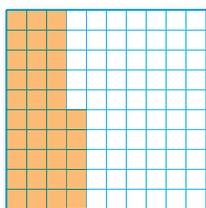
Algebra $n\% = \frac{n}{100}$

EXAMPLE 1 Writing Percents as Fractions

Study Tip

Equivalent fractions and percents represent the same number using different notations.

a. Write 35% as a fraction in simplest form.



$$35\% = \frac{35}{100}$$

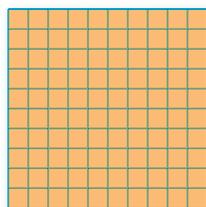
$$= \frac{7}{20}$$

Write as a fraction with a denominator of 100.

Simplify.

So, $35\% = \frac{7}{20}$.

b. Write 100% as a fraction in simplest form.



$$100\% = \frac{100}{100}$$

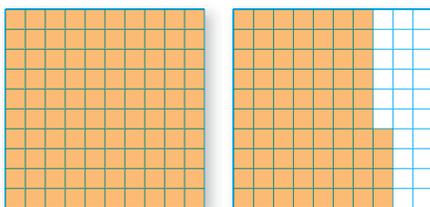
$$= 1$$

Write as a fraction with a denominator of 100.

Simplify.

So, $100\% = 1$.

c. Write 174% as a mixed number in simplest form.



$$174\% = \frac{174}{100}$$

$$= \frac{87}{50}, \text{ or } 1\frac{37}{50}$$

Write as a fraction with a denominator of 100.

Simplify.

So, $174\% = 1\frac{37}{50}$.

On Your Own

Write the percent as a fraction or mixed number in simplest form.

1. 5%

2. 168%

3. 36%

4. 83%

Now You're Ready
Exercises 8–19

Key Idea

Writing Fractions as Percents

Words Write an equivalent fraction with a denominator of 100. Then write the numerator with the percent symbol.

Numbers $\frac{1}{4} = \frac{25}{100} = 25\%$

EXAMPLE 2 Writing a Fraction as a Percent

Write $\frac{3}{50}$ as a percent.

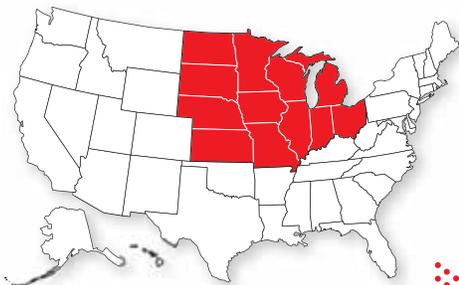
$$\frac{3}{50} = \frac{6}{100} = 6\%$$

Because $50 \times 2 = 100$, multiply the numerator and denominator by 2. Write the numerator with a percent symbol.

EXAMPLE 3 Real-Life Application

A drought affects 9 out of 12 midwestern states. What percent of the midwestern states are affected by the drought?

Midwestern United States



$$\frac{9}{12} = \frac{3}{4}$$

Simplify.

$$= \frac{75}{100}$$

$$\frac{3 \times 25}{4 \times 25} = \frac{75}{100}$$

$$= 75\%$$

Write the numerator with a percent symbol.

So, 75% of the midwestern states are affected by the drought.

On Your Own

Write the fraction or mixed number as a percent.

5. $\frac{31}{50}$

6. $\frac{7}{25}$

7. $\frac{19}{20}$

8. $1\frac{1}{2}$

9. **WHAT IF?** In Example 3, it rains in all the midwestern states. In what percent of the states affected by drought does it rain?

Now You're Ready
Exercises 21–28



Vocabulary and Concept Check

- WRITING** Explain how you can use a 10-by-10 grid to model 42%.
- WHICH ONE DOESN'T BELONG?** Which one does *not* have the same value as the other three? Explain your reasoning.

$$\frac{10}{100}$$

$$10\%$$

$$\frac{1}{10}$$

$$0.01$$

- OPEN-ENDED** Write three different fractions that are less than 40%.
- NUMBER SENSE** Can $1\frac{1}{4}$ be written as a percent? Explain.



Practice and Problem Solving

Use a 10-by-10 grid to model the percent.

5. 10%

6. 55%

7. 35%

Write the percent as a fraction or mixed number in simplest form.

1

8. 45%

9. 90%

10. 15%

11. 7%

12. 34%

13. 79%

14. 77.5%

15. 188%

16. 8%

17. 224%

18. 0.25%

19. 0.4%

20. **ERROR ANALYSIS** Describe and correct the error in writing 225% as a fraction.



$$225\% = \frac{225}{1000} = \frac{9}{40}$$

Write the fraction or mixed number as a percent.

2

21. $\frac{1}{10}$

22. $\frac{1}{5}$

23. $\frac{11}{20}$

24. $\frac{2}{25}$

25. $\frac{27}{50}$

26. $\frac{18}{25}$

27. $1\frac{17}{20}$

28. $2\frac{41}{50}$

29. **ERROR ANALYSIS** Describe and correct the error in writing $\frac{14}{25}$ as a percent.



$$\frac{14}{25} = \frac{14 \times 4}{25 \times 4} = \frac{56}{100} = 0.56\%$$

- LEFT-HANDED** Of the students in your class, 12% are left-handed. What *fraction* of the students are left-handed? Are there more right-handed or left-handed students? Explain.
- ARCADE** You have 125% of the tickets required for a souvenir. What *fraction* of the required tickets do you have? Do you need more tickets for the souvenir? Explain.

Find the percent.

32. 3 is what percent of 8? 33. 13 is what percent of 16?
34. 9 is what percent of 16? 35. 33 is what percent of 40?

36. **SOCIAL NETWORKING** A survey asked students to choose their favorite social networking website. The results are shown in the table.

Social Networking Website	Number of Students
Website A	35
Website B	13
Website C	22
Website D	10

- a. What fraction of the students chose Website A?
b. What percent of the students chose Website C?

37. **GEOGRAPHY** The percent of the total area of the United States that is in each of four states is shown.



- a. Write the percents as fractions in simplest form.
b. How many times larger is Illinois than Hawaii?
c. Compared to the map of Florida, is the map of Alaska the correct size? Explain your reasoning.
d. **RESEARCH** Which of the 50 states are larger than Illinois?
38. **CRITICAL THINKING** A school fundraiser raised 120% of its goal last year and 125% of its goal this year. Did the fundraiser raise more money this year? Explain your reasoning.
39. **CRITICAL THINKING** How can you use a 10-by-10 grid to model $\frac{1}{2}\%$?
40. **Reasoning** Write $\frac{1}{12}$ as a percent. Explain how you found your answer.



Fair Game Review what you learned in previous grades & lessons

Divide. Write the answer in simplest form. (Section 2.2)

41. $\frac{1}{6} \div \frac{1}{3}$ 42. $9 \div \frac{3}{4}$ 43. $10 \div \frac{5}{8}$ 44. $\frac{1}{6} \div 2$

45. **MULTIPLE CHOICE** Which of the following is *not* equal to 15? (Section 2.1)

- (A) $\frac{3}{4} \cdot 20$ (B) $\frac{5}{9} \cdot 27$ (C) $35 \cdot \frac{3}{7}$ (D) $28 \cdot \frac{5}{7}$

5.6 Solving Percent Problems

Essential Question How can you use mental math to find the percent of a number?



"I have a secret way for finding 21% of 80."



"10% is 8, and 1% is 0.8."



"So, 21% is $8 + 8 + 0.8 = 16.8$."

1 ACTIVITY: Finding 10% of a Number

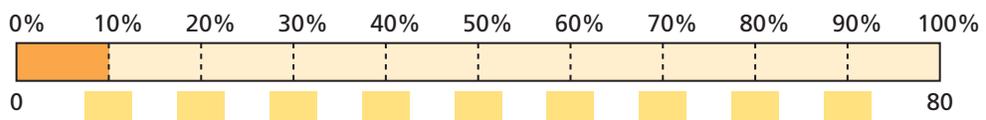
Work with a partner.

- a. How did Newton know that 10% of 80 is 8?

Write 10% as a fraction. $10\% = \frac{\square}{\square} = \frac{1}{\square}$

Labels: 10 (top), per (middle), cent (bottom)

Method 1: Use a model.



Method 2: Use multiplication.

$$10\% \text{ of } 80 = \frac{\square}{10} \text{ of } 80 = \frac{\square}{10} \times \square = \frac{\square}{10} = \square$$

- b. How do you move the decimal point to find 10% of a number?

Move the decimal point one place to the \square . $10\% \text{ of } 80. = \square$

2 ACTIVITY: Finding 1% of a Number

Work with a partner.

- a. How did Newton know that 1% of 80 is 0.8?
 b. How do you move the decimal point to find 1% of a number?



Percents

- In this lesson, you will
- find percents of numbers.
 - find the whole given the part and the percent.

Learning Standard
 MACC.6.RP.1.3c

3 ACTIVITY: Using Mental Math

Math Practice 8

Evaluate Results

Does your answer seem reasonable? How can you check your answer?

Work with a partner. Use mental math to find each percent of a number.

a. 12% of 40

Think: $12\% = 10\% + 1\% + 1\%$

$$\begin{array}{c} 10\% \text{ of } 40 = \square \\ 1\% \text{ of } 40 = \square \\ \square + \square + \square = \square \end{array}$$

b. 19% of 50

Think: $19\% = 10\% + 10\% - 1\%$

$$\begin{array}{c} 10\% \text{ of } 50 = \square \\ 1\% \text{ of } 50 = \square \\ \square + \square - \square = \square \end{array}$$

4 ACTIVITY: Using Mental Math

Work with a partner. Use mental math to find each percent of a number.



a. 20% tip for a \$30 meal

b. 18% tip for a \$30 meal

c. 6% sales tax on a \$20 shirt

d. 9% sales tax on a \$20 shirt



e. 6% service charge for a \$200 boxing ticket

f. 2% delivery fee for a \$200 boxing ticket



g. 21% bonus on a total of 40,000 points

h. 38% bonus on a total of 80,000 points



What Is Your Answer?

- IN YOUR OWN WORDS** How can you use mental math to find the percent of a number?
- Describe two real-life examples of finding a percent of a number.
- How can you use 10% of a number to find 20% of the number? 30%? Explain your reasoning.

Practice

Use what you learned about finding the percent of a number to complete Exercises 3–10 on page 229.

Key Idea

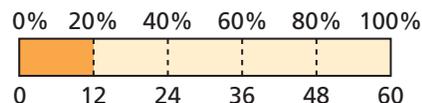
Finding the Percent of a Number

Words Write the percent as a fraction. Then multiply by the whole. The percent times the whole equals the part.

Numbers 20% of 60 is 12.

$$\frac{1}{5} \times 60 = 12$$

Model



EXAMPLE 1 Finding the Percent of a Number

25% of 40 is what number?

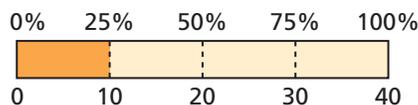
$$\begin{aligned} 25\% \text{ of } 40 &= \frac{1}{4} \cdot 40 \\ &= \frac{1 \cdot \cancel{40}^{10}}{1 \cdot \cancel{4}} \\ &= 10 \end{aligned}$$

Write the percent as a fraction and multiply.

Divide out the common factor.

Simplify.

So, 25% of 40 is 10.



Study Tip

You can use mental math to check your answer in Example 1.

$$10\% \text{ of } 40 = 4$$

$$5\% \text{ of } 40 = 2$$

So, 25% of 40 is

$$4 + 4 + 2 = 10.$$

You can also use a ratio table to find the percent of a number.

EXAMPLE 2 Finding the Percent of a Number Using a Ratio Table

60% of 150 is what number?

Use a ratio table to find the part. Let one row be the *part*, and let the other be the *whole*. Find an equivalent ratio with 150 as the whole.

The first column represents the percent.

$$\frac{\text{part}}{\text{whole}} = \frac{60}{100} = 60\%$$

Part	60	30	90
Whole	100	50	150

Annotations: $\div 2$ (from 60 to 30), $\times 3$ (from 30 to 90), $\div 2$ (from 100 to 50), $\times 3$ (from 50 to 150).

So, 60% of 150 is 90.

On Your Own

Find the percent of the number. Explain your method.

- 90% of 20
- 75% of 32
- 10% of 110
- 30% of 75

Now You're Ready
Exercises 3–22

You can use a related division equation to find the whole given the part and the percent.

Key Idea

Finding the Whole

Write the percent as a fraction. Then divide the part by the fraction.

Words The part divided by the percent equals the whole.

Numbers 20% of 60 is 12.

$$\frac{1}{5} \times 60 = 12 \longrightarrow 12 \div \frac{1}{5} = 60$$

Multiplication equation

Related division equation

EXAMPLE 3 Finding the Whole

75% of what number is 48?

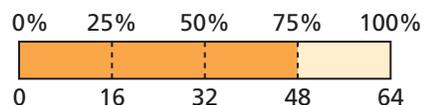
$$\begin{aligned} 48 \div 75\% &= 48 \div \frac{3}{4} \\ &= 48 \cdot \frac{4}{3} \\ &= 64 \end{aligned}$$

Write the percent as a fraction and divide.

Multiply by the reciprocal.

Simplify.

So, 75% of 64 is 48.



EXAMPLE 4 Finding the Whole Using a Ratio Table

120% of what number is 72?

Use a ratio table to find the whole. Find an equivalent ratio with 72 as the part.

The first column represents the percent.

$$\frac{\text{part}}{\text{whole}} = \frac{120}{100} = 120\%$$

Part	120	6	72
Whole	100	5	60

$\div 20$ (from 120 to 6)
 $\times 12$ (from 6 to 72)
 $\div 20$ (from 100 to 5)
 $\times 12$ (from 5 to 60)

So, 120% of 60 is 72.

On Your Own

Find the whole. Explain your method.

5. 5% of what number is 10? 6. 62% of what number is 31?

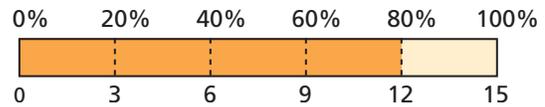
EXAMPLE 5 Real-Life Application

The width of a rectangular room is 80% of its length. What is the area of the room?

Find 80% of 15 feet.



$$\begin{aligned}80\% \text{ of } 15 &= \frac{4}{5} \times 15 \\ &= \frac{4 \times \overset{3}{\cancel{15}}}{\underset{1}{\cancel{5}}} \\ &= 12\end{aligned}$$



The width is 12 feet.

Use the formula for the area A of a rectangle.

$$A = 15 \times 12 = 180$$

So, the area of the room is 180 square feet.

On Your Own

7. The width of a rectangular stage is 55% of its length. The stage is 120 feet long. What is the area?

EXAMPLE 6 Real-Life Application

You win an online auction for concert tickets. Your winning bid is 60% of your maximum bid. How much more were you willing to pay for the tickets than you actually paid?

- (A) \$72 (B) \$80 (C) \$120 (D) \$200

Your maximum bid is the *whole*, and your winning bid is the *part*. Find your maximum bid by dividing the part by the percent.

$$120 \div 60\% = 120 \div \frac{3}{5} \quad \text{Divide the part by the percent.}$$

$$= 120 \cdot \frac{5}{3} \quad \text{Multiply by the reciprocal.}$$

$$= 200 \quad \text{Simplify.}$$

Your maximum bid is \$200, and your winning bid is \$120. So, you were willing to pay $200 - 120 = \$80$ more for the tickets.

The correct answer is (B).

On Your Own

8. **WHAT IF?** Your winning bid is 96% of your maximum bid. How much more were you willing to pay for the tickets than you actually paid?



Vocabulary and Concept Check

1. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

What is twenty percent of 30?

What is one-fifth of 30?

Twenty percent of what number is 30?

What is two-tenths of 30?

2. **NUMBER SENSE** If 52 is 130% of a number, is the number greater or less than 52? Explain.

Practice and Problem Solving

Find the percent of the number. Explain your method.

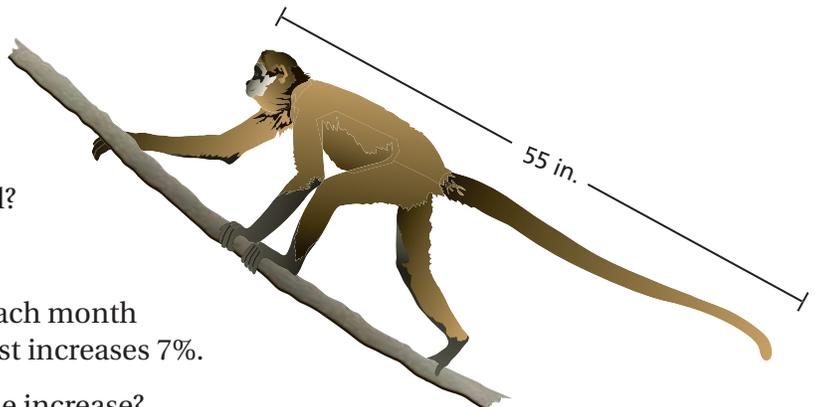
- 1 2 3. 20% of 60 4. 10% of 40 5. 50% of 70 6. 30% of 30
 7. 10% of 90 8. 15% of 20 9. 25% of 50 10. 5% of 60
 11. 30% of 70 12. 75% of 48 13. 45% of 45 14. 92% of 19
 15. 40% of 60 16. 38% of 22 17. 70% of 20 18. 87% of 55
 19. 140% of 60 20. 120% of 33 21. 175% of 54 22. 250% of 146
23. **ERROR ANALYSIS** Describe and correct the error in finding 40% of 75.



$$40\% \text{ of } 75 = 40\% \times 75 = 3000$$

24. **PINE TREES** A town had about 2120 acres of pine trees 40 years ago. Only about 13% of the pine trees remain. How many acres of pine trees remain?

25. **SPIDER MONKEY** The tail of the spider monkey is 64% of the length shown. What is the length of its tail?



26. **CABLE** A family pays \$45 each month for cable television. The cost increases 7%.
- How many dollars is the increase?
 - What is the new monthly cost?

Find the whole. Explain your method.

- 3 4 27. 10% of what number is 14?
 29. 25% of what number is 21?
 31. 15% of what number is 12?
 33. 140% of what number is 35?
 35. 125% of what number is 25?
28. 20% of what number is 18?
 30. 75% of what number is 27?
 32. 85% of what number is 17?
 34. 160% of what number is 32?
 36. 175% of what number is 42?

37. **ERROR ANALYSIS** Describe and correct the error in finding the whole.

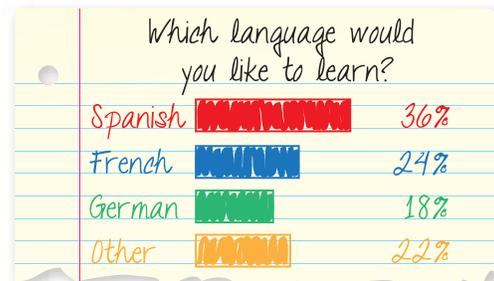
20% of what number is 5?

X $5 \div 20\% = \frac{5}{20}$
 $= \frac{1}{4}$



38. **COUPON** You have a coupon for a restaurant. You save \$3 on a meal. What was the original cost of the meal?

39. **SURVEY** The results of a survey are shown at the right. In the survey, 12 students said that they would like to learn French.
- How many students were surveyed?
 - How many of the students surveyed would like to learn Spanish?



40. **WEIGHT** A sixth grader weighs 90 pounds, which is 120% of what he weighed in fourth grade. How much did he weigh in fourth grade?
41. **PARKING LOT** In a parking lot, 16% of the cars are blue. There are 4 blue cars in the parking lot. How many cars in the parking lot are *not* blue?



42. **LOTION** A bottle contains 20 fluid ounces of lotion and sells for \$5.80. The 20-fluid-ounce bottle contains 125% of the lotion in the next smallest size, which sells for \$5.12. Which is the better buy? Explain.

Copy and complete the statement using $<$, $>$, or $=$.

43. 80% of 60 60% of 80

44. 20% of 30 30% of 40

45. 120% of 5 0.8% of 250

46. 85% of 40 25% of 136

47. **TIME** How many minutes is 40% of 2 hours?

48. **LENGTH** How many inches is 78% of 3 feet?

49. **GEOMETRY** The width of the rectangle is 75% of its length.

- a. What is the area of the rectangle?
- b. The length of the rectangle is doubled. What percent of the length is the width now? Explain your reasoning.



24 in.

50. **BASKETBALL** To pass inspection, a new basketball should bounce back to between 68% and 75% of the starting height. A new ball is dropped from 6 feet and bounces back 4 feet 1 inch. Does the ball pass inspection? Explain.



51. **REASONING** You know that 15% of a number n is 12. How can you use this to find 30% of n ? 45% of n ? Explain.

52. **SURFBOARD** You have a coupon for 10% off the sale price of a surfboard. Which is the better buy? Explain your reasoning.

- 40% off the regular price
- 30% off the regular price and then 10% off the sale price

53. **Number Sense** On three 150-point geography tests, you earned grades of 88%, 94%, and 90%. The final test is worth 250 points. What *percent* do you need on the final to earn 93% of the total points on all tests?



Fair Game Review what you learned in previous grades & lessons

Multiply. (Section 2.5)

54. 0.6×8

55. 3.3×5

56. 0.74×9

57. 2.19×12

58. **MULTIPLE CHOICE** What is the quotient of 75 and 2.4? (Section 2.6)

(A) 0.032

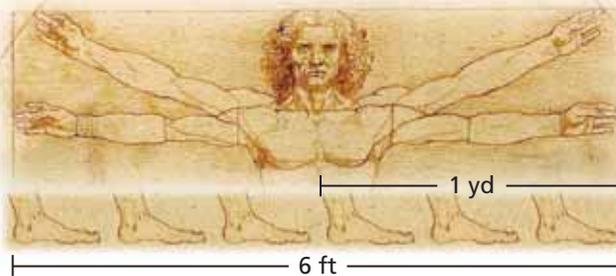
(B) 0.3125

(C) 3.2

(D) 31.25

5.7 Converting Measures

Essential Question How can you compare lengths between the customary and metric systems?



1 ACTIVITY: Customary Measure History

Work with a partner.

- a. Match the measure of length with its historical beginning.

<i>Length</i>	<i>Historical Beginning</i>
Inch	The length of a human foot
Foot	The width of a human thumb
Yard	The distance a human can walk in 1000 paces (1 pace = 2 steps)
Mile	The distance from a human nose to the end of an outstretched human arm

- b. Use a ruler to measure your thumb, arm, and foot. How do your measurements compare to your answers from part (a)? Are they close to the historical measures?

You know how to convert measures within the customary and metric systems.

Equivalent Customary Lengths

$$1 \text{ ft} = 12 \text{ in.} \quad 1 \text{ yd} = 3 \text{ ft} \quad 1 \text{ mi} = 5280 \text{ ft}$$

Equivalent Metric Lengths

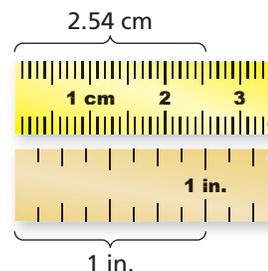
$$1 \text{ m} = 1000 \text{ mm} \quad 1 \text{ m} = 100 \text{ cm} \quad 1 \text{ km} = 1000 \text{ m}$$

You will learn how to convert between the two systems.

Converting Between Systems

$$1 \text{ in.} = 2.54 \text{ cm}$$

$$1 \text{ mi} \approx 1.61 \text{ km}$$



Converting Measures

In this lesson, you will

- use conversion factors (rates) to convert units of measurement.

Learning Standard
MACC.6.RP.1.3d

2 ACTIVITY: Comparing Measures

Math Practice 1

Analyze Givens

What is the relationship between the given quantities? What are you trying to find?

Work with a partner. Answer each question. Explain your answer. Use a diagram in your explanation.

	<i>Metric</i>	<i>Customary</i>
a. Car Speed: Which is faster?	80 km/h	60 mi/h
b. Trip Distance: Which is farther?	200 km	200 mi
c. Human Height: Who is taller?	180 cm	5 ft 8 in.
d. Wrench Width: Which is wider?	8 mm	5/16 in.
e. Swimming Pool Depth: Which is deeper?	1.4 m	4 ft

3 ACTIVITY: Changing Units in a Rate

Work with a partner. Change the units of the rate by multiplying by a “Magic One.” Write your answer as a unit rate. Show your work.

<i>Original Rate</i>	<i>Magic One</i>	<i>New Units</i>	<i>Unit Rate</i>
a. Sample:			
$\frac{\cancel{\$120}}{\cancel{h}}$	$\times \frac{1 \cancel{h}}{60 \text{ min}}$	$= \frac{\$120}{60 \text{ min}}$	$= \frac{\$2}{1 \text{ min}}$
b. $\frac{\$3}{\text{min}}$	$\times \frac{1 \text{ h}}{60 \text{ min}}$	$= \frac{\$3}{60 \text{ min}}$	$= \frac{\$ \square}{1 \text{ h}}$
c. $\frac{12 \text{ in.}}{\text{ft}}$	$\times \frac{1 \text{ yd}}{36 \text{ in.}}$	$= \frac{12 \text{ in.}}{36 \text{ in.}}$	$= \frac{\square \text{ in.}}{1 \text{ yd}}$
d. $\frac{2 \text{ ft}}{\text{week}}$	$\times \frac{1 \text{ yr}}{52 \text{ weeks}}$	$= \frac{2 \text{ ft}}{52 \text{ weeks}}$	$= \frac{\square \text{ ft}}{1 \text{ yr}}$

What Is Your Answer?

- One problem-solving strategy is called *Working Backwards*. What does this mean? How can you use this strategy to find the rates in Activity 3?
- IN YOUR OWN WORDS** How can you compare lengths between the customary and the metric systems? Give examples with your description.

Practice

Use what you learned about converting measures between systems to complete Exercises 4 and 5 on page 236.

Key Vocabulary

U.S. customary system, p. 234
 metric system, p. 234
 conversion factor, p. 234
 unit analysis, p. 234

The **U.S. customary system** is a system of measurement that contains units for length, capacity, and weight. The **metric system** is a decimal system of measurement, based on powers of 10, that contains units for length, capacity, and mass.

To convert from one unit of measure to another, multiply by one or more *conversion factors*. A conversion factor can be written using fraction notation.

Key Idea
Conversion Factor

A **conversion factor** is a rate that equals 1.

*Relationship**Conversion Factors***Example** $1 \text{ m} \approx 3.28 \text{ ft}$

$$\frac{1 \text{ m}}{3.28 \text{ ft}} \text{ and } \frac{3.28 \text{ ft}}{1 \text{ m}}$$

You can use **unit analysis** to decide which conversion factor will produce the appropriate units.

EXAMPLE 1 **Converting Units**

- a. Convert 36 quarts to gallons.

Use a conversion factor.

$$36 \cancel{\text{qt}} \cdot \frac{1 \text{ gal}}{4 \cancel{\text{qt}}} = \frac{36 \cdot 1 \text{ gal}}{4} = 9 \text{ gal}$$

$$1 \text{ gal} = 4 \text{ qt}$$

∴ So, 36 quarts is 9 gallons.

- b. Convert 20 centimeters to inches.

Use a conversion factor.

$$20 \cancel{\text{cm}} \cdot \frac{1 \text{ in.}}{2.54 \cancel{\text{cm}}} \approx 7.87 \text{ in.}$$

$$1 \text{ in.} = 2.54 \text{ cm}$$

∴ So, 20 centimeters is about 7.87 inches.

On Your Own

Copy and complete the statement. Round to the nearest hundredth if necessary.

1. $48 \text{ ft} = \square \text{ yd}$ 2. $7 \text{ lb} = \square \text{ oz}$ 3. $5 \text{ g} = \square \text{ mg}$
 4. $7 \text{ mi} \approx \square \text{ km}$ 5. $12 \text{ qt} \approx \square \text{ L}$ 6. $25 \text{ kg} \approx \square \text{ lb}$

Now You're Ready
 Exercises 6–17

EXAMPLE 2 Comparing Units

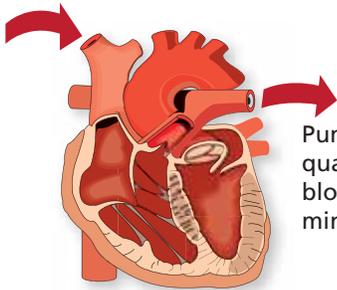
Copy and complete the statement using $<$ or $>$: 25 oz 2 kg.

Convert 25 ounces to kilograms.

$$25 \text{ oz} \times \frac{1 \text{ lb}}{16 \text{ oz}} \times \frac{0.45 \text{ kg}}{1 \text{ lb}} = \frac{25 \cdot 1 \cdot 0.45 \text{ kg}}{16 \cdot 1} \approx 0.70 \text{ kg}$$

Because 0.70 kilogram is less than 2 kilograms, $25 \text{ oz} < 2 \text{ kg}$.

EXAMPLE 3 Converting a Rate: Changing One Unit



Pumps 5 quarts of blood per minute

How many liters does the human heart pump per minute?

$$\frac{5 \text{ qt}}{1 \text{ min}} \cdot \frac{0.95 \text{ L}}{1 \text{ qt}} \approx \frac{4.75 \text{ L}}{1 \text{ min}}$$

The rate of 5 quarts per minute is about 4.75 liters per minute.

EXAMPLE 4 Converting a Speed: Changing Both Units

You are riding on a zip line. Your speed is 15 miles per hour. What is your speed in feet per second?

$$\begin{aligned} \frac{15 \text{ mi}}{1 \text{ hr}} \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ sec}} \right) &= \frac{15 \cdot 5280 \text{ ft}}{3600 \text{ sec}} \\ &= \frac{79,200 \text{ ft}}{3600 \text{ sec}} \\ &= \frac{22 \text{ ft}}{1 \text{ sec}} \end{aligned}$$

Your speed is 22 feet per second.

On Your Own

Copy and complete the statement using $<$ or $>$.

7. 7 cm 3 in. 8. 8 c 2 L 9. 3 oz 70 g

10. An oil tanker is leaking oil at a rate of 300 gallons per minute. What is this rate in gallons per second?

11. A tennis ball travels at a speed of 120 miles per hour. What is this rate in feet per second?

Now You're Ready
Exercises 20–31

Vocabulary and Concept Check

- VOCABULARY** Is $\frac{10 \text{ mm}}{1 \text{ cm}}$ a conversion factor? Explain.
- WRITING** Describe how to convert 2 liters per hour to milliliters per second.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

Convert 5 inches to centimeters.

Find the number of inches in 5 centimeters.

How many centimeters are in 5 inches?

Five inches equals how many centimeters?

Practice and Problem Solving

Answer the question. Explain your answer.

- Which juice container is larger: 2 L or 1 gal?
- Which person is heavier: 75 kg or 110 lb?

Copy and complete the statement. Round to the nearest hundredth if necessary.

- | | | |
|---|--|--|
| 1 6. 3 pt = <input type="text"/> c | 7. 1500 mL = <input type="text"/> L | 8. 40 oz = <input type="text"/> lb |
| 9. 12 L \approx <input type="text"/> qt | 10. 14 m \approx <input type="text"/> ft | 11. 4 ft \approx <input type="text"/> m |
| 12. 64 lb \approx <input type="text"/> kg | 13. 0.3 km \approx <input type="text"/> mi | 14. 75.2 in. \approx <input type="text"/> cm |
| 15. 17 kg \approx <input type="text"/> lb | 16. 15 cm \approx <input type="text"/> in. | 17. 9 mi \approx <input type="text"/> km |

- ERROR ANALYSIS** Describe and correct the error in converting the units.
- BRIDGE** The Mackinac Bridge in Michigan is the third-longest suspension bridge in the United States.
 - How high above the water is the roadway in meters?
 - The bridge has a length of 26,372 feet. What is the length in kilometers?

$$\begin{aligned}
 & \times \quad 8 \text{ L} \approx 8 \text{ L} \cdot \frac{0.95 \text{ qt}}{1 \text{ L}} \\
 & = 8 \cancel{\text{L}} \cdot \frac{0.95 \text{ qt}}{1 \cancel{\text{L}}} \\
 & = 7.6 \text{ qt}
 \end{aligned}$$



Copy and complete the statement using < or >.

20. 8 kg 30 oz

21. 6 ft 300 cm

22. 3 gal 6 L

23. 10 in. 200 mm

24. 1200 g 5 lb

25. 1500 m 3000 ft

Copy and complete the statement.

26. $\frac{13 \text{ km}}{\text{h}} \approx \frac{\text{mi}}{\text{h}}$

27. $\frac{22 \text{ L}}{\text{min}} = \frac{\text{L}}{\text{h}}$

28. $\frac{63 \text{ mi}}{\text{h}} = \frac{\text{mi}}{\text{sec}}$

29. $\frac{3 \text{ km}}{\text{min}} \approx \frac{\text{mi}}{\text{h}}$

30. $\frac{17 \text{ gal}}{\text{h}} \approx \frac{\text{qt}}{\text{min}}$

31. $\frac{6 \text{ cm}}{\text{min}} = \frac{\text{m}}{\text{sec}}$

32. **BOTTLE** Can you pour the water from a full 2-liter bottle into a 2-quart pitcher without spilling any? Explain.

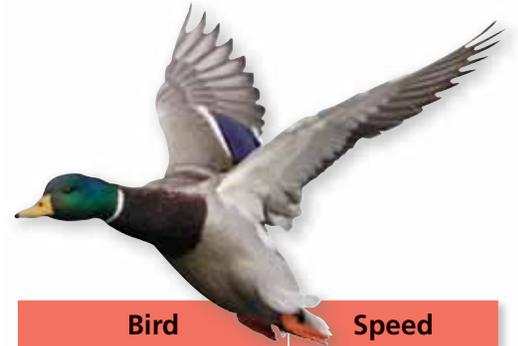


33. **AUTOBAHN** Germany suggests a speed limit of 130 kilometers per hour on highways.

- Is the speed shown greater than the suggested limit?
- Suppose the speed shown drops 30 miles per hour. Is the new speed below the suggested limit?

34. **BIRDS** The table shows the flying speeds of several birds.

- Which bird is the fastest? Which is the slowest?
- The peregrine falcon has a dive speed of 322 kilometers per hour. Is the dive speed of the peregrine falcon faster than the flying speed of any of the birds? Explain.



Bird	Speed
Spine-tailed swift	2843.2 m/min
Spur-winged goose	129.1 ft/sec
Eider duck	31.3 m/sec
Mallard	65 mi/h

35. **SPEED OF LIGHT** The speed of light is about 300,000 kilometers per second. Convert the speed to miles per hour.

36. **Critical Thinking** One liter of paint covers 100 square feet. How many gallons of paint does it take to cover a room whose walls have an area of 800 square meters?



Fair Game Review what you learned in previous grades & lessons

Find the percent of the number. (Section 5.6)

37. 25% of 120

38. 65% of 47

39. 120% of 15

40. 3.2% of 80

41. **MULTIPLE CHOICE** What is the area of a parallelogram with a base of 15 centimeters and a height of 12 centimeters? (Section 4.1)

(A) 90 cm²

(B) 175 cm²

(C) 180 cm²

(D) 205 cm²

5 Chapter Review

Review Key Vocabulary

ratio, p. 192

equivalent ratios, p. 198

ratio table, p. 198

rate, p. 206

unit rate, p. 206

equivalent rates, p. 206

percent, p. 220

U.S. customary system, p. 234

metric system, p. 234

conversion factor, p. 234

unit analysis, p. 234

Review Examples and Exercises

5.1 Ratios (pp. 190–195)

Write the ratio of apples to oranges.
Explain what the ratio means.

3 apples → 3 to 5 ← 5 oranges

So, the ratio of apples to oranges is 3 to 5, or 3 : 5. That means that for every 3 apples, there are 5 oranges.



Exercises

Write the ratio. Explain what the ratio means.

1. butterflies : caterpillars



2. saxophones : trumpets



5.2 Ratio Tables (pp. 196–203)

Find the missing values in the ratio table.
Then write the equivalent ratios.

You can use multiplication to find the missing values.

The equivalent ratios are 2 : 5, 6 : 15, and 12 : 30.

Trees	2	6	
Birds	5		30

Trees	2	6	12
Birds	5	15	30

$\times 3$ $\times 2$
 $\times 3$ $\times 2$

Exercises

Find the missing values in the ratio table. Then write the equivalent ratios.

3.

Levers	6		18
Pulleys	3	6	

4.

Cars	3	6	
Trucks	4		24

5.3 Rates (pp. 204–209)

A horse can run 165 feet in 3 seconds. At this rate, how far can the horse run in 5 seconds?

Using a ratio table, divide to find the unit rate. Then multiply to find the distance that the horse can run in 5 seconds.

Distance (feet)	165	55	275
Time (seconds)	3	1	5

$\div 3$ (from 165 to 55) $\times 5$ (from 55 to 275)
 $\div 3$ (from 3 to 1) $\times 5$ (from 1 to 5)

So, the horse can run 275 feet in 5 seconds.

Exercises

Write a unit rate for the situation.

- 12 stunts in 4 movies
- 3600 stitches in 3 minutes
- MUSIC** A song has 28 beats in 4 seconds. At this rate, how many beats are there in 30 seconds?

5.4 Comparing and Graphing Ratios (pp. 210–215)

There are 24 grams of sugar in 6 fluid ounces of Soft Drink A, and there are 15 grams of sugar in 4 fluid ounces of Soft Drink B. Which soft drink contains more sugar in a 12-ounce can?

Use ratio tables to compare the soft drinks.

Soft Drink A

Sugar (grams)	24	48
Volume (fluid ounces)	6	12

$\times 2$ (from 24 to 48)
 $\times 2$ (from 6 to 12)

Soft Drink B

Sugar (grams)	15	45
Volume (fluid ounces)	4	12

$\times 3$ (from 15 to 45)
 $\times 3$ (from 4 to 12)

The tables show that a 12-ounce can of Soft Drink A has $48 - 45 = 3$ more grams of sugar than Soft Drink B.

So, a 12-ounce can of Soft Drink A has more sugar.

Exercises

- TUNA** A 5-ounce can of tuna costs \$0.90. A 12-ounce can of tuna costs \$2.40. Which is the better buy?

5.5 Percents (pp. 218–223)

Write $\frac{3}{20}$ as a percent.

$$\frac{3}{20} = \frac{15}{100} = 15\%$$

Because $20 \times 5 = 100$, multiply the numerator and denominator by 5. Write the numerator with a percent symbol.

Exercises

Write the percent as a fraction or mixed number in simplest form.

9. 12%

10. 88%

11. 0.8%

Write the fraction or mixed number as a percent.

12. $\frac{3}{5}$

13. $\frac{43}{25}$

14. $1\frac{21}{50}$

5.6 Solving Percent Problems (pp. 224–231)

a. 75% of 80 is what number?

$$75\% \text{ of } 80 = \frac{3}{4} \times 80 = \frac{3 \times \cancel{80}^{20}}{\cancel{4}_1} = 60$$

∴ So, 75% of 80 is 60.

b. 30% of what number is 27?

$$27 \div 30\% = 27 \div \frac{3}{10} = \cancel{27}^9 \cdot \frac{10}{\cancel{3}_1} = 90$$

∴ So, 30% of 90 is 27.

Exercises

Find the percent of the number. Explain your method.

15. 60% of 80

16. 80% of 55

17. 150% of 48

Find the whole. Explain your method.

18. 70% of what number is 35?

19. 140% of what number is 56?

5.7 Converting Measures (pp. 232–237)

Convert 8 kilometers to miles.

$$8 \text{ km} \times \frac{1 \text{ mi}}{1.6 \text{ km}} \approx 5 \text{ mi}$$

Because $1 \text{ mi} \approx 1.6 \text{ km}$, use the ratio $\frac{1 \text{ mi}}{1.6 \text{ km}}$.

Exercises

Copy and complete the statement. Round to the nearest hundredth if necessary.

20. 3 L \approx qt

21. 9.2 in. \approx cm

22. 15 lb \approx kg

5 Chapter Test

Write the ratio. Explain what the ratio means.

1. scooters : bikes



2. starfish : seashells



Find the missing values in the ratio table. Then write the equivalent ratios.

3.

Lemons	4		36
Limes	2	6	

4.

Rabbits	2	4	
Hamsters	9		54

Write a unit rate for the situation.

5. \$54.00 for 3 tickets

6. 210 miles in 3 hours

Write the fraction or mixed number as a percent.

7. $\frac{21}{25}$

8. $\frac{17}{20}$

9. $1\frac{2}{5}$

Find the percent of the number. Explain your method.

10. 80% of 90

11. 30% of 50

12. 120% of 75

Find the whole. Explain your method.

13. 34 is 40% of what number?

14. 52 is 130% of what number?

Copy and complete the statement. Round to the nearest hundredth if necessary.

15. 5 L \approx qt

16. 56 lb \approx kg

17. **SOUP** There are 600 milligrams of sodium in 4 ounces of Soup A, and there are 720 milligrams of sodium in 6 ounces of Soup B. You prepare an 18-ounce bowl of each soup. Which bowl of soup contains more sodium?



18. **ORANGE JUICE** A 48-fluid-ounce container of orange juice costs \$2.40. A 60-fluid-ounce container of orange juice costs \$3.60. Which is the better buy?

Video Download

Progress...

46%

2 min remaining 736 KB of 1.6 MB copied

Destination
C:\Files\Video

19. **DOWNLOAD** Your computer displays the progress of a downloading video. What fraction of the video is downloaded? Write your answer in simplest form.
20. **GLASSES** In a class of 20 students, 40% are boys. Twenty-five percent of the boys and 50% of the girls wear glasses. How many students wear glasses?

6. What property was used to simplify the expression? (MACC.6.EE.1.3)

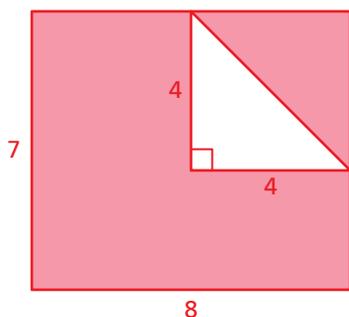
$$\begin{aligned} 12 \times 47 &= 12 \times (40 + 7) \\ &= 12 \times 40 + 12 \times 7 \\ &= 480 + 84 \\ &= 564 \end{aligned}$$

- F. Distributive Property
G. Identity Property of Addition
H. Commutative Property of Addition
I. Associative Property of Multiplication
7. What is 15% of 36? (MACC.6.RP.1.3c)



8. If 5 dogs share equally a bag of dog treats, each dog gets 24 treats. Suppose 8 dogs share equally the bag of treats. How many treats does each dog get? (MACC.6.RP.1.3b)
- A. 3
B. 15
C. 21
D. 38

9. The figure below consists of a rectangle and a right triangle. (MACC.6.G.1.1)



What is the area of the shaded region?

- F. 23 units²
G. 40 units²
H. 48 units²
I. 60 units²

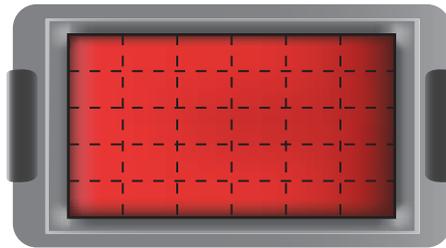


10. What is the area, in square inches, of the trapezoid-shaped award? (MACC.6.G.1.1)



11. Your friend evaluated an expression using $k = 0.5$ and $p = 1.6$ and got an answer of 12. Which expression did your friend evaluate? (MACC.6.EE.1.2c)
- A. $5p + 8k$ C. $0.5k + 1.6p$
 B. $8p + 5k$ D. $0.8k + 0.5p$

12. For a party, you made a gelatin dessert in a rectangular pan and cut the dessert into equal-sized pieces as shown below.



The dessert consisted of 5 layers of equal height. Each layer was a different flavor, as shown below by a side view of the pan. (MACC.6.RP.1.3c)



Your guests ate $\frac{3}{5}$ of the pieces of the dessert.

Part A Write the amount of cherry gelatin eaten by your guests as a fraction of the total dessert. Justify your answer.

Part B Write the amount of cherry gelatin eaten by your guests as a percent of the total dessert. Justify your answer.