10.2 Product of Powers Property

Essential Question: How can you use inductive reasoning to observe patterns and write general rules involving properties of exponents?

1 ACTIVITY: Finding Products of Powers

Work with a partner.

a. Copy and complete the table.

<table>
<thead>
<tr>
<th>Product</th>
<th>Repeated Multiplication Form</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^2 \cdot 2^4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-3)^2 \cdot (-3)^4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$7^3 \cdot 7^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5.1^1 \cdot 5.1^6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(-4)^2 \cdot (-4)^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^3 \cdot 10^5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^5$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. **INDUCTIVE REASONING** Describe the pattern in the table. Then write a general rule for multiplying two powers that have the same base.

$$a^m \cdot a^n = a^{m+n}$$

c. Use your rule to simplify the products in the first column of the table above. Does your rule give the results in the third column?

d. Most calculators have exponent keys that you can use to evaluate powers. Use a calculator with an exponent key to evaluate the products in part (a).

2 ACTIVITY: Writing a Rule for Powers of Powers

Work with a partner. Write the expression as a single power. Then write a general rule for finding a power of a power.

a. $(3^2)^3 = (3 \cdot 3)(3 \cdot 3)(3 \cdot 3) =$

b. $(2^3)^4 =$

c. $(7^3)^2 =$

d. $(y^3)^3 =$

e. $(x^4)^2 =$
ACTIVITY: Writing a Rule for Powers of Products

Work with a partner. Write the expression as the product of two powers. Then write a general rule for finding a power of a product.

a. \((2 \cdot 3)^3 = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = 
\)

b. \((2 \cdot 5)^2 = 
\)

c. \((5 \cdot 4)^3 = 
\)

d. \((6a)^4 = 
\)

e. \((3x)^2 = 
\)

ACTIVITY: The Penny Puzzle

Work with a partner.

- The rows \(y\) and columns \(x\) of a chessboard are numbered as shown.
- Each position on the chessboard has a stack of pennies. (Only the first row is shown.)
- The number of pennies in each stack is \(2^x \cdot 2^y\).

a. How many pennies are in the stack in location \((3, 5)\)?

b. Which locations have 32 pennies in their stacks?

c. How much money (in dollars) is in the location with the tallest stack?

d. A penny is about 0.06 inch thick. About how tall (in inches) is the tallest stack?

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use inductive reasoning to observe patterns and write general rules involving properties of exponents?

Use what you learned about properties of exponents to complete Exercises 3–5 on page 420.
Key Ideas

Product of Powers Property
Words To multiply powers with the same base, add their exponents.

Numbers \(4^2 \cdot 4^3 = 4^{2+3} = 4^5\)

Algebra \(a^m \cdot a^n = a^{m+n}\)

Power of a Power Property
Words To find a power of a power, multiply the exponents.

Numbers \((4^6)^3 = 4^{6 \cdot 3} = 4^{18}\)

Algebra \((a^m)^n = a^{mn}\)

Power of a Product Property
Words To find a power of a product, find the power of each factor and multiply.

Numbers \((3 \cdot 2)^5 = 3^5 \cdot 2^5\)

Algebra \((ab)^m = a^m b^m\)

EXAMPLE 1 Multiplying Powers with the Same Base

a. \(2^4 \cdot 2^5 = 2^{4+5}\)

\[= 2^9\]

Product of Powers Property

Simplify.

b. \(-5 \cdot (-5)^6 = (-5)^1 \cdot (-5)^6\)

\[= (-5)^{1+6}\]

\[= (-5)^7\]

Rewrite \(-5\) as \((-5)^1\).

Product of Powers Property

Simplify.

c. \(x^3 \cdot x^7 = x^{3+7}\)

\[= x^{10}\]

Product of Powers Property

Simplify.

EXAMPLE 2 Finding a Power of a Power

a. \((3^4)^3 = 3^{4 \cdot 3}\)

\[= 3^{12}\]

Power of a Power Property

Simplify.

b. \((w^5)^4 = w^{5 \cdot 4}\)

\[= w^{20}\]

Power of a Power Property

Simplify.
EXAMPLE 3 Finding a Power of a Product

a. \((2x)^3 = 2^3 \cdot x^3\)  
   \[= 8x^3\]  
   Power of a Product Property  
   Simplify.

b. \((3xy)^2 = 3^2 \cdot x^2 \cdot y^2\)  
   \[= 9x^2y^2\]  
   Power of a Product Property  
   Simplify.

On Your Own
Simplify the expression.

1. \(6^2 \cdot 6^4\)
2. \(\left(-\frac{1}{2}\right)^3 \cdot \left(-\frac{1}{2}\right)^6\)
3. \(z \cdot z^{12}\)
4. \((4^4)^3\)
5. \((y^2)^4\)
6. \((-4^3)^2\)
7. \((5y)^4\)
8. \((ab)^5\)
9. \((0.5mn)^2\)

EXAMPLE 4 Simplifying an Expression

A gigabyte (GB) of computer storage space is \(2^{30}\) bytes. The details of a computer are shown. How many bytes of total storage space does the computer have?

\[
\begin{array}{c|c}
\text{A} & 2^{34} \\
\hline
\text{B} & 2^{36} \\
\text{C} & 2^{180} \\
\text{D} & 128^{30}
\end{array}
\]

The computer has 64 gigabytes of total storage space. Notice that you can write 64 as a power, \(2^6\). Use a model to solve the problem.

\[
\text{Total number of bytes} = \text{Number of bytes in a gigabyte} \times \text{Number of gigabytes}
\]

\[= 2^{30} \cdot 2^6 \quad \text{Substitute.}\]

\[= 2^{30+6} \quad \text{Product of Powers Property}\]

\[= 2^{36} \quad \text{Simplify.}\]

\[\therefore \text{The computer has } 2^{36} \text{ bytes of total storage space. The correct answer is } (B).\]

On Your Own

10. How many bytes of free storage space does the computer have?
10.2 Exercises

Vocabulary and Concept Check:

1. **REASONING** When should you use the Product of Powers Property?

2. **CRITICAL THINKING** Can you use the Product of Powers Property to multiply $5^2 \cdot 6^4$? Explain.

Practice and Problem Solving

Simplify the expression. Write your answer as a power.

1. $3^2 \cdot 3^2$
2. $8^{10} \cdot 8^4$
3. $a^3 \cdot a^3$
4. $h^6 \cdot h$
5. $(-4)^5 \cdot (-4)^7$
6. $\left(\frac{-5}{7}\right)^8 \cdot \left(\frac{-5}{7}\right)^9$
7. $(-2.9) \cdot (-2.9)^7$
8. $(b^{12})^3$
9. $(3.8^3)^4$
10. $\left(\frac{3}{4}\right)^2$
11. $(5^4)^3$
12. $(2\sqrt{3})^2$
13. $(\sqrt[3]{5})^9$
14. $(1.2m)^4$
15. $(rt)^{12}$
16. $\left(\frac{1}{5}\right)^2$
17. $(6g)^3$
18. $(-3v)^5$
19. $\left(\frac{1}{5}\right)^2$
20. $(1.2m)^4$
21. $(rt)^{12}$
22. $\left(\frac{3}{4}\right)^3$
23. **PRECISION** Is $3^2 + 3^3$ equal to $3^5$? Explain.

24. **ARTIFACT** A display case for the artifact is in the shape of a cube. Each side of the display case is three times longer than the width of the artifact.
   a. Write an expression for the volume of the case. Write your answer as a power.
   b. Simplify the expression.
Simplify the expression.

25.  \(2^4 \cdot 2^5 - (2^2)^2\)  
26.  \(16 \left( \frac{1}{2}x \right)^4\)  
27.  \(5^2(5^3 \cdot 5^2)\)

28. **CLOUDS** The lowest altitude of an altocumulus cloud is about \(3^8\) feet. The highest altitude of an altocumulus cloud is about 3 times the lowest altitude. What is the highest altitude of an altocumulus cloud? Write your answer as a power.

29. **PYTHON EGG** The volume \(V\) of a python egg is given by the formula \(V = \frac{4}{3} \pi abc\).
   
   For the python eggs shown, \(a = 2\) inches, \(b = 2\) inches, and \(c = 3\) inches.
   
   a. Find the volume of a python egg.
   
   b. Square the dimensions of the python egg. Then evaluate the formula. How does this volume compare to your answer in part (a)?

30. **PYRAMID** A square pyramid has a height \(h\) and a base with side length \(b\). The side lengths of the base increase by 50%. Write a formula for the volume of the new pyramid in terms of \(b\) and \(h\).

31. **MAIL** The United States Postal Service delivers about \(2^8 \cdot 5^2\) pieces of mail each second. There are \(2^8 \cdot 3^4 \cdot 5^2\) seconds in 6 days. How many pieces of mail does the United States Postal Service deliver in 6 days? Write your answer as an expression involving powers.

32. **Critical Thinking** Find the value of \(x\) in the equation without evaluating the power.
   
   a.  \(2^5 \cdot 2^x = 256\)
   
   b.  \((\frac{1}{3})^2 \cdot (\frac{1}{3})^x = \frac{1}{729}\)

**Fair Game Review** What you learned in previous grades & lessons

33. \(\frac{4 \cdot 4}{4}\)  
34. \(\frac{5 \cdot 5 \cdot 5}{5}\)  
35. \(\frac{2 \cdot 3}{2}\)  
36. \(\frac{8 \cdot 6 \cdot 6}{6 \cdot 8}\)

37. **MULTIPLE CHOICE** What is the measure of each interior angle of the regular polygon?  
   
   - (A) 45°  
   - (B) 135°  
   - (C) 1080°  
   - (D) 1440°