# 8.3 Volumes of Spheres

## Essential Question How can you find the volume of a sphere?

A **sphere** is the set of all points in space that are the same distance from a point called the *center*. The *radius r* is the distance from the center to any point on the sphere.

radius, *r*

center

A sphere is different from the other solids you have studied so far because it does not have a base. To discover the volume of a sphere, you can use an activity similar to the one in the previous section.
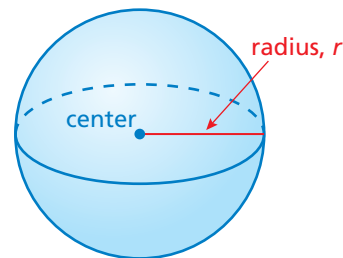
### 1 ACTIVITY: Exploring the Volume of a Sphere

**Work with a partner. Use a plastic ball similar to the one shown.**

- Estimate the diameter and the radius of the ball.

- Use these measurements to draw a net for a cylinder with a diameter and a height equal to the diameter of the ball. How is the height *h* of the cylinder related to the radius *r* of the ball? Explain.

- Cut out the net. Then fold and tape it to form an open cylinder. Make two marks on the cylinder that divide it into thirds, as shown.
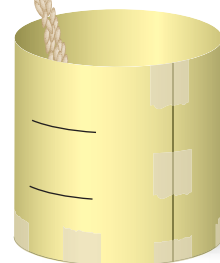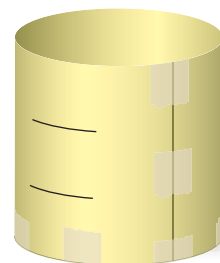
- Cover the ball with aluminum foil or tape. Leave one hole open. Fill the ball with rice. Then pour the rice into the cylinder. What fraction of the cylinder is filled with rice?

**2  ACTIVITY: Deriving the Formula for the Volume of a Sphere**

**Work with a partner. Use the results from Activity 1 and the formula for the volume of a cylinder to complete the steps.**

$$V = \pi r^2 h$$  Write formula for volume of a cylinder.

$$= \frac{\boxed{\phantom{x}}}{\boxed{\phantom{x}}} \pi r^2 h$$  Multiply by $\dfrac{\boxed{\phantom{x}}}{\boxed{\phantom{x}}}$ because the volume of a sphere

is $\dfrac{\boxed{\phantom{x}}}{\boxed{\phantom{x}}}$ of the volume of the cylinder.

$$= \frac{\boxed{\phantom{x}}}{\boxed{\phantom{x}}} \pi r^2 \boxed{\phantom{x}}$$  Substitute $\boxed{\phantom{x}}$ for $h$.

$$= \frac{\boxed{\phantom{x}}}{\boxed{\phantom{x}}} \pi \boxed{\phantom{x}}$$  Simplify.

**3  ACTIVITY: Deriving the Formula for the Volume of a Sphere**
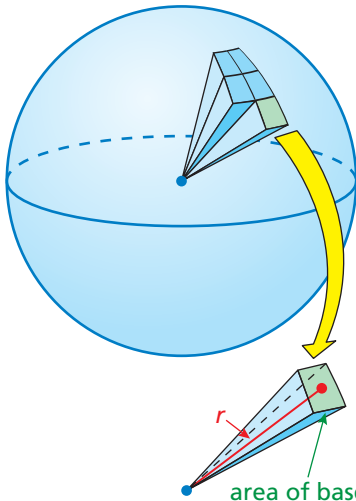
**Work with a partner. Imagine filling the inside of a sphere with $n$ small pyramids. The vertex of each pyramid is at the center of the sphere. The height of each pyramid is approximately equal to $r$, as shown. Complete the steps. (The surface area of a sphere is equal to $4\pi r^2$.)**

$r$

area of base, $B$

$$V = \frac{1}{3}Bh$$  Write formula for volume of a pyramid.

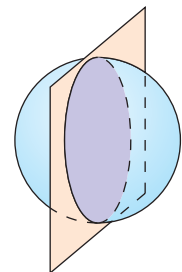$$= n\frac{1}{3}B \boxed{\phantom{x}}$$  Multiply by the number of small pyramids $n$ and substitute $\boxed{\phantom{x}}$ for $h$.

$$= \frac{1}{3}(4\pi r^2)\boxed{\phantom{x}}$$  $4\pi r^2 \approx n \cdot \boxed{\phantom{x}}$

**Show how this result is equal to the result in Activity 2.**

# What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the volume of a sphere?

5. Describe the intersection of the plane and the sphere. Then explain how to find the volume of each section of the solid.

**Practice**  Use what you learned about the volumes of spheres to complete Exercises 3–5 on page 352.

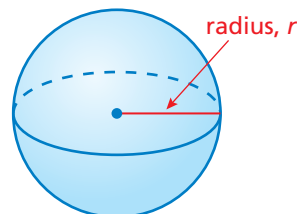**Key Vocabulary** 🔊
sphere, *p. 348*
hemisphere, *p. 351*
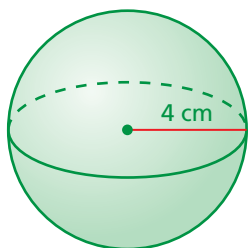
🔑 **Key Idea**

**Volume of a Sphere**

**Words**   The volume $V$ of a sphere is the product of $\frac{4}{3}\pi$ and the cube of the radius of the sphere.

radius, $r$

**Algebra**   $V = \frac{4}{3}\pi r^3$

⤷ Cube of radius of sphere

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**EXAMPLE** **1**   **Finding the Volume of a Sphere**

**Find the volume of the sphere. Round your answer to the nearest tenth.**

4 cm

$$V = \frac{4}{3}\pi r^3 \qquad \text{Write formula for volume.}$$

$$= \frac{4}{3}\pi (4)^3 \qquad \text{Substitute 4 for } r.$$

$$= \frac{256}{3}\pi \qquad \text{Simplify.}$$

$$\approx 268.1 \qquad \text{Use a calculator.}$$

⸫   The volume is about 268.1 cubic centimeters.

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**EXAMPLE** **2**   **Finding the Radius of a Sphere**

**Find the radius of the sphere.**

Volume = $288\pi$ in.$^3$

$r$

$$V = \frac{4}{3}\pi r^3 \qquad \text{Write formula.}$$

$$288\pi = \frac{4}{3}\pi r^3 \qquad \text{Substitute.}$$

$$288\pi = \frac{4\pi}{3}r^3 \qquad \text{Multiply.}$$

$$\frac{3}{4\pi} \cdot 288\pi = \frac{3}{4\pi} \cdot \frac{4\pi}{3}r^3 \qquad \text{Multiplication Property of Equality}$$

$$216 = r^3 \qquad \text{Simplify.}$$

$$6 = r \qquad \text{Take the cube root of each side.}$$

⸫   The radius is 6 inches.

🔊 **Multi-Language Glossary at BigIdeasMath ✓com**
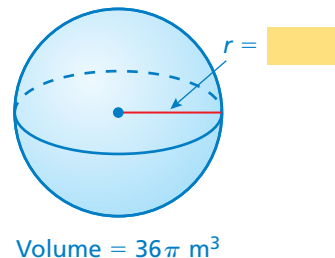
**Find the volume $V$ or radius $r$ of the sphere. Round your answer to the nearest tenth, if necessary.**

**1.**

16 ft

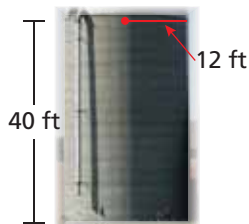$V \approx$ [ ]

**2.**

$r =$ [ ]

Volume $= 36\pi$ m$^3$
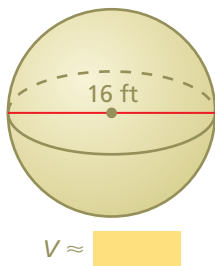
## EXAMPLE 3 Finding the Volume of a Composite Solid

A **hemisphere** is one-half of a sphere. The top of the silo is a hemisphere with a radius of 12 feet. What is the volume of the silo? Round your answer to the nearest thousand.
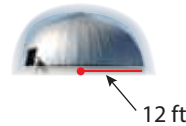
52 ft

The silo is made up of a cylinder and a hemisphere. Find the volume of each solid.

**Cylinder**

12 ft

40 ft

$$V = Bh$$

$$= \pi(12)^2(40)$$

$$= 5760\pi$$

**Hemisphere**

12 ft

$$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$$

$$= \frac{1}{2} \cdot \frac{4}{3}\pi(12)^3$$

$$= 1152\pi$$

So, the volume is $5760\pi + 1152\pi = 6912\pi \approx 22{,}000$ cubic feet.

## On Your Own

**Find the volume of the composite solid. Round your answer to the nearest tenth.**
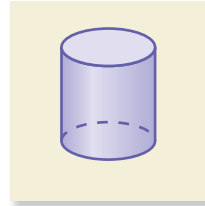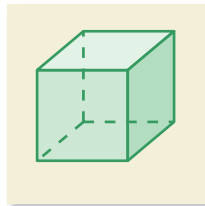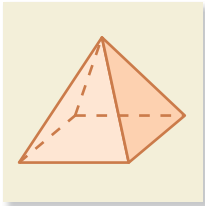
**3.**

2 in.

8 in.

**4.**
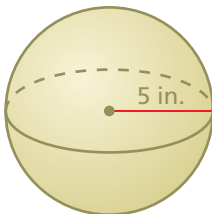
9 m

3 m

5 m
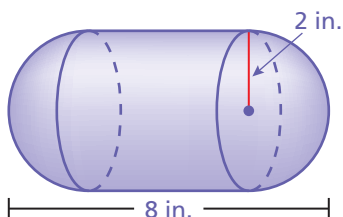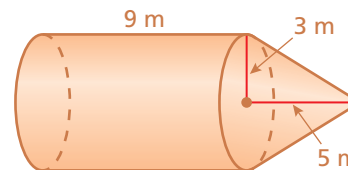
## ✓ Vocabulary and Concept Check

1. **VOCABULARY** How is a sphere different from a hemisphere?

2. **WHICH ONE DOESN'T BELONG?** Which figure does *not* belong with the other three? Explain your reasoning.
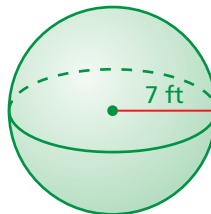
## Practice and Problem Solving

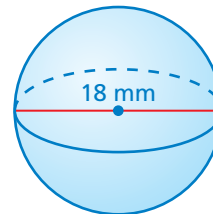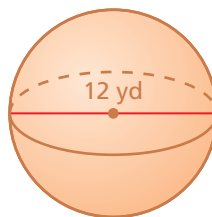**Find the volume of the sphere. Round your answer to the nearest tenth.**

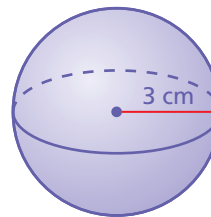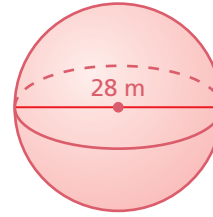**3.**

5 in.

**4.**

7 ft

**5.**

18 mm

**6.**

12 yd

**7.**

3 cm

**8.**

28 m

**Find the radius of the sphere with the given volume.**

9. Volume $= 972\pi \text{ mm}^3$

10. Volume $= 4.5\pi \text{ cm}^3$
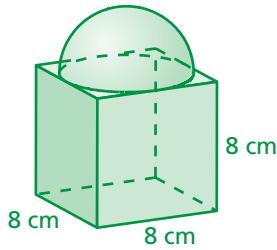
11. Volume $= 121.5\pi \text{ ft}^3$

12. **GLOBE** The globe of the Moon has a radius of 10 inches. Find the volume of the globe. Round your answer to the nearest whole number.
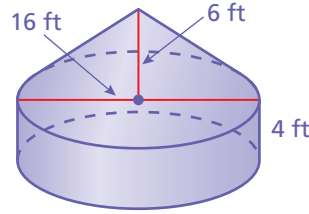
13. **SOFTBALL** A softball has a volume of $\frac{125}{6}\pi$ cubic inches. Find the radius of the softball.

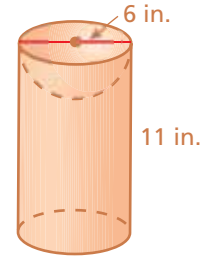**Find the volume of the composite solid. Round your answer to the nearest tenth.**

**③ 14.**

8 cm

8 cm   8 cm

**15.**

16 ft   6 ft

4 ft

**16.**

6 in.

11 in.

**17. REASONING** A sphere and a right cylinder have the same radius and volume. Find the radius $r$ in terms of the height $h$ of the cylinder.

**18. PACKAGING** A cylindrical container of three rubber balls has a height of 18 centimeters and a diameter of 6 centimeters. Each ball in the container has a radius of 3 centimeters. Find the amount of space in the container that is not occupied by rubber balls. Round your answer to the nearest whole number.

**19. BASKETBALL** The basketball shown is packaged in a box that is in the shape of a cube. The edge length of the box is equal to the diameter of the basketball. What is the surface area and the volume of the box?
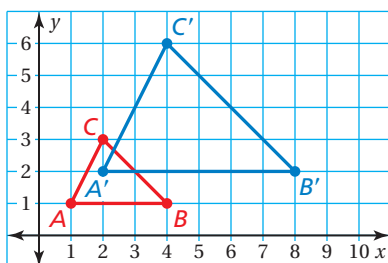
Volume $= 4500\pi$ in.³

**20.** **Logic** Your friend says that the volume of a sphere with radius $r$ is four times the volume of a cone with radius $r$. When is this true? Justify your answer.
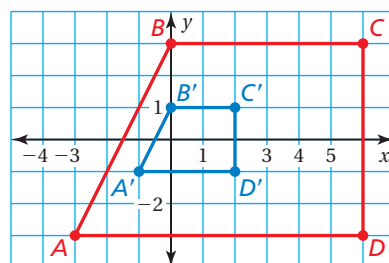
**Fair Game Review** *What you learned in previous grades & lessons*

**The blue figure is a dilation of the red figure. Identify the type of dilation and find the scale factor.** *(Section 2.7)*

**21.**

**22.**

**23. MULTIPLE CHOICE** A person who is 5 feet tall casts a 6-foot-long shadow. A nearby flagpole casts a 30-foot-long shadow. What is the height of the flagpole? *(Section 3.4)*

   **Ⓐ** 25 ft       **Ⓑ** 29 ft       **Ⓒ** 36 ft       **Ⓓ** 40 ft