7.5 Using the Pythagorean Theorem

Essential Question In what other ways can you use the Pythagorean Theorem?

The *converse* of a statement switches the hypothesis and the conclusion.

Statement:	Converse of the statement:
If p, then q.	If q, then p.

1 ACTIVITY: Analyzing Converses of Statements

Work with a partner. Write the converse of the true statement. Determine whether the converse is *true* or *false*. If it is true, justify your reasoning. If it is false, give a counterexample.

a. If $a = b$, then $a^2 = b^2$.

b. If $a = b$, then $a^3 = b^3$.

c. If one figure is a translation of another figure, then the figures are congruent.

d. If two triangles are similar, then the triangles have the same angle measures.

Is the converse of a true statement always true? always false? Explain.

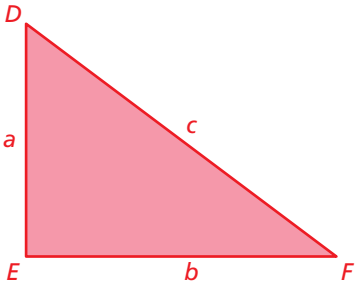
2 ACTIVITY: The Converse of the Pythagorean Theorem

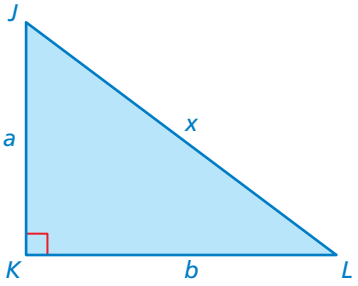
Work with a partner. The converse of the Pythagorean Theorem states: "If the equation $a^2 + b^2 = c^2$ is true for the side lengths of a triangle, then the triangle is a right triangle."

a. Do you think the converse of the Pythagorean Theorem is *true* or *false*? How could you use deductive reasoning to support your answer?

b. Consider $\triangle DEF$ with side lengths a, b, and c, such that $a^2 + b^2 = c^2$. Also consider $\triangle JKL$ with leg lengths a and b, where $\angle K = 90°$.

- What does the Pythagorean Theorem tell you about $\triangle JKL$?
- What does this tell you about c and x?
- What does this tell you about $\triangle DEF$ and $\triangle JKL$?
- What does this tell you about $\angle E$?
- What can you conclude?





Common Core

Pythagorean Theorem

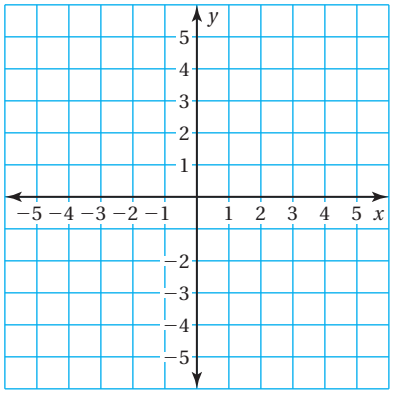
In this lesson, you will

- use the converse of the Pythagorean Theorem to identify right triangles.
- use the Pythagorean Theorem to find distances in a coordinate plane.
- solve real-life problems.

Learning Standards
8.EE.2
8.G.6
8.G.7
8.G.8

3 ACTIVITY: Developing the Distance Formula

Work with a partner. Follow the steps below to write a formula that you can use to find the distance between any two points in a coordinate plane.



Step 1: Choose two points in the coordinate plane that do not lie on the same horizontal or vertical line. Label the points (x_1, y_1) and (x_2, y_2).

Step 2: Draw a line segment connecting the points. This will be the hypotenuse of a right triangle.

Step 3: Draw horizontal and vertical line segments from the points to form the legs of the right triangle.

Step 4: Use the x-coordinates to write an expression for the length of the horizontal leg.

Step 5: Use the y-coordinates to write an expression for the length of the vertical leg.

Step 6: Substitute the expressions for the lengths of the legs into the Pythagorean Theorem.

Step 7: Solve the equation in Step 6 for the hypotenuse c.

What does the length of the hypotenuse tell you about the two points?

Math Practice 6

Communicate Precisely

What steps can you take to make sure that you have written the distance formula accurately?

What Is Your Answer?

4. **IN YOUR OWN WORDS** In what other ways can you use the Pythagorean Theorem?

5. What kind of real-life problems do you think the converse of the Pythagorean Theorem can help you solve?

Practice Use what you learned about the converse of a true statement to complete Exercises 3 and 4 on page 322.

7.5 Lesson

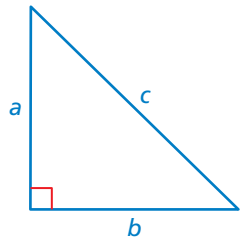
Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
distance formula, *p. 320*

Key Ideas

Converse of the Pythagorean Theorem

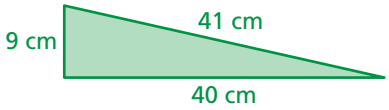
If the equation $a^2 + b^2 = c^2$ is true for the side lengths of a triangle, then the triangle is a right triangle.



EXAMPLE 1 Identifying a Right Triangle

Tell whether each triangle is a right triangle.

a.



$a^2 + b^2 = c^2$

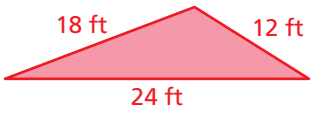
$9^2 + 40^2 \stackrel{?}{=} 41^2$

$81 + 1600 \stackrel{?}{=} 1681$

$1681 = 1681$ ✓

∴ It *is* a right triangle.

b.



$a^2 + b^2 = c^2$

$12^2 + 18^2 \stackrel{?}{=} 24^2$

$144 + 324 \stackrel{?}{=} 576$

$468 \neq 576$ ✗

∴ It is *not* a right triangle.

Study Tip

A *Pythagorean triple* is a set of three positive integers a, b, and c, where $a^2 + b^2 = c^2$.

Common Error

When using the converse of the Pythagorean Theorem, always substitute the length of the longest side for c.

Now You're Ready
Exercises 5–10

On Your Own

Tell whether the triangle with the given side lengths is a right triangle.

1. 28 in., 21 in., 20 in.

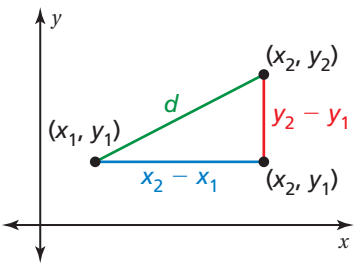
2. 1.25 mm, 1 mm, 0.75 mm

On page 319, you used the Pythagorean Theorem to develop the *distance formula*. You can use the **distance formula** to find the distance between any two points in a coordinate plane.

Key Idea

Distance Formula

The distance d between any two points (x_1, y_1) and (x_2, y_2) is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



EXAMPLE 2 Finding the Distance Between Two Points

Find the distance between (1, 5) and (−4, −2).

Let $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (-4, -2)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Write the distance formula.

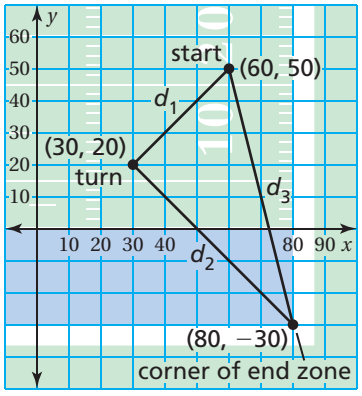
$$= \sqrt{(-4 - 1)^2 + (-2 - 5)^2}$$ Substitute.

$$= \sqrt{(-5)^2 + (-7)^2}$$ Simplify.

$$= \sqrt{25 + 49}$$ Evaluate powers.

$$= \sqrt{74}$$ Add.

EXAMPLE 3 Real-Life Application



You design a football play in which a player runs down the field, makes a 90° turn, and runs to the corner of the end zone. Your friend runs the play as shown. Did your friend make a 90° turn? Each unit of the grid represents 10 feet.

Use the distance formula to find the lengths of the three sides.

$$d_1 = \sqrt{(60 - 30)^2 + (50 - 20)^2} = \sqrt{30^2 + 30^2} = \sqrt{1800} \text{ feet}$$

$$d_2 = \sqrt{(80 - 30)^2 + (-30 - 20)^2} = \sqrt{50^2 + (-50)^2} = \sqrt{5000} \text{ feet}$$

$$d_3 = \sqrt{(80 - 60)^2 + (-30 - 50)^2} = \sqrt{20^2 + (-80)^2} = \sqrt{6800} \text{ feet}$$

Use the converse of the Pythagorean Theorem to determine if the side lengths form a right triangle.

$$(\sqrt{1800})^2 + (\sqrt{5000})^2 \stackrel{?}{=} (\sqrt{6800})^2$$

$$1800 + 5000 \stackrel{?}{=} 6800$$

$$6800 = 6800 \checkmark$$

The sides form a right triangle.

∴ So, your friend made a 90° turn.

On Your Own

Now You're Ready
Exercises 11–16

Find the distance between the two points.

3. (0, 0), (4, 5) **4.** (7, −3), (9, 6) **5.** (−2, −3), (−5, 1)

6. WHAT IF? In Example 3, your friend made the turn at (20, 10). Did your friend make a 90° turn?

7.5 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** Describe two ways to find the distance between two points in a coordinate plane.

2. **WHICH ONE DOESN'T BELONG?** Which set of numbers does *not* belong with the other three? Explain your reasoning.

3, 6, 8	6, 8, 10	5, 12, 13	7, 24, 25

Practice and Problem Solving

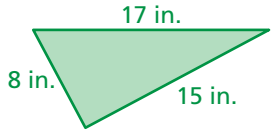
Write the converse of the true statement. Determine whether the converse is *true* or *false*. If it is true, justify your reasoning. If it is false, give a counterexample.

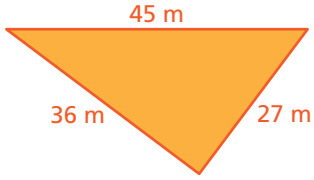
3. If a is an odd number, then a^2 is odd.

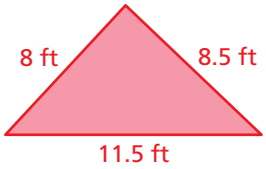
4. If $ABCD$ is a square, then $ABCD$ is a parallelogram.

Tell whether the triangle with the given side lengths is a right triangle.

1

5. 

6. 

7. 

8. 14 mm, 19 mm, 23 mm

9. $\frac{9}{10}$ mi, $1\frac{1}{5}$ mi, $1\frac{1}{2}$ mi

10. 1.4 m, 4.8 m, 5 m

Find the distance between the two points.

2

11. (1, 2), (7, 6)

12. (4, −5), (−1, 7)

13. (2, 4), (7, 2)

14. (−1, −3), (1, 3)

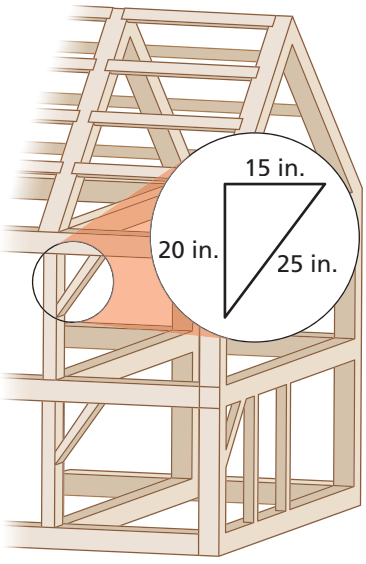
15. (−6, −7), (0, 0)

16. (12, 5), (−12, −2)

17. **ERROR ANALYSIS** Describe and correct the error in finding the distance between the points (−3, −2) and (7, 4).

✗
$$d = \sqrt{[7 - (-3)]^2 - [4 - (-2)]^2}$$
$$= \sqrt{100 - 36}$$
$$= \sqrt{64} = 8$$

18. **CONSTRUCTION** A post and beam frame for a shed is shown in the diagram. Does the brace form a right triangle with the post and beam? Explain.

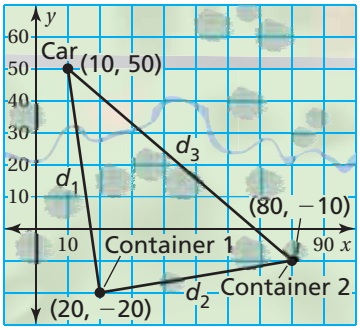


Tell whether a triangle with the given side lengths is a right triangle.

19. $\sqrt{63}, 9, 12$ **20.** $4, \sqrt{15}, 6$ **21.** $\sqrt{18}, \sqrt{24}, \sqrt{42}$

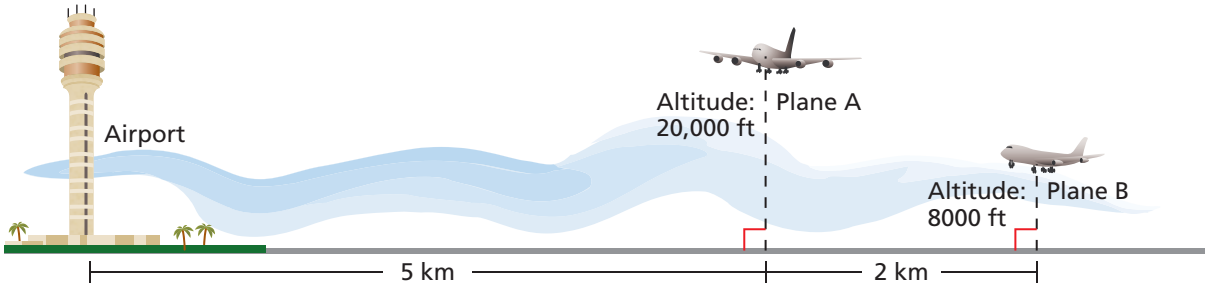
22. **REASONING** Plot the points $(-1, 3)$, $(4, -2)$, and $(1, -5)$ in a coordinate plane. Are the points the vertices of a right triangle? Explain.

23. **GEOCACHING** You spend the day looking for hidden containers in a wooded area using a Global Positioning System (GPS). You park your car on the side of the road, and then locate Container 1 and Container 2 before going back to the car. Does your path form a right triangle? Explain. Each unit of the grid represents 10 yards.



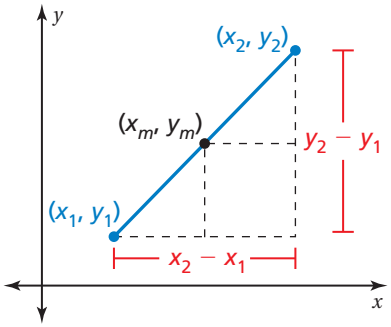
24. **REASONING** Your teacher wants the class to find the distance between the two points (2, 4) and (9, 7). You use (2, 4) for (x_1, y_1), and your friend uses (9, 7) for (x_1, y_1). Do you and your friend obtain the same result? Justify your answer.

25. **AIRPORT** Which plane is closer to the base of the airport tower? Explain.



Not drawn to scale

26. **Structure** Consider the two points (x_1, y_1) and (x_2, y_2) in the coordinate plane. How can you find the point (x_m, y_m) located in the middle of the two given points? Justify your answer using the distance formula.



Fair Game Review What you learned in previous grades & lessons

Find the mean, median, and mode of the data. *(Skills Review Handbook)*

27. 12, 9, 17, 15, 12, 13 **28.** 21, 32, 16, 27, 22, 19, 10 **29.** 67, 59, 34, 71, 59

30. **MULTIPLE CHOICE** What is the sum of the interior angle measures of an octagon? *(Section 3.3)*

(A) 720° (B) 1080° (C) 1440° (D) 1800°