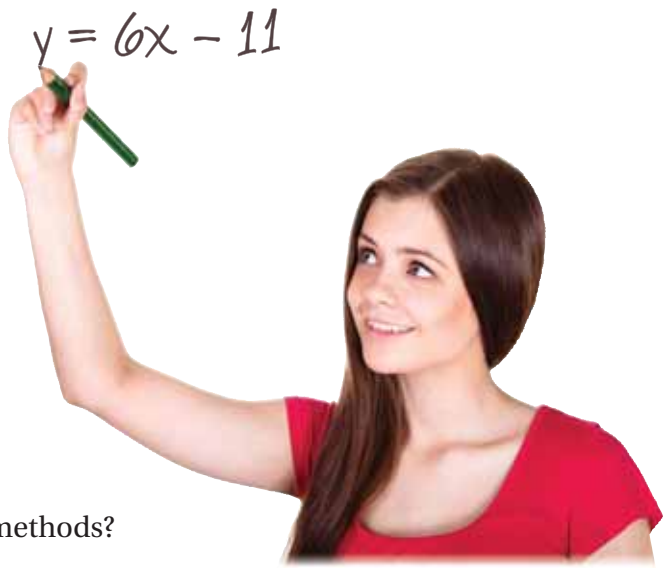
5.2 Solving Systems of Linear Equations by Substitution

Essential Question How can you use substitution to solve a system of linear equations?

1 ACTIVITY: Using Substitution to Solve a System

Work with a partner. Solve each system of linear equations by using two methods.



Method 1: Solve for x first.

Solve for x in one of the equations. Use the expression for x to find the solution of the system. Explain how you did it.

Method 2: Solve for y first.

Solve for y in one of the equations. Use the expression for y to find the solution of the system. Explain how you did it.

Is the solution the same using both methods?

a. $6x - y = 11$
$2x + 3y = 7$

b. $2x - 3y = -1$
$x - y = 1$

c. $3x + y = 5$
$5x - 4y = -3$

d. $5x - y = 2$
$3x - 6y = 12$

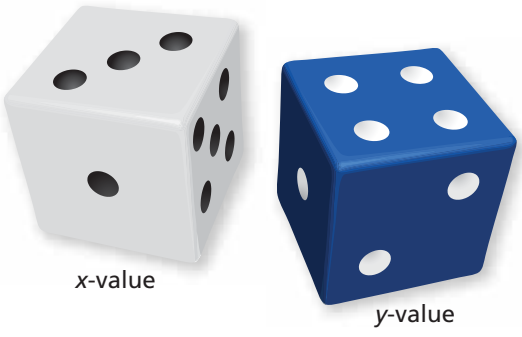
e. $x + y = -1$
$5x + y = -13$

f. $2x - 6y = -6$
$7x - 8y = 5$

2 ACTIVITY: Writing and Solving a System of Equations

Work with a partner.

a. Roll a pair of number cubes that have different colors. Then write the ordered pair shown by the number cubes. The ordered pair at the right is (3, 4).



b. Write a system of linear equations that has this ordered pair as its solution.

c. Exchange systems with your partner. Use one of the methods from Activity 1 to solve the system.

Common Core

Systems of Equations

In this lesson, you will

- write and solve systems of linear equations by substitution.
- solve real-life problems.

Learning Standards
8.EE.8b
8.EE.8c

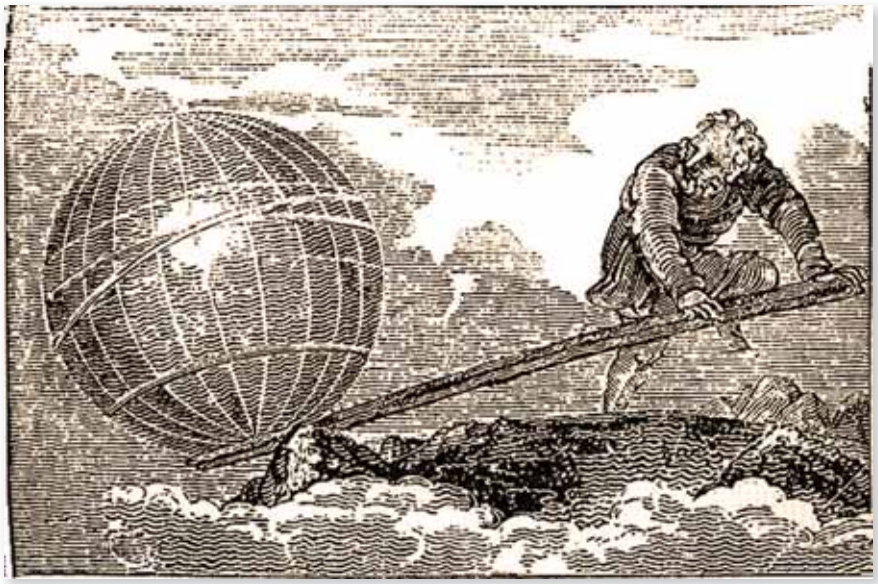
3 ACTIVITY: Solving a Secret Code

Math Practice 1

Check Progress

As you complete each system of equations, how do you know your answer is correct?

Work with a partner. Decode the quote by Archimedes.



$\underline{-8}\ \underline{-7}\ \underline{7}\ \underline{-5}\quad \underline{-4}\ \underline{-5}\quad \underline{-3}\quad \underline{-2}\ \underline{-1}\ \underline{-3}\ \underline{0}\ \underline{-5}\quad \underline{1}\ \underline{2}\quad \underline{3}\ \underline{1}\ \underline{-3}\ \underline{4}\ \underline{5},$

$\underline{-3}\ \underline{4}\ \underline{5}\quad \underline{-7}\quad \underline{6}\ \underline{-7}\ \underline{-1}\ \underline{-1}\quad \underline{-4}\ \underline{2}\ \underline{7}\ \underline{-5}\quad \underline{1}\ \underline{8}\ \underline{-5}\quad \underline{-5}\ \underline{-3}\ \underline{9}\ \underline{1}\ \underline{8}.$

(A, C) $x + y = -3$, $x - y = -3$

(D, E) $x + y = 0$, $x - y = 10$

(G, H) $x + y = 0$, $x - y = -16$

(I, L) $x + 2y = -9$, $2x - y = -13$

(M, N) $x + 2y = 4$, $2x - y = -12$

(O, P) $x + 2y = -2$, $2x - y = 6$

(R, S) $2x + y = 21$, $x - y = 6$

(T, U) $2x + y = -7$, $x - y = 10$

(V, W) $2x + y = 20$, $x - y = 1$

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use substitution to solve a system of linear equations?

Practice Use what you learned about systems of linear equations to complete Exercises 4–6 on page 212.

5.2 Lesson

Check It Out
Lesson Tutorials
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Another way to solve systems of linear equations is to use substitution.

Key Idea

Solving a System of Linear Equations by Substitution

Step 1: Solve one of the equations for one of the variables.

Step 2: Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3: Substitute the value from Step 2 into one of the original equations and solve.

EXAMPLE 1 Solving a System of Linear Equations by Substitution

Solve the system by substitution.

$$y = 2x - 4 \quad \text{Equation 1}$$
$$7x - 2y = 5 \quad \text{Equation 2}$$

Step 1: Equation 1 is already solved for y.

Step 2: Substitute $2x - 4$ for y in Equation 2.

$7x - 2y = 5$ Equation 2

$7x - 2(2x - 4) = 5$ Substitute $2x - 4$ for y.

$7x - 4x + 8 = 5$ Distributive Property

$3x + 8 = 5$ Combine like terms.

$3x = -3$ Subtract 8 from each side.

$x = -1$ Divide each side by 3.

Step 3: Substitute -1 for x in Equation 1 and solve for y.

$y = 2x - 4$ Equation 1

$= 2(-1) - 4$ Substitute -1 for x.

$= -2 - 4$ Multiply.

$= -6$ Subtract.

∴ The solution is $(-1, -6)$.

Check

Equation 1

$y = 2x - 4$

$-6 \stackrel{?}{=} 2(-1) - 4$

$-6 = -6$ ✓

Equation 2

$7x - 2y = 5$

$7(-1) - 2(-6) \stackrel{?}{=} 5$

$5 = 5$ ✓

On Your Own

Now You're Ready
Exercises 10–15

Solve the system of linear equations by substitution. Check your solution.

1. $y = 2x + 3$
$y = 5x$

2. $4x + 2y = 0$
$y = \frac{1}{2}x - 5$

3. $x = 5y + 3$
$2x + 4y = -1$

EXAMPLE 2 Real-Life Application

You buy a total of 50 turkey burgers and veggie burgers for \$90. You pay \$2 per turkey burger and \$1.50 per veggie burger. Write and solve a system of linear equations to find the number x of turkey burgers and the number y of veggie burgers you buy.

Use a verbal model to write a system of linear equations.

Number of turkey burgers, x	+	Number of veggie burgers, y	=	Total number of burgers

Cost per turkey burger	•	Number of turkey burgers, x	+	Cost per veggie burger	•	Number of veggie burgers, y	=	Total cost

The system is:

$x + y = 50$ Equation 1

$2x + 1.5y = 90$ Equation 2

Step 1: Solve Equation 1 for x.

$x + y = 50$ Equation 1

$x = 50 - y$ Subtract y from each side.

Study Tip

It is easiest to solve for a variable that has a coefficient of 1 or -1.

Step 2: Substitute $50 - y$ for x in Equation 2.

$2x + 1.5y = 90$ Equation 2

$2(50 - y) + 1.5y = 90$ Substitute $50 - y$ for x.

$100 - 2y + 1.5y = 90$ Distributive Property

$-0.5y = -10$ Simplify.

$y = 20$ Divide each side by -0.5.

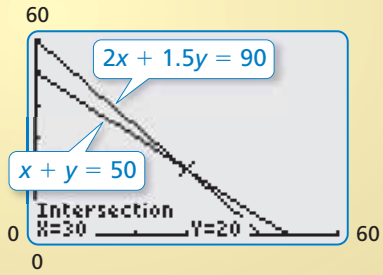
Step 3: Substitute 20 for y in Equation 1 and solve for x.

$x + y = 50$ Equation 1

$x + 20 = 50$ Substitute 20 for y.

$x = 30$ Subtract 20 from each side.

Check



∴ You buy 30 turkey burgers and 20 veggie burgers.

On Your Own

Now You're Ready
Exercises 18–20

4. You sell lemonade for \$2 per cup and orange juice for \$3 per cup. You sell a total of 100 cups for \$240. Write and solve a system of linear equations to find the number of cups of lemonade and the number of cups of orange juice you sold.

5.2 Exercises

Check It Out
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Vocabulary and Concept Check

1. **WRITING** Describe how to solve a system of linear equations by substitution.
2. **NUMBER SENSE** When solving a system of linear equations by substitution, how do you decide which variable to solve for in Step 1?
3. **REASONING** Does solving a system of linear equations by graphing give the same solution as solving by substitution? Explain your reasoning.

Practice and Problem Solving

Write a system of linear equations that has the ordered pair as its solution. Use a method from Activity 1 to solve the system.

4. 
5. 
6. 

Tell which equation you would choose to solve for one of the variables when solving the system by substitution. Explain your reasoning.

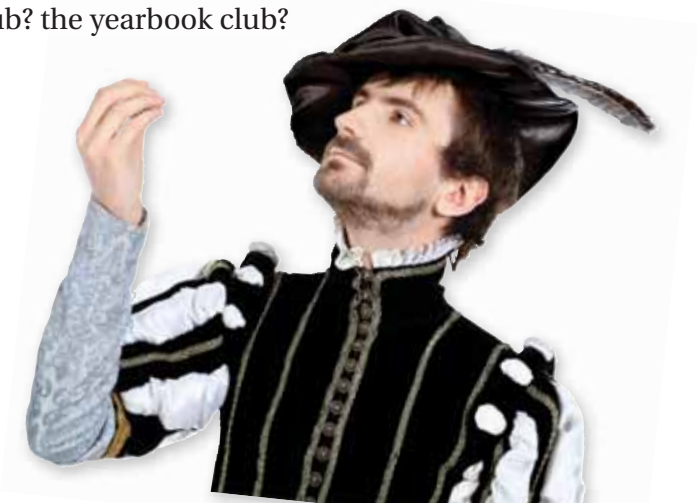
7. $2x + 3y = 5$
   $4x - y = 3$
8. $\frac{2}{3}x + 5y = -1$
   $x + 6y = 0$
9. $2x + 10y = 14$
   $5x - 9y = 1$

Solve the system of linear equations by substitution. Check your solution.

1 10. $y = x - 4$
    $y = 4x - 10$
11. $y = 2x + 5$
    $y = 3x - 1$
12. $x = 2y + 7$
    $3x - 2y = 3$
13. $4x - 2y = 14$
    $y = \frac{1}{2}x - 1$
14. $2x = y - 10$
    $x + 7 = y$
15. $8x - \frac{1}{3}y = 0$
    $12x + 3 = y$

16. **SCHOOL CLUBS** There are a total of 64 students in a drama club and a yearbook club. The drama club has 10 more students than the yearbook club.
    a. Write a system of linear equations that represents this situation.
    b. How many students are in the drama club? the yearbook club?

17. **THEATER** A drama club earns $1040 from a production. It sells a total of 64 adult tickets and 132 student tickets. An adult ticket costs twice as much as a student ticket.
    a. Write a system of linear equations that represents this situation.
    b. What is the cost of each ticket?



Solve the system of linear equations by substitution. Check your solution.

18. $y - x = 0$
$2x - 5y = 9$

19. $x + 4y = 14$
$3x + 7y = 22$

20. $-2x - 5y = 3$
$3x + 8y = -6$

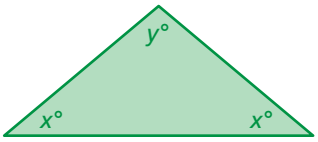
21. ERROR ANALYSIS Describe and correct the error in solving the system of linear equations.

$2x + y = 5$ Equation 1
$3x - 2y = 4$ Equation 2

Step 1:
$2x + y = 5$
$y = -2x + 5$

Step 2:
$2x + (-2x + 5) = 5$
$2x - 2x + 5 = 5$
$5 = 5$

22. STRUCTURE The measure of the obtuse angle in the isosceles triangle is two and a half times the measure of one base angle. Write and solve a system of linear equations to find the measures of all the angles.



23. ANIMAL SHELTER An animal shelter has a total of 65 abandoned cats and dogs. The ratio of cats to dogs is 6 : 7. How many cats are in the shelter? How many dogs are in the shelter? Justify your answers.

24. NUMBER SENSE The sum of the digits of a two-digit number is 8. When the digits are reversed, the number increases by 36. Find the original number.

25. Repeated Reasoning A DJ has a total of 1075 dance, rock, and country songs on her system. The dance selection is three times the size of the rock selection. The country selection has 105 more songs than the rock selection. How many songs on the system are dance? rock? country?

Fair Game Review What you learned in previous grades & lessons

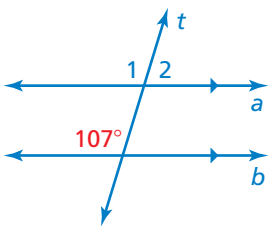
Write the equation in standard form. *(Section 4.5)*

26. $3x - 9 = 7y$

27. $8 - 5y = -2x$

28. $6x = y + 3$

29. MULTIPLE CHOICE Use the figure to find the measure of $\angle 2$. *(Section 3.1)*



(A) 17°
(B) 73°
(C) 83°
(D) 107°