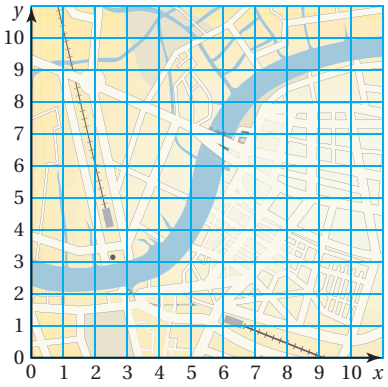
4.4 Polygons in the Coordinate Plane

Essential Question How can you find the lengths of line segments in a coordinate plane?

1 ACTIVITY: Finding Distances on a Map

Work with a partner. The coordinate grid shows a portion of a city. Each square on the grid represents one square mile.



a. A public library is located at (4, 5). City Hall is located at (7, 5). Plot and label these points.

b. How far is the public library from City Hall?

c. A stadium is located 4 miles from the public library. Give the coordinates of several possible locations of the stadium. Justify your answers by graphing.

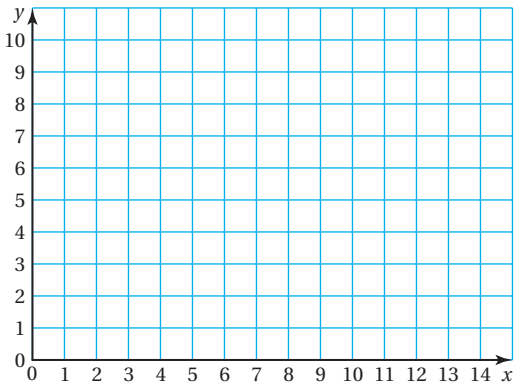
d. Connect the three locations of the public library, City Hall, and the stadium using your answers in part (c). What shapes are formed?

2 ACTIVITY: Graphing Polygons

Work with a partner. Plot and label each set of points in the coordinate plane. Then connect each set of points to form a polygon.

Rectangle: $A(2, 3)$, $B(2, 10)$, $C(6, 10)$, $D(6, 3)$

Triangle: $E(8, 3)$, $F(14, 8)$, $G(14, 3)$



Common Core

Geometry

In this lesson, you will
- draw polygons in the coordinate plane.
- find distances in the coordinate plane.
- solve real-life problems.

Learning Standard 6.G.3

3 ACTIVITY: Finding Distances in a Coordinate Plane

Work with a partner.

a. Find the length of each horizontal line segment in Activity 2.

b. **STRUCTURE** What relationship do you notice between the lengths of the line segments in part (a) and the coordinates of their endpoints? Explain.

c. Find the length of each vertical line segment in Activity 2.

d. **STRUCTURE** What relationship do you notice between the lengths of the line segments in part (c) and the coordinates of their endpoints? Explain.

e. Plot and label the points below in the coordinate plane. Then connect each pair of points with a line segment. Use the relationships you discovered in parts (b) and (d) above to find the length of each line segment. Show your work.

$S(3, 1)$ and $T(14, 1)$ $U(9, 8)$ and $V(9, 0)$

$W(0, 7)$ and $X(0, 10)$ $Y(1, 9)$ and $Z(7, 9)$

f. Check your answers in part (e) by counting grid lines.

Math Practice 8

Repeat Calculations

What calculations are repeated? How can you use this information to write a rule about the length of a line segment?

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the lengths of line segments in a coordinate plane? Give examples to support your explanation.

5. Do the methods you used in Activity 3 work for diagonal line segments? Explain why or why not.

6. Use the Internet or some other reference to find an example of how "finding distances in a coordinate plane" is helpful in each of the following careers.

a.

Archaeologist

b.

Surveyor

c.

Pilot

Practice Use what you learned about finding the lengths of line segments to complete Exercises 3–5 on page 178.

4.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

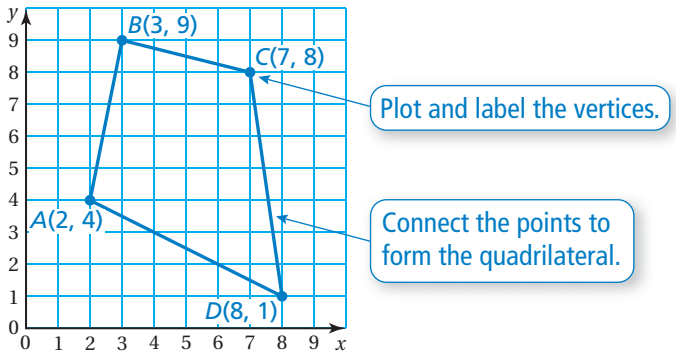
You can use ordered pairs to represent vertices of polygons. To draw a polygon in a coordinate plane, plot and connect the ordered pairs.

EXAMPLE 1 Drawing a Polygon in a Coordinate Plane

The vertices of a quadrilateral are $A(2, 4)$, $B(3, 9)$, $C(7, 8)$, and $D(8, 1)$. Draw the quadrilateral in a coordinate plane.

Study Tip

After you plot the vertices, connect them *in order* to draw the polygon.



On Your Own

Now You're Ready
Exercises 6–11

Draw the polygon with the given vertices in a coordinate plane.

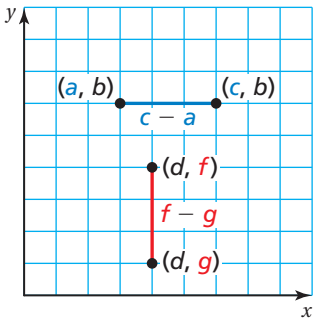
1. $A(0, 0)$, $B(5, 7)$, $C(7, 4)$
2. $W(4, 4)$, $X(7, 4)$, $Y(7, 1)$, $Z(4, 1)$
3. $F(1, 3)$, $G(3, 6)$, $H(5, 6)$, $J(3, 3)$
4. $P(1, 4)$, $Q(3, 5)$, $R(7, 3)$, $S\left(6, \frac{1}{2}\right)$, $T\left(2, \frac{1}{2}\right)$

Key Idea

Finding Distances in the First Quadrant

You can find the length of a horizontal or vertical line segment in a coordinate plane by using the coordinates of the endpoints.

- When the x-coordinates are the same, the vertical distance between the points is the difference of the y-coordinates.
- When the y-coordinates are the same, the horizontal distance between the points is the difference of the x-coordinates.



Be sure to subtract the lesser coordinate from the greater coordinate.

EXAMPLE 2 Finding a Perimeter

The vertices of a rectangle are $F(1, 6)$, $G(7, 6)$, $H(7, 2)$, and $J(1, 2)$. Draw the rectangle in a coordinate plane and find its perimeter.

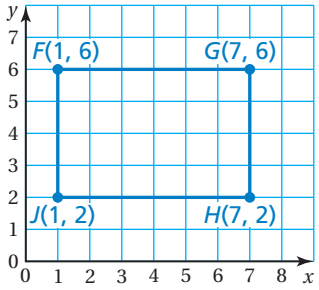
Draw the rectangle and use the vertices to find its dimensions.

The length is the horizontal distance between $F(1, 6)$ and $G(7, 6)$, which is the difference of the x-coordinates.

$$\text{length} = 7 - 1 = 6 \text{ units}$$

The width is the vertical distance between $G(7, 6)$ and $H(7, 2)$, which is the difference of the y-coordinates.

$$\text{width} = 6 - 2 = 4 \text{ units}$$



So, the perimeter of the rectangle is $2(6) + 2(4) = 20$ units.

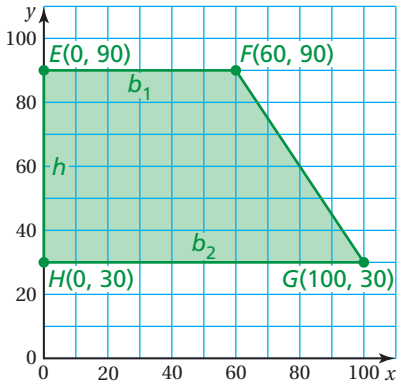
Study Tip

You can also find the length using vertices H and J. You can find the width using vertices F and J.

EXAMPLE 3 Real-Life Application

In a grid of the exhibits at a zoo, the vertices of the giraffe exhibit are $E(0, 90)$, $F(60, 90)$, $G(100, 30)$, and $H(0, 30)$. The coordinates are measured in feet. What is the area of the giraffe exhibit?

Plot and connect the vertices using a coordinate grid to form a trapezoid. Use the coordinates to find the lengths of the bases and the height.



$$b_1 = 60 - 0 = 60$$

$$b_2 = 100 - 0 = 100$$

$$h = 90 - 30 = 60$$

Use the formula for the area of a trapezoid.

$$A = \frac{1}{2}(60)(60 + 100)$$

$$= \frac{1}{2}(60)(160) = 4800$$

The area of the giraffe exhibit is 4800 square feet.

Common Error

You can count grid lines to find the dimensions, but make sure you consider the scale of the axes.

On Your Own

Now You're Ready
Exercises 12–15

5. The vertices of a rectangle are $J(2, 7)$, $K(4, 7)$, $L(4, 1.5)$, and $M(2, 1.5)$. Find the perimeter and the area of the rectangle.
6. **WHAT IF?** In Example 3, the giraffe exhibit is enlarged by moving vertex F to $(80, 90)$. How does this affect the area? Explain.

4.4 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **WRITING** How can you use a coordinate plane to draw a polygon?
2. **WRITING** How can you find the perimeter of a rectangle in a coordinate plane?

Practice and Problem Solving

Plot and label each pair of points in a coordinate plane. Find the length of the line segment connecting the points.

3. $C(0, 1), D(8, 1)$
4. $K(5, 2), L(5, 6)$
5. $Q(3, 4), R(3, 9)$

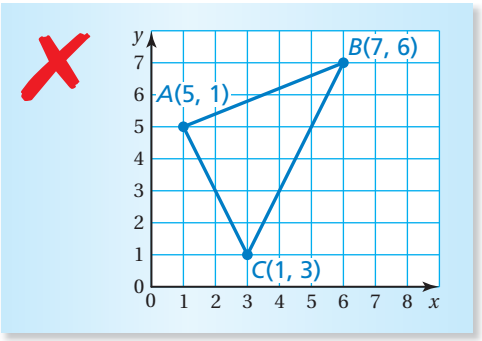
Draw the polygon with the given vertices in a coordinate plane.

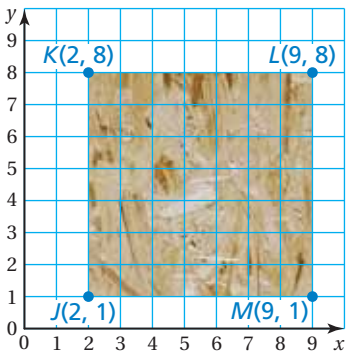
1 6. $A(4, 7), B(6, 2), C(0, 0)$
7. $D\left(\frac{1}{2}, 2\right), E(5, 5), F(4, 1)$
8. $G\left(1\frac{1}{2}, 4\right), H\left(1\frac{1}{2}, 8\right), J(5, 8), K(5, 4)$
9. $L(3, 2), M(3, 5), N(9, 5), P(9, 2)$
10. $Q(0, 4), R(10, 8), S(7, 4), T(10, 2), U(5, 0)$
11. $V(2, 2), W\left(3, 7\frac{1}{2}\right), X\left(8, 7\frac{1}{2}\right), Y(10, 4), Z(7, 0)$

Find the perimeter and the area of the polygon with the given vertices.

2 12. $C(1, 1), D(1, 4), E(4, 4), F(4, 1)$
13. $J(1, 2), K(7, 2), L(7, 8), M(1, 8)$
14. $N(0, 2), P(5, 2), Q(5, 5), R(0, 5)$
15. $S(3, 0), T(3, 9), U(8, 9), V(8, 0)$

16. **ERROR ANALYSIS** Describe and correct the error in drawing a triangle with vertices $A(5, 1)$, $B(7, 6)$, and $C(1, 3)$.





17. **TREE HOUSE** You design a tree house using a coordinate plane. You plot the vertices of the floor at $J(2, 1)$, $K(2, 8)$, $L(9, 8)$, and $M(9, 1)$. The coordinates are measured in feet.
   a. What is the shape of the floor?
   b. What are the perimeter and the area of the floor?

OPEN-ENDED **Draw a polygon with the given conditions in a coordinate plane.**

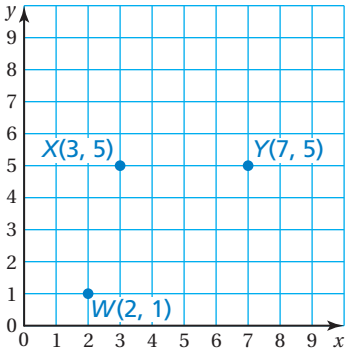
18. a square with a perimeter of 20 units

19. a rectangle with a perimeter of 18 units

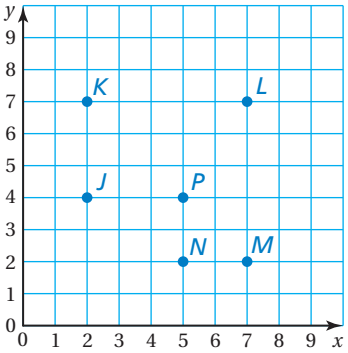
20. a rectangle with an area of 24 units^2

21. a triangle with an area of 15 units^2

22. **STRUCTURE** The coordinate plane shows three vertices of a parallelogram. Find two possible points that could represent the fourth vertex.



23. **BUS ROUTE** Polygon *JKLMNP* represents a bus route. Each grid square represents 9 square miles. What is the shortest distance, in miles, from station *P* to station *L* using the bus route? Explain your reasoning.



24. **CITY LIMITS** In a topographical map of a city, the vertices of the city limits are $A(10, 9)$, $B(18, 9)$, $C(18, 2)$, $D(14, 4.5)$, and $E(10, 4.5)$. The coordinates are measured in miles. What is the area of the city?

25. **BACKYARD** The vertices of a backyard are $W(10, 30)$, $X(10, 100)$, $Y(110, 100)$, and $Z(50, 30)$. The coordinates are measured in feet. The line segment XZ separates the backyard into a lawn and a garden. The area of the lawn is greater than the area of the garden. How many times larger is the lawn than the garden?

26. **Precision** The vertices of a rectangle are $(1, 0)$, $(1, a)$, $(5, a)$, and $(5, 0)$. The vertices of a parallelogram are $(1, 0)$, $(2, b)$, $(6, b)$, and $(5, 0)$. The value of a is greater than the value of b. Which polygon has a greater area? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Divide. Write the answer in simplest form. *(Section 2.3)*

27. $1\frac{1}{3} \div \frac{2}{3}$

28. $6\frac{3}{5} \div \frac{3}{4}$

29. $2\frac{1}{2} \div 8$

30. $4\frac{1}{6} \div 1\frac{1}{8}$

31. **MULTIPLE CHOICE** You are filling bottles from 5 gallons of lemonade. How many bottles can you fill when each bottle is $\frac{3}{8}$ of a gallon? *(Section 2.2)*

A $1\frac{7}{8}$ **B** 3 **C** 8 **D** $13\frac{1}{3}$