

# 4.3 Areas of Trapezoids

**Essential Question** How can you derive a formula for the area of a trapezoid?

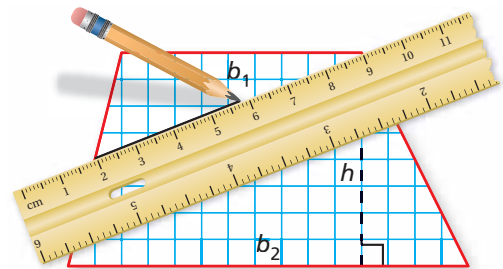
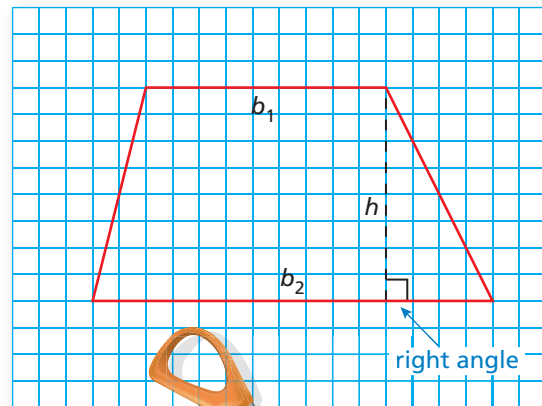
## 1 ACTIVITY: Deriving the Area Formula of a Trapezoid

Work with a partner. Use a piece of centimeter grid paper.

- Draw *any* trapezoid so that its base lies on one of the horizontal lines of the paper.
- Estimate the area of your trapezoid (in square centimeters) by counting unit squares.

Area  $\approx$   Estimate

- Label the height and the bases *inside* the trapezoid.
- Cut out the trapezoid. Mark the midpoint of the side opposite the height. Draw a line from the midpoint to the opposite upper vertex.
- Cut along the line. You will end up with a triangle and a quadrilateral. Arrange these two figures to form a figure whose area you know.

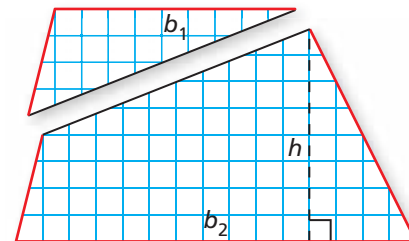


- Use your result to write a *formula* for the area of a trapezoid.

Area =  Formula

- Use your formula to find the area of your trapezoid (in square centimeters).

Area =  Exact Area



- Compare this area with your estimate in part (b).



### Geometry

- In this lesson, you will
- find areas of trapezoids.
  - solve real-life problems.

Learning Standard  
6.G.1

## 2 ACTIVITY: Writing a Math Lesson

Work with a partner. Use your results from Activity 1 to write a lesson on finding the area of a trapezoid.

### Math Practice 6

#### Use Clear Definitions

Do your steps for the *Key Idea* help another person understand how to solve the problem? Do the examples follow your steps?

Describe steps you can use to find the area of a trapezoid.

Write two examples for finding the area of a trapezoid. Include a drawing for each.

Write two exercises for finding the area of a trapezoid. Include an answer sheet.

*Area of a Trapezoid*

**Key Idea** Use the following steps to find the area of a trapezoid.

- 1.
- 2.
- 3.

**Examples**

a. \_\_\_\_\_ b. \_\_\_\_\_

**Exercises**

Find the area.

1. \_\_\_\_\_ 2. \_\_\_\_\_

## What Is Your Answer?

- IN YOUR OWN WORDS** How can you derive a formula for the area of a trapezoid?
- In this chapter, you used deductive reasoning to derive new area formulas from area formulas you have already learned. Describe a real-life career in which deductive reasoning is important.

### Practice

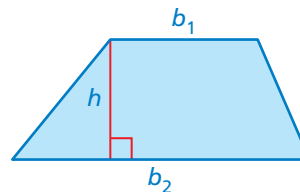
Use what you learned about the areas of trapezoids to complete Exercises 4–6 on page 170.

## Key Idea

### Area of a Trapezoid

**Words** The area  $A$  of a trapezoid is one-half the product of its height  $h$  and the sum of its bases  $b_1$  and  $b_2$ .

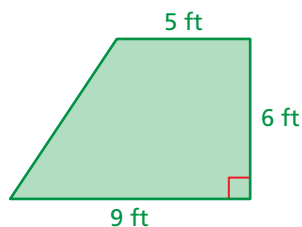
**Algebra**  $A = \frac{1}{2}h(b_1 + b_2)$



## EXAMPLE 1 Finding Areas of Trapezoids

Find the area of each trapezoid.

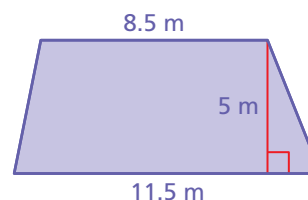
a.



$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) && \text{Write formula.} \\ &= \frac{1}{2}(6)(5 + 9) && \text{Substitute.} \\ &= \frac{1}{2}(6)(14) && \text{Add.} \\ &= 42 && \text{Multiply.} \end{aligned}$$

∴ The area of the trapezoid is 42 square feet.

b.



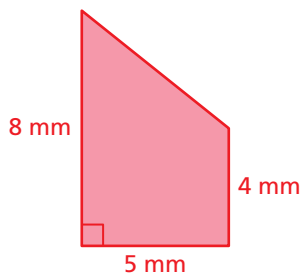
$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) \\ &= \frac{1}{2}(5)(8.5 + 11.5) \\ &= \frac{1}{2}(5)(20) \\ &= 50 \end{aligned}$$

∴ The area of the trapezoid is 50 square meters.

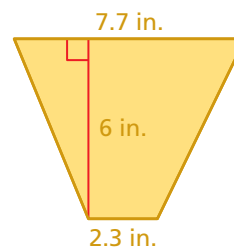
## On Your Own

Find the area of the trapezoid.

1.

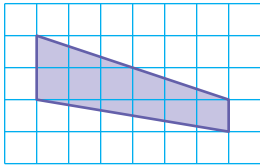


2.



Now You're Ready  
Exercises 7–9

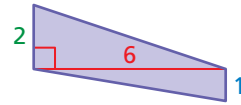
## EXAMPLE 2 Finding the Area of a Trapezoid on a Grid



What is the area of the trapezoid?

- (A) 6 units<sup>2</sup>      (B) 7 units<sup>2</sup>      (C) 9 units<sup>2</sup>      (D) 12 units<sup>2</sup>

Count grid lines to find the dimensions. The height  $h$  is 6 units, base  $b_1$  is 1 unit, and base  $b_2$  is 2 units.

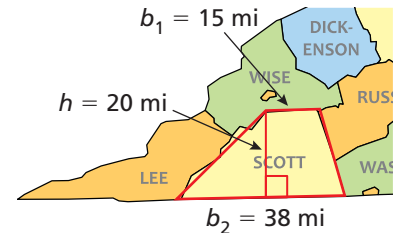


$$\begin{aligned}
 A &= \frac{1}{2}h(b_1 + b_2) && \text{Write formula.} \\
 &= \frac{1}{2}(6)(1 + 2) && \text{Substitute values.} \\
 &= \frac{1}{2}(6)(3) && \text{Add.} \\
 &= 9 && \text{Multiply.}
 \end{aligned}$$

- ∴ The area of the trapezoid is 9 square units. The correct answer is (C).

## EXAMPLE 3 Real-Life Application

You can use a trapezoid to approximate the shape of Scott County, Virginia. The population is about 23,200. About how many people are there per square mile?



Find the area of Scott County.

$$\begin{aligned}
 A &= \frac{1}{2}h(b_1 + b_2) && \text{Write formula for area of a trapezoid.} \\
 &= \frac{1}{2}(20)(15 + 38) && \text{Substitute 20 for } h, 15 \text{ for } b_1, \text{ and } 38 \text{ for } b_2. \\
 &= \frac{1}{2}(20)(53) = 530 && \text{Simplify.}
 \end{aligned}$$

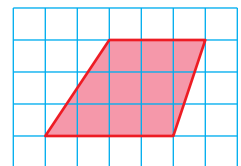
The area of Scott County is about 530 square miles. Divide the population by the area to find the number of people per square mile.

- ∴ So, there are about  $\frac{23,200 \text{ people}}{530 \text{ mi}^2} \approx 44$  people per square mile.

### On Your Own

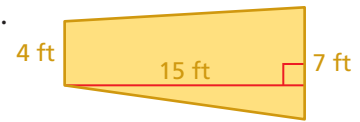
Now You're Ready  
Exercises 11–13

- Find the area of the trapezoid.
- WHAT IF?** In Example 3, the population of Scott County decreases by 550. By how much does the number of people per square mile change? Explain.



## Vocabulary and Concept Check

- VOCABULARY** Identify the bases and the height of the trapezoid.
- REASONING** What measures do you need to find the area of a trapezoid?
- WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.



$$\frac{1}{2}bh$$

$$\ell w$$

$$2\ell + 2w$$

$$\frac{1}{2}h(b_1 + b_2)$$

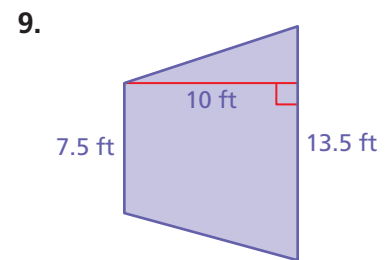
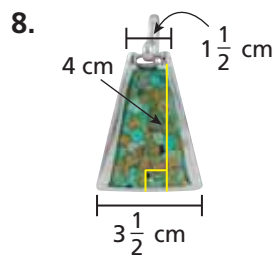
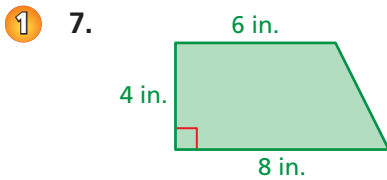
## Practice and Problem Solving

Find the area of the trapezoid.

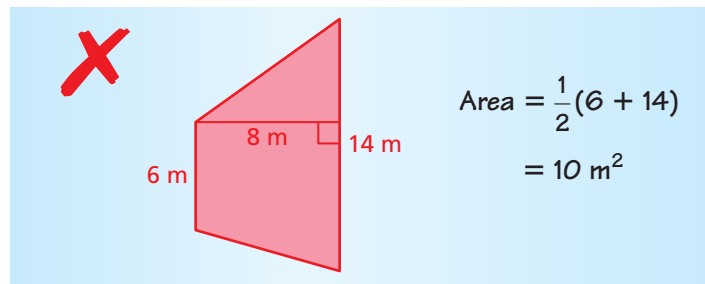
4.  $b_1 = 4, b_2 = 8, h = 2$

5.  $b_1 = 5, b_2 = 7, h = 4$

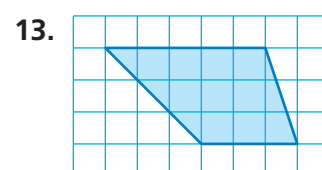
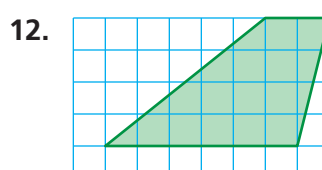
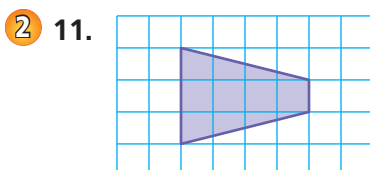
6.  $b_1 = 12, b_2 = 6, h = 3$



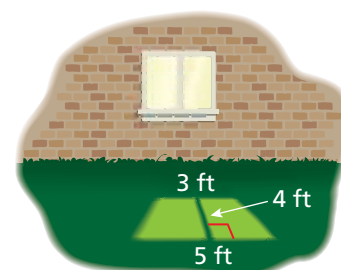
10. **ERROR ANALYSIS** Describe and correct the error in finding the area of the trapezoid.



Find the area of the trapezoid.



14. **LIGHT** Light shines through a window. What is the area of the trapezoid-shaped region created by the light?



Find the area of a trapezoid with height  $h$  and bases  $b_1$  and  $b_2$ .

15.  $h = 6$  in.

$b_1 = 9$  in.

$b_2 = 11$  in.

16.  $h = 22$  cm

$b_1 = 10.5$  cm

$b_2 = 12.5$  cm

17.  $h = 12$  mi

$b_1 = 5.6$  mi

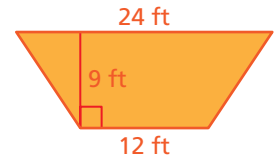
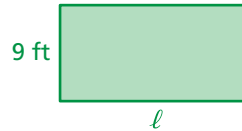
$b_2 = 7.4$  mi

18.  $h = 14$  m

$b_1 = 21$  m

$b_2 = 22$  m

19. **REASONING** The rectangle and the trapezoid have the same area. What is the length  $\ell$  of the rectangle?



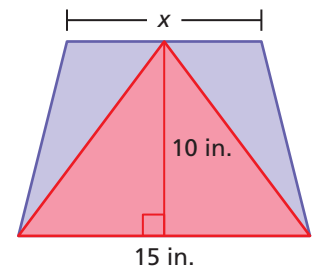
20. **OPEN-ENDED** The area of the trapezoidal student election sign is 5 square feet. Find two possible values for each base length.

21. **AUDIO** How many times greater is the area of the floor covered by the larger speaker than by the smaller speaker?



22. **Critical Thinking** The triangle and the trapezoid share a 15-inch base and a height of 10 inches.

- The area of the trapezoid is less than twice the area of the triangle. Find the values of  $x$ . Explain your reasoning.
- Can the area of the *trapezoid* be exactly twice the area of the triangle? Explain your reasoning.



## Fair Game Review what you learned in previous grades & lessons

Plot the ordered pair in a coordinate plane. (*Skills Review Handbook*)

23.  $(5, 0)$

24.  $(2, 4)$

25.  $(0, 3)$

26.  $(6, 1)$

27. **MULTIPLE CHOICE** Which expression represents “6 more than  $x$ ”?

(*Section 3.2*)

(A)  $6 - x$

(B)  $6x$

(C)  $x + 6$

(D)  $\frac{6}{x}$