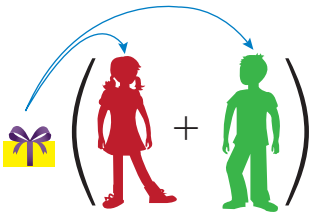
3.4 The Distributive Property

Essential Question How do you use mental math to multiply two numbers?

The Meaning of a Word Distribute

When you **distribute** something to each person in a group, you give that thing to each person in the group.



1 ACTIVITY: Modeling a Property

Work with a partner.

a. **MODELING** Draw two rectangles of the same width but with different lengths on a piece of grid paper. Label the dimensions.

b. Write an expression for the total area of the rectangles.

$$(\square \times \square) + (\square \times \square)$$

c. Rearrange the rectangles by aligning the shortest sides to form one rectangle. Label the dimensions. Write an expression for the area.

$$\square \times (\square + \square)$$

d. Can the expressions from parts (b) and (c) be set equal to each other? Explain.

e. **REPEATED REASONING** Repeat this activity using different rectangles. Explain how this illustrates the Distributive Property. Write a rule for the Distributive Property.

Common Core

Equivalent Expressions

In this lesson, you will
- use the Distributive Property to find products.
- use the Distributive Property to simplify algebraic expressions.

Learning Standards
6.NS.4
6.EE.2b
6.EE.3
6.EE.4

2 ACTIVITY: Using Mental Math

Math Practice 1

Find Entry Points

How can you rewrite the larger number as the sum of two numbers so that you can use mental math?

Work with a partner. Use the method shown to find the product.

a. Sample: 23×6

$$\begin{array}{rl} 23 & \leftarrow \text{23 is 20 + 3.} \\ \times\ 6 & \\ 120 & \text{Multiply 20 and 6.} \\ +\ 18 & \text{Multiply 3 and 6.} \\ 138 & \text{Add.} \end{array}$$

So, $23 \times 6 = 138$.

b. 33×7

c. 47×9

d. 28×5

e. 17×4

3 ACTIVITY: Using Mental Math

Work with a partner. Use the Distributive Property and mental math to find the product.

a. Sample: 6×23

$$\begin{aligned} 6 \times 23 &= 6 \times (20 + 3) && \text{Write 23 as the sum of 20 and 3.} \\ &= (6 \times 20) + (6 \times 3) && \text{Distribute the 6 over the sum.} \\ &= 120 + 18 && \text{Find the products.} \\ &= 138 && \text{Add.} \end{aligned}$$

So, $6 \times 23 = 138$.

b. 5×17

c. 8×26

d. 20×19

e. 40×29

f. 25×39

g. 15×47

What Is Your Answer?

4. Compare the methods in Activities 2 and 3.

5. IN YOUR OWN WORDS How do you use mental math to multiply two numbers? Give examples to support your explanation.

Practice Use what you learned about the Distributive Property to complete Exercises 5–8 on page 137.

3.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
like terms, *p. 136*

Key Idea

Distributive Property

Words To multiply a sum or difference by a number, multiply each number in the sum or difference by the number outside the parentheses. Then evaluate.

Numbers
$3(7 + 2) = 3 \times 7 + 3 \times 2$
$3(7 - 2) = 3 \times 7 - 3 \times 2$

Algebra
$a(b + c) = ab + ac$
$a(b - c) = ab - ac$

EXAMPLE 1 Using Mental Math

Use the Distributive Property and mental math to find 8×53.

$8 \times 53 = 8(50 + 3)$ Write 53 as 50 + 3.
$= 8(50) + 8(3)$ Distributive Property
$= 400 + 24$ Multiply.
$= 424$ Add.

EXAMPLE 2 Using the Distributive Property

Use the Distributive Property to find $\frac{1}{2} \times 2\frac{3}{4}$.

$\frac{1}{2} \times 2\frac{3}{4} = \frac{1}{2} \times \left(2 + \frac{3}{4}\right)$ Rewrite $2\frac{3}{4}$ as the sum $2 + \frac{3}{4}$.
$= \left(\frac{1}{2} \times 2\right) + \left(\frac{1}{2} \times \frac{3}{4}\right)$ Distributive Property
$= 1 + \frac{3}{8}$ Multiply.
$= 1\frac{3}{8}$ Add.

On Your Own

Now You're Ready
Exercises 5–16

Use the Distributive Property to find the product.

1. 5×41
2. 9×19
3. $6(37)$
4. $\frac{2}{3} \times 1\frac{1}{2}$
5. $\frac{1}{4} \times 4\frac{1}{5}$
6. $\frac{2}{7} \times 3\frac{3}{4}$

EXAMPLE 3 Simplifying Algebraic Expressions

Use the Distributive Property to simplify the expression.

a. $4(n + 5)$

$4(n + 5) = 4(n) + 4(5)$ Distributive Property

$= 4n + 20$ Multiply.

b. $12(2y - 3)$

$12(2y - 3) = 12(2y) - 12(3)$ Distributive Property

$= 24y - 36$ Multiply.

c. $9(6 + x + 2)$

$9(6 + x + 2) = 9(6) + 9(x) + 9(2)$ Distributive Property

$= 54 + 9x + 18$ Multiply.

$= 9x + 54 + 18$ Commutative Property of Addition

$= 9x + 72$ Add 54 and 18.

Study Tip

You can use the Distributive Property when there are more than two terms in the sum or difference.

On Your Own

Now You're Ready
Exercises 17–32

Use the Distributive Property to simplify the expression.

7. $7(a + 2)$ **8.** $3(d - 11)$ **9.** $7(2 + 6 - 4d)$

EXAMPLE 4 Real-Life Application

José is x years old. His brother, Felipe, is 2 years older than José. Their aunt, Maria, is three times as old as Felipe. Write and simplify an expression that represents Maria's age in years.

Name	Description	Expression
José	He is x years old.	x
Felipe	He is 2 years *older* than José. So, *add* 2 to x.	$x + 2$
Maria	She is three *times* as old as Felipe. So, *multiply* 3 and $(x + 2)$.	$3(x + 2)$

$3(x + 2) = 3(x) + 3(2)$ Distributive Property

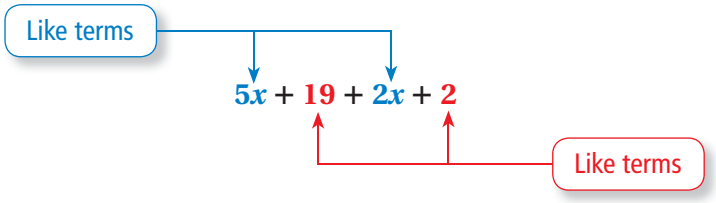
$= 3x + 6$ Multiply.

∴ Maria's age in years is represented by the expression $3x + 6$.

On Your Own

10. Alexis is x years old. Her sister, Gloria, is 7 years older than Alexis. Their grandfather is five times as old as Gloria. Write and simplify an expression that represents their grandfather's age in years.

In an algebraic expression, **like terms** are terms that have the same variables raised to the same exponents. Constant terms are also like terms.



Use the Distributive Property to *combine* like terms.

EXAMPLE 5 Combining Like Terms

Simplify each expression.

a. $3x + 9 + 2x - 5$

$3x + 9 + 2x - 5 = 3x + 2x + 9 - 5$ Commutative Property of Addition

$= (3 + 2)x + 9 - 5$ Distributive Property

$= 5x + 4$ Simplify.

b. $y + y + y$

$y + y + y = 1y + 1y + 1y$ Multiplication Property of One

$= (1 + 1 + 1)y$ Distributive Property

$= 3y$ Add coefficients.

c. $7z + 2(z - 5y)$

$7z + 2(z - 5y) = 7z + 2(z) - 2(5y)$ Distributive Property

$= 7z + 2z - 10y$ Multiply.

$= (7 + 2)z - 10y$ Distributive Property

$= 9z - 10y$ Add coefficients.

On Your Own

Now You're Ready
Exercises 39–53

Simplify the expression.

11. $8 + 3z - z$ **12.** $3(b + 5) + b + 2$

3.4 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** One meaning of the word *distribute* is "to give something to each member of a group." How can this help you remember the Distributive Property?
2. **OPEN-ENDED** Write an algebraic expression in which you use the Distributive Property and then the Associative Property of Addition to simplify.
3. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$2(x + 2)$ $5(x - 8)$ $4 + (x \cdot 4)$ $8(9 - x)$

4. Identify the like terms in the expression $8x + 1 + 7x + 4$.

Practice and Problem Solving

Use the Distributive Property and mental math to find the product.

1 **5.** 3×21 **6.** 9×76 **7.** $12(43)$ **8.** $5(88)$

9. 18×52 **10.** 8×27 **11.** $8(63)$ **12.** $7(28)$

Use the Distributive Property to find the product.

2 **13.** $\frac{1}{4} \times 2\frac{2}{7}$ **14.** $\frac{5}{6} \times 2\frac{2}{5}$ **15.** $\frac{5}{9} \times 4\frac{1}{2}$ **16.** $\frac{2}{15} \times 5\frac{5}{8}$

Use the Distributive Property to simplify the expression.

3 **17.** $3(x + 4)$ **18.** $10(b - 6)$ **19.** $6(s - 9)$ **20.** $7(8 + y)$

21. $8(12 + a)$ **22.** $9(2n + 1)$ **23.** $12(6 - k)$ **24.** $18(5 - 3w)$

25. $9(3 + c + 4)$ **26.** $7(8 + x + 2)$ **27.** $8(5g + 5 - 2)$ **28.** $6(10 + z + 3)$

29. $4(x + y)$ **30.** $25(x - y)$ **31.** $7(p + q + 9)$ **32.** $13(n + 4 + 7m)$

33. ERROR ANALYSIS Describe and correct the error in rewriting the expression.

✗ $6(y + 8) = 6y + 8$

34. ART MUSEUM A class of 30 students visits an art museum and a special exhibit while there.

a. Use the Distributive Property to write and simplify an expression for the cost.

b. Estimate a reasonable value for x. Explain.

c. Use your estimate for x to evaluate the original expression and the simplified expression in part (a). Are the values the same?

35. FITNESS Each day, you run on a treadmill for r minutes and lift weights for 15 minutes. Which expressions can you use to find how many minutes of exercise you do in 5 days? Explain your reasoning.

$5(r + 15)$ $\quad 5r + 5 \cdot 15$ $\quad 5r + 15$ $\quad r(5 + 15)$

36. SPEED A cheetah can run 103 feet per second. A zebra can run x feet per second. Use the Distributive Property to write and simplify an expression for how much farther the cheetah can run in 10 seconds.

UNIFORMS Your baseball team has 16 players. Use the Distributive Property to write and simplify an expression for the total cost of buying the items shown for all the players.

37.

Pants: \$10 $\quad$ Belt: \$$x$

38.

Jersey: \$12 $\quad$ Socks: \$4 $\quad$ Hat: \$$x$

5 Simplify the expression.

39. $6(x + 4) + 1$ $\quad$ **40.** $5 + 8(3 + x)$ $\quad$ **41.** $7(8 + 4k) + 12$

42. $x + 3 + 5x$ $\quad$ **43.** $7y + 6 - 1 + 12y$ $\quad$ **44.** $w + w + 5w$

45. $4d + 9 - d - 8$ $\quad$ **46.** $n + 3(n - 1)$ $\quad$ **47.** $2v + 8v - 5v$

48. $5(z + 4) + 5(2 - z)$ $\quad$ **49.** $2.7(w - 5.2)$ $\quad$ **50.** $\frac{2}{3}y + \frac{1}{6}y + y$

51. $\frac{3}{4}\left(z + \frac{2}{5}\right) + 2z$ $\quad$ **52.** $7(x + y) - 7x$ $\quad$ **53.** $4x + 9y + 3(x + y)$

54. ERROR ANALYSIS Describe and correct the error in simplifying the expression.

$$8x - 2x + 5x = 8x - 7x$$
$$= (8 - 7)x$$
$$= x$$

ALGEBRA Find the value of x that makes the expressions equivalent.

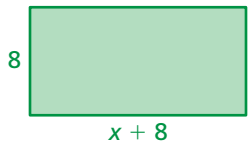
55. $4(x - 5)$; $32 - 20$ $\quad$ **56.** $2(x + 9)$; $30 + 18$ $\quad$ **57.** $7(8 - x)$; $56 - 21$

58. REASONING Simplify the expressions and compare. What do you notice? Explain.

$4(x + 6)$ $\quad (x + 6) + (x + 6) + (x + 6) + (x + 6)$

GEOMETRY **Write and simplify expressions for the area and perimeter of the rectangle.**

59.



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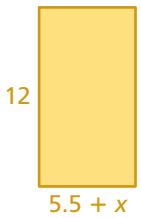

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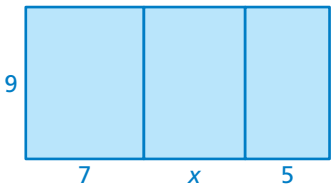


62. **FUNDRAISER** An art club sells 42 large candles and 56 small candles.

a. Use the Distributive Property to write and simplify an expression for the profit.

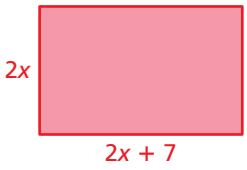
b. A large candle costs \$5, and a small candle costs \$3. What is the club's profit?

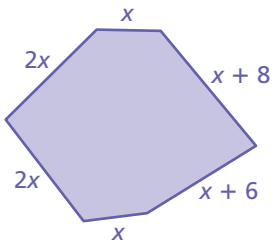
Profit = Price − Cost

63. **REASONING** Evaluate each expression by (1) using the Distributive Property and (2) evaluating inside the parentheses first. Which method do you prefer? Is your preference the same for both expressions? Explain your reasoning.

a. $2(3.22 - 0.12)$ **b.** $12\left(\frac{1}{2} + \frac{2}{3}\right)$

64. **REASONING** Write and simplify an expression for the difference between the perimeters of the rectangle and the hexagon. Interpret your answer.





65. **Puzzle** Add one set of parentheses to the expression $7 \cdot x + 3 + 8 \cdot x + 3 \cdot x + 8 - 9$ so that it is equivalent to $2(9x + 10)$.

Fair Game Review What you learned in previous grades & lessons

Evaluate the expression. *(Section 2.4, Section 2.5, and Section 2.6)*

66. $4.871 + 7.4 - 1.63$

67. $25.06 - 0.049 + 8.995$

68. $15.3 \cdot 9.1 - 4.017$

69. $29.24 \div 3.4 \cdot 0.045$

70. **MULTIPLE CHOICE** What is the GCF of 48, 80, and 96? *(Section 1.5)*

(A) 12 (B) 16

(C) 24 (D) 480