2.2 Dividing Fractions

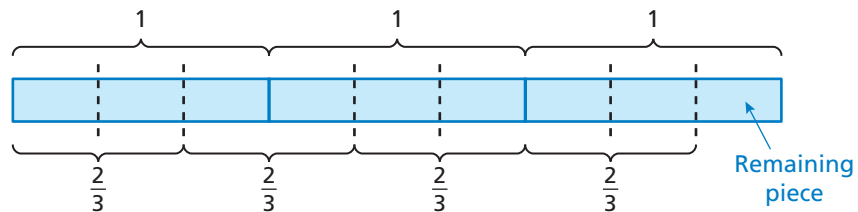**Essential Question** How can you divide by a fraction?

1 ACTIVITY: Dividing by a Fraction

Work with a partner. Write the division problem and solve it using a model.

a. How many two-thirds are in three?

The division problem is $\boxed{} \div \dfrac{\boxed{}}{\boxed{}}$.

$$
\underbrace{\hspace{3cm}}_{} \quad 1 \qquad 1 \qquad 1
$$

Remaining piece

$\dfrac{2}{3} \qquad \dfrac{2}{3} \qquad \dfrac{2}{3} \qquad \dfrac{2}{3}$

How many groups of $\dfrac{2}{3}$ are in 3? $\boxed{}$

The remaining piece represents $\dfrac{\boxed{}}{\boxed{}}$ of $\dfrac{2}{3}$.

So, there are $\boxed{}\,\dfrac{\boxed{}}{\boxed{}}$ groups of $\dfrac{2}{3}$ in 3.

So, $\boxed{} \div \dfrac{\boxed{}}{\boxed{}} = \boxed{}\,\dfrac{\boxed{}}{\boxed{}}$.

b. How many halves are in five halves?

$$
1 \qquad\qquad 1 \qquad\qquad \dfrac{1}{2}
$$
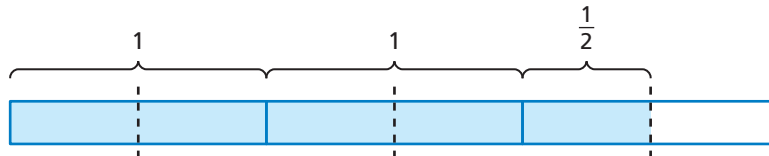
c. How many four-fifths are in eight?

d. How many one-thirds are in seven halves?

e. How many three-fourths are in five halves?

Work with a partner.

a. Complete each table.

Division Table   **Multiplication Table**

$8 \div 16$	$\dfrac{1}{2}$
$8 \div 8$	1
$8 \div 4$	2
$8 \div 2$	4
$8 \div 1$	8
$8 \div \dfrac{1}{2}$	
$8 \div \dfrac{1}{4}$	
$8 \div \dfrac{1}{8}$	

$8 \times \dfrac{1}{16}$	$\dfrac{1}{2}$
$8 \times \dfrac{1}{8}$	1
$8 \times \dfrac{1}{4}$	2
$8 \times \dfrac{1}{2}$	4
8×1	8
8×2	
8×4	
8×8	

Math Practice 4

Analyze Relationships

How is multiplying numbers similar to dividing numbers?

b. Describe the relationship between the red numbers in the division table and the red numbers in the multiplication table.

c. Describe the relationship between the blue numbers in the division table and the blue numbers in the multiplication table.

d. **STRUCTURE** Make a conjecture about how you can use multiplication to divide by a fraction.

e. Test your conjecture using the problems in Activity 1.

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you divide by a fraction? Give an example.

4. How many halves are in a fourth? Explain how you found your answer.

Practice Use what you learned about dividing fractions to complete Exercises 11–18 on page 67.

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Key Vocabulary 🔊
reciprocals, *p. 64*

Two numbers whose product is 1 are **reciprocals**. To write the reciprocal of a number, write the number as a fraction. Then invert the fraction. So, the reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$, where a and $b \neq 0$.

The Meaning of a Word ● Invert

When you **invert** a glass, you turn it over.

Study Tip

The product of a nonzero number and its reciprocal is 1.
$$\frac{a}{b} \cdot \frac{b}{a} = 1$$
This is called the *Multiplicative Inverse Property*. You will learn more about this property in Chapter 7.

EXAMPLE 1 — Writing Reciprocals

Study Tip

When any number is multiplied by 0, the product is 0. So, the number 0 does not have a reciprocal.

	Original Number	*Fraction*	*Reciprocal*	*Check*
a.	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{3}$	$\frac{3}{5} \times \frac{5}{3} = 1$
b.	$\frac{9}{5}$	$\frac{9}{5}$	$\frac{5}{9}$	$\frac{9}{5} \times \frac{5}{9} = 1$
c.	2	$\frac{2}{1}$	$\frac{1}{2}$	$\frac{2}{1} \times \frac{1}{2} = 1$

● On Your Own

Now You're Ready
Exercises 7–10

Write the reciprocal of the number.

1. $\frac{3}{4}$ **2.** 5   **3.** $\frac{7}{2}$ **4.** $\frac{4}{9}$

🔑 Key Idea

Dividing Fractions

Words To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Numbers $\frac{1}{5} \div \frac{3}{4} = \frac{1}{5} \times \frac{4}{3} = \frac{1 \times 4}{5 \times 3}$

Algebra $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$, where b, c, and $d \neq 0$

🔊 Multi-Language Glossary at BigIdeasMath⟋com

EXAMPLE **2** **Dividing a Fraction by a Fraction**

Find $\dfrac{1}{6} \div \dfrac{2}{3}$.

$$\frac{1}{6} \div \frac{2}{3} = \frac{1}{6} \times \frac{3}{2}$$
Multiply by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$.
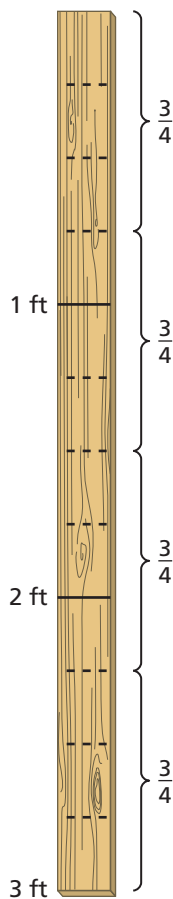
$$= \frac{1 \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \times 2}$$
Multiply fractions. Divide out the common factor 3.

$$= \frac{1}{4}$$
Simplify.

EXAMPLE **3** **Dividing a Whole Number by a Fraction**

A piece of wood is 3 feet long. How many $\dfrac{3}{4}$-foot pieces can you cut from the piece of wood?

Method 1: Draw a diagram. Mark each foot on the diagram. Then divide each foot into $\dfrac{1}{4}$-foot sections.

Count the number of $\dfrac{3}{4}$-foot pieces of wood. There are four.

∴ So, you can cut four $\dfrac{3}{4}$-foot pieces from the piece of wood.

Method 2: Divide 3 by $\dfrac{3}{4}$ to find the number of $\dfrac{3}{4}$-foot pieces.

$$3 \div \frac{3}{4} = 3 \times \frac{4}{3}$$
Multiply by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$$= \frac{\overset{1}{\cancel{3}} \times 4}{\underset{1}{\cancel{3}}}$$
Multiply. Divide out the common factor 3.

$$= 4$$
Simplify.

∴ So, you can cut four $\dfrac{3}{4}$-foot pieces from the piece of wood.

🔴 **On Your Own**

Divide. Write the answer in simplest form.

5. $\dfrac{2}{7} \div \dfrac{1}{3}$ **6.** $\dfrac{1}{2} \div \dfrac{1}{8}$ **7.** $\dfrac{3}{8} \div \dfrac{1}{4}$ **8.** $\dfrac{2}{5} \div \dfrac{3}{10}$

9. How many $\dfrac{1}{2}$-foot pieces can you cut from a 7-foot piece of wood?

EXAMPLE 4   Dividing a Fraction by a Whole Number

Find $\dfrac{4}{5} \div 2$.

$$\dfrac{4}{5} \div 2 = \dfrac{4}{5} \div \dfrac{2}{1}$$ Write 2 as an improper fraction.

$$= \dfrac{4}{5} \times \dfrac{1}{2}$$ Multiply by the reciprocal of $\dfrac{2}{1}$, which is $\dfrac{1}{2}$.

$$= \dfrac{\overset{2}{\cancel{4}} \times 1}{5 \times \underset{1}{\cancel{2}}}$$ Multiply fractions. Divide out the common factor 2.

$$= \dfrac{2}{5}$$ Simplify.

On Your Own

Now You're Ready
Exercises 11–26

Divide. Write the answer in simplest form.

10. $\dfrac{1}{2} \div 3$

11. $\dfrac{2}{3} \div 10$

12. $\dfrac{5}{8} \div 4$

13. $\dfrac{6}{7} \div 4$

EXAMPLE 5   Using Order of Operations

Evaluate $\dfrac{3}{8} + \dfrac{5}{6} \div 5$.

$$\dfrac{3}{8} + \dfrac{5}{6} \div 5 = \dfrac{3}{8} + \dfrac{5}{6} \times \dfrac{1}{5}$$ Multiply by the reciprocal of 5, which is $\dfrac{1}{5}$.

$$= \dfrac{3}{8} + \dfrac{\overset{1}{\cancel{5}} \times 1}{6 \times \underset{1}{\cancel{5}}}$$ Multiply $\dfrac{5}{6}$ and $\dfrac{1}{5}$. Divide out the common factor 5.

$$= \dfrac{3}{8} + \dfrac{1}{6}$$ Simplify.

$$= \dfrac{18}{48} + \dfrac{8}{48}$$ Rewrite fractions using a common denominator.

$$= \dfrac{26}{48}, \text{ or } \dfrac{13}{24}$$ Simplify.

Study Tip

You can use the LCD, 24, to add the fractions in Example 5.

$\dfrac{3}{8} + \dfrac{1}{6} = \dfrac{9}{24} + \dfrac{4}{24} = \dfrac{13}{24}$

On Your Own

Now You're Ready
Exercises 43–51

Evaluate the expression. Write the answer in simplest form.

14. $\dfrac{4}{5} + \dfrac{2}{5} \div 4$

15. $\dfrac{3}{8} \div \dfrac{3}{4} - \dfrac{1}{6}$

16. $\dfrac{8}{9} \div 2 \div 8$

Vocabulary and Concept Check

1. **OPEN-ENDED** Write a fraction and its reciprocal.

2. **WHICH ONE DOESN'T BELONG?** Which of the following does *not* belong with the other three? Explain your reasoning.

$$\frac{1}{3} \qquad \frac{1}{6} \qquad \frac{2}{9} \qquad \frac{1}{8}$$

MATCHING Match the expression with its value.

3. $\frac{2}{5} \div \frac{8}{15}$

4. $\frac{8}{15} \div \frac{2}{5}$

5. $\frac{2}{15} \div \frac{8}{5}$

6. $\frac{8}{5} \div \frac{2}{15}$

A. $\frac{1}{12}$

B. $\frac{3}{4}$

C. 12

D. $1\frac{1}{3}$

Practice and Problem Solving

Write the reciprocal of the number.

① 7. 8

8. $\frac{6}{7}$

9. $\frac{2}{5}$

10. $\frac{8}{11}$

Divide. Write the answer in simplest form.

② ③ ④ 11. $\frac{1}{8} \div \frac{1}{4}$

12. $\frac{5}{6} \div \frac{2}{7}$

13. $12 \div \frac{3}{4}$

14. $8 \div \frac{2}{5}$

15. $\frac{3}{7} \div 6$

16. $\frac{12}{25} \div 4$

17. $\frac{2}{9} \div \frac{2}{3}$

18. $\frac{8}{15} \div \frac{4}{5}$

19. $\frac{1}{3} \div \frac{1}{9}$

20. $\frac{7}{10} \div \frac{3}{8}$

21. $\frac{14}{27} \div 7$

22. $\frac{5}{8} \div 15$

23. $\frac{27}{32} \div \frac{7}{8}$

24. $\frac{4}{15} \div \frac{10}{13}$

25. $9 \div \frac{4}{9}$

26. $10 \div \frac{5}{12}$

**ERROR ANALYSIS** Describe and correct the error in finding the quotient.

27.

$$\frac{4}{7} \div \frac{13}{28} = \frac{4}{7} \times \frac{13}{28}$$
$$= \frac{\overset{1}{\cancel{4}} \times 13}{7 \times \cancel{28}_{7}}$$
$$= \frac{13}{49}$$

28.

$$\frac{2}{5} \div \frac{8}{9} = \frac{5}{2} \times \frac{8}{9}$$
$$= \frac{5 \times \overset{4}{\cancel{8}}}{\underset{1}{\cancel{2}} \times 9}$$
$$= \frac{20}{9}$$

29. **REASONING** How can you use estimation to show that the quotient in Exercise 28 is incorrect?

30. APPLE PIE You have $\frac{3}{5}$ of an apple pie. You divide the remaining pie into 5 equal slices. What fraction of the original pie is each slice?

31. ANIMALS How many times longer is the baby alligator than the baby gecko?

$\frac{2}{15}$ ft

$\frac{3}{4}$ ft
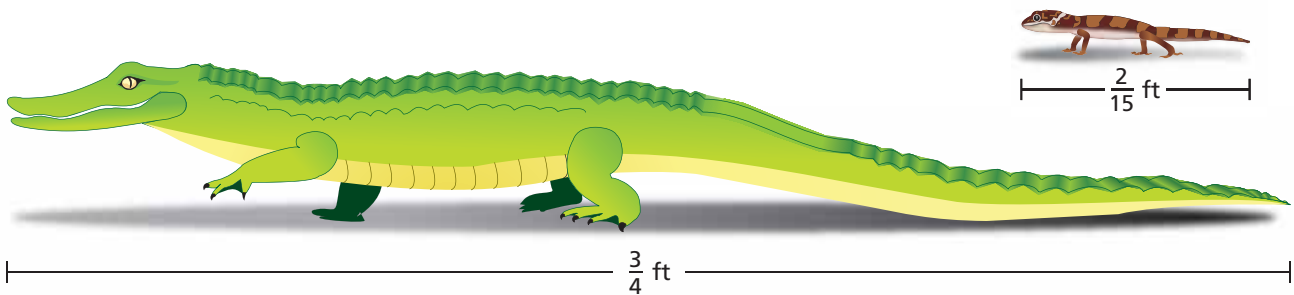
Determine whether the numbers are reciprocals. If not, write the reciprocal of each number.

32. $9, \frac{1}{9}$

33. $\frac{4}{5}, \frac{10}{8}$

34. $\frac{5}{6}, \frac{15}{18}$

35. $\frac{6}{5}, \frac{5}{6}$

Copy and complete the statement.

36. $\frac{5}{12} \times \boxed{} = 1$

37. $3 \times \boxed{} = 1$

38. $7 \div \boxed{} = 56$

Without finding the quotient, copy and complete the statement using <, >, or =. Explain your reasoning.

39. $5 \div \frac{7}{9} \boxed{} 5$

40. $\frac{3}{7} \div 1 \boxed{} \frac{3}{7}$

41. $8 \div \frac{3}{4} \boxed{} 8$

42. $\frac{5}{6} \div \frac{7}{8} \boxed{} \frac{5}{6}$

Evaluate the expression. Write the answer in simplest form.

⑤ **43.** $\frac{1}{6} \div 6 \div 6$

44. $\frac{7}{12} \div 14 \div 6$

45. $\frac{3}{5} \div \frac{4}{7} \div \frac{9}{10}$

46. $4 \div \frac{8}{9} - \frac{1}{2}$

47. $\frac{3}{4} + \frac{5}{6} \div \frac{2}{3}$

48. $\frac{7}{8} - \frac{3}{8} \div 9$

49. $\frac{9}{16} \div \frac{3}{4} \cdot \frac{2}{13}$

50. $\frac{3}{14} \cdot \frac{2}{5} \div \frac{6}{7}$

51. $\frac{10}{27} \cdot \left(\frac{3}{8} \div \frac{5}{24} \right)$

52. REASONING Use a model to evaluate the quotient $\frac{1}{2} \div \frac{1}{6}$. Explain.

53. VIDEO CHATTING You use $\frac{1}{8}$ of your battery for every $\frac{2}{5}$ of an hour that you video chat. You use $\frac{3}{4}$ of your battery video chatting. How long did you video chat?

54. NUMBER SENSE When is the reciprocal of a fraction a whole number? Explain.

55. BUDGETS The table shows the portions of a family budget that are spent on several expenses.

Expense	Portion of Budget
Housing	$\dfrac{1}{4}$
Food	$\dfrac{1}{12}$
Automobiles	$\dfrac{1}{15}$
Recreation	$\dfrac{1}{40}$

   a. How many times more is the expense for housing than for automobiles?

   b. How many times more is the expense for food than for recreation?

   c. The expense for automobile fuel is $\dfrac{1}{60}$ of the total expenses. What fraction of the automobile expense is spent on fuel?

56. PROBLEM SOLVING You have 6 pints of glaze. It takes $\dfrac{7}{8}$ of a pint to glaze a bowl and $\dfrac{9}{16}$ of a pint to glaze a plate.

   a. How many bowls could you glaze? How many plates could you glaze?

   b. You want to glaze 5 bowls, and then use the rest for plates. How many plates can you glaze? How much glaze will be left over?

   c. How many of each object could you glaze so that there is no glaze left over? Explain how you found your answer.

57. 《Reasoning》 A water tank is $\dfrac{1}{8}$ full. The tank is $\dfrac{3}{4}$ full when 42 gallons of water are added to the tank.

   a. How much water can the tank hold?

   b. How much water was originally in the tank?

   c. How much water is in the tank when it is $\dfrac{1}{2}$ full?

Fair Game Review What you learned in previous grades & lessons

Find the GCF of the numbers. *(Section 1.5)*

58. 8, 16   **59.** 24, 66   **60.** 48, 80   **61.** 15, 45, 100

62. MULTIPLE CHOICE How many inches are in $5\dfrac{1}{2}$ yards? *(Skills Review Handbook)*

   Ⓐ $15\dfrac{1}{2}$ **Ⓑ** $16\dfrac{1}{2}$ **Ⓒ** 66   **Ⓓ** 198