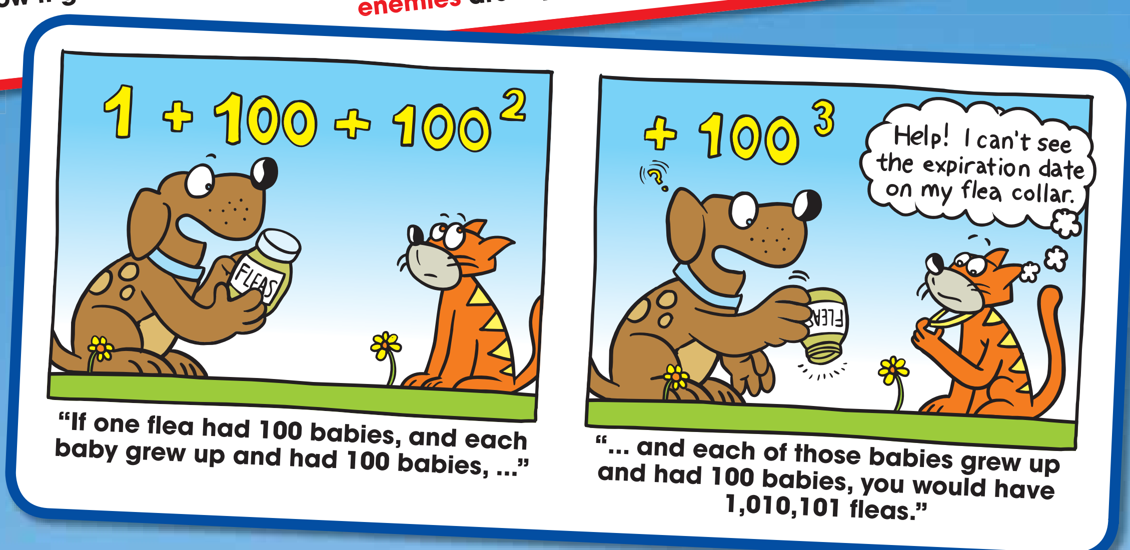
10 Exponents and Scientific Notation



What You Learned Before

"It's called the Power of Negative One, Descartes!"

Using Order of Operations (6.EE.1)

Example 1 Evaluate $6^2 \div 4 - 2(9 - 5)$.

First:	**P**arentheses	$6^2 \div 4 - 2(9 - 5) = 6^2 \div 4 - 2 \cdot 4$
Second:	**E**xponents	$= 36 \div 4 - 2 \cdot 4$
Third:	**M**ultiplication and **D**ivision (from left to right)	$= 9 - 8$
Fourth:	**A**ddition and **S**ubtraction (from left to right)	$= 1$

Try It Yourself

Evaluate the expression.

1. $15\left(\frac{8}{4}\right) + 2^2 - 3 \cdot 7$

2. $5^2 \cdot 2 \div 10 + 3 \cdot 2 - 1$

3. $3^2 - 1 + 2(4(3 + 2))$

Multiplying and Dividing Decimals (6.NS.3)

Example 2 Find $2.1 \cdot 0.35$.

$$\begin{array}{r l}
2.1 & \leftarrow \text{1 decimal place} \\
\times\ 0.35 & \leftarrow \underline{+\ \text{2 decimal places}} \\
\hline
105 & \\
63 & \\
\hline
0.735 & \leftarrow \text{3 decimal places}
\end{array}$$

Example 3 Find $1.08 \div 0.9$.

$0.9\overline{)1.08}$ Multiply each number by 10.

$$\begin{array}{r}
1.2 \\
9\overline{)10.8} \\
-\ 9 \\
\hline
1\ 8 \\
-\ 1\ 8 \\
\hline
0
\end{array}$$

Place the decimal point above the decimal point in the dividend 10.8.

Try It Yourself

Find the product or quotient.

4. $1.75 \cdot 0.2$

5. $1.4 \cdot 0.6$

6. $\begin{array}{r} 7.03 \\ \times\ 4.3 \\ \hline \end{array}$

7. $\begin{array}{r} 0.894 \\ \times\ 0.2 \\ \hline \end{array}$

8. $5.40 \div 0.09$

9. $4.17 \div 0.3$

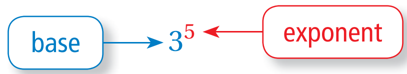
10. $0.15\overline{)3.6}$

11. $0.004\overline{)7.2}$

10.1 Exponents

Essential Question How can you use exponents to write numbers?

The expression 3^5 is called a *power*. The *base* is 3. The *exponent* is 5.



1 ACTIVITY: Using Exponent Notation

Work with a partner.

a. Copy and complete the table.

Power	Repeated Multiplication Form	Value
$(-3)^1$	-3	-3
$(-3)^2$	$(-3) \cdot (-3)$	9
$(-3)^3$		
$(-3)^4$		
$(-3)^5$		
$(-3)^6$		
$(-3)^7$		

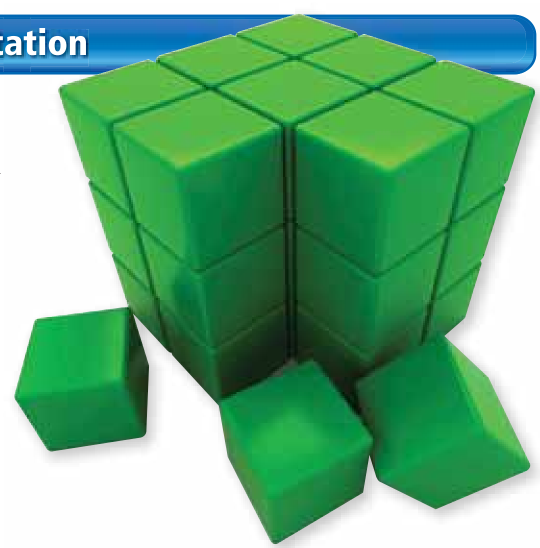
b. **REPEATED REASONING** Describe what is meant by the expression $(-3)^n$. How can you find the value of $(-3)^n$?

2 ACTIVITY: Using Exponent Notation

Work with a partner.

a. The cube at the right has \$3 in each of its small cubes. Write a power that represents the total amount of money in the large cube.

b. Evaluate the power to find the total amount of money in the large cube.



COMMON CORE

Exponents

In this lesson, you will

- write expressions using integer exponents.
- evaluate expressions involving integer exponents.

Learning Standard 8.EE.1

3 ACTIVITY: Writing Powers as Whole Numbers

Work with a partner. Write each distance as a whole number. Which numbers do you know how to write in words? For instance, in words, 10^3 is equal to *one thousand*.

a. 10^{26} meters: diameter of observable universe

b. 10^{21} meters: diameter of Milky Way galaxy

c. 10^{16} meters: diameter of solar system

d. 10^{7} meters: diameter of Earth

e. 10^{6} meters: length of Lake Erie shoreline

f. 10^{5} meters: width of Lake Erie

4 ACTIVITY: Writing a Power

Math Practice 1

Analyze Givens

What information is given in the poem? What are you trying to find?

Work with a partner. Write the number of kits, cats, sacks, and wives as a power.

As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
Each sack had seven cats
Each cat had seven kits
Kits, cats, sacks, wives
How many were going to St. Ives?

Nursery Rhyme, 1730

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use exponents to write numbers? Give some examples of how exponents are used in real life.

Practice

Use what you learned about exponents to complete Exercises 3–5 on page 414.

10.1 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Vocabulary
power, *p. 412*
base, *p. 412*
exponent, *p. 412*

A **power** is a product of repeated factors. The **base** of a power is the common factor. The **exponent** of a power indicates the number of times the base is used as a factor.

base exponent

$$\underbrace{\left(\frac{1}{2}\right)^5}_{\text{power}} = \underbrace{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}}_{\frac{1}{2}\text{ is used as a factor 5 times.}}$$

EXAMPLE 1 Writing Expressions Using Exponents

Study Tip
Use parentheses to write powers with negative bases.

Write each product using exponents.

a. $(-7) \cdot (-7) \cdot (-7)$

Because -7 is used as a factor 3 times, its exponent is 3.

So, $(-7) \cdot (-7) \cdot (-7) = (-7)^3$.

b. $\pi \cdot \pi \cdot r \cdot r \cdot r$

Because π is used as a factor 2 times, its exponent is 2. Because r is used as a factor 3 times, its exponent is 3.

So, $\pi \cdot \pi \cdot r \cdot r \cdot r = \pi^2 r^3$.

On Your Own

Now You're Ready
Exercises 3–10

Write the product using exponents.

1. $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$

2. $0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot x \cdot x$

EXAMPLE 2 Evaluating Expressions

Evaluate each expression.

a. $(-2)^4$

The base is -2.

$(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2)$ Write as repeated multiplication.

$= 16$ Simplify.

b. -2^4

The base is 2.

$-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2)$ Write as repeated multiplication.

$= -16$ Simplify.

EXAMPLE 3 Using Order of Operations

Evaluate each expression.

a. $3 + 2 \cdot 3^4 = 3 + 2 \cdot 81$ — Evaluate the power.

$= 3 + 162$ — Multiply.

$= 165$ — Add.

b. $3^3 - 8^2 \div 2 = 27 - 64 \div 2$ — Evaluate the powers.

$= 27 - 32$ — Divide.

$= -5$ — Subtract.

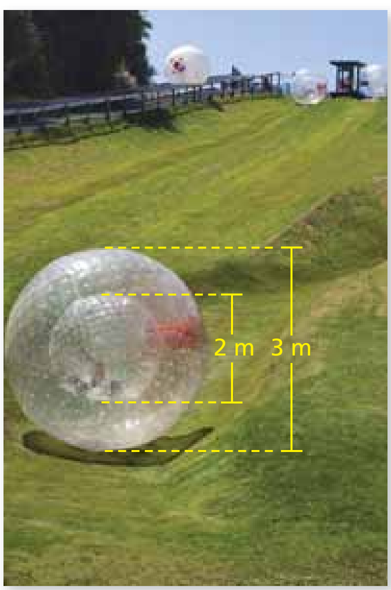
On Your Own

Now You're Ready
Exercises 11–16
and 21–26

Evaluate the expression.

3. -5^4 **4.** $\left(-\frac{1}{6}\right)^3$ **5.** $\left|-3^3 \div 27\right|$ **6.** $9 - 2^5 \cdot 0.5$

EXAMPLE 4 Real-Life Application



In sphering, a person is secured inside a small, hollow sphere that is surrounded by a larger sphere. The space between the spheres is inflated with air. What is the volume of the inflated space?

You can find the radius of each sphere by dividing each diameter given in the diagram by 2.

Outer Sphere		*Inner Sphere*
$V = \frac{4}{3}\pi r^3$	Write formula.	$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi\left(\frac{3}{2}\right)^3$	Substitute.	$= \frac{4}{3}\pi(1)^3$
$= \frac{4}{3}\pi\left(\frac{27}{8}\right)$	Evaluate the power.	$= \frac{4}{3}\pi(1)$
$= \frac{9}{2}\pi$	Multiply.	$= \frac{4}{3}\pi$

So, the volume of the inflated space is $\frac{9}{2}\pi - \frac{4}{3}\pi = \frac{19}{6}\pi$, or about 10 cubic meters.

On Your Own

7. **WHAT IF?** The diameter of the inner sphere is 1.8 meters. What is the volume of the inflated space?

10.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **NUMBER SENSE** Describe the difference between -3^4 and $(-3)^4$.

2. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

5^3 The exponent is 3.	5^3 The power is 5.	5^3 The base is 5.	5^3 Five is used as a factor 3 times.

Practice and Problem Solving

Write the product using exponents.

1 3. $3 \cdot 3 \cdot 3 \cdot 3$

4. $(-6) \cdot (-6)$

5. $\left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right) \cdot \left(-\frac{1}{2}\right)$

6. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

7. $\pi \cdot \pi \cdot \pi \cdot x \cdot x \cdot x \cdot x$

8. $(-4) \cdot (-4) \cdot (-4) \cdot y \cdot y$

9. $6.4 \cdot 6.4 \cdot 6.4 \cdot 6.4 \cdot b \cdot b \cdot b$

10. $(-t) \cdot (-t) \cdot (-t) \cdot (-t) \cdot (-t)$

Evaluate the expression.

2 11. 5^2

12. -11^3

13. $(-1)^6$

14. $\left(\frac{1}{2}\right)^6$

15. $\left(-\frac{1}{12}\right)^2$

16. $-\left(\frac{1}{9}\right)^3$

17. **ERROR ANALYSIS** Describe and correct the error in evaluating the expression.

$$-6^2 = (-6) \cdot (-6) = 36$$

18. **PRIME FACTORIZATION** Write the prime factorization of 675 using exponents.

19. **STRUCTURE** Write $-\left(\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}\right)$ using exponents.

20. **RUSSIAN DOLLS** The largest doll is 12 inches tall. The height of each of the other dolls is $\frac{7}{10}$ the height of the next larger doll. Write an expression involving a power for the height of the smallest doll. What is the height of the smallest doll?

Evaluate the expression.

3 **21.** $5 + 3 \cdot 2^3$ **22.** $2 + 7 \cdot (-3)^2$ **23.** $(13^2 - 12^2) \div 5$

24. $\frac{1}{2}(4^3 - 6 \cdot 3^2)$ **25.** $\left| \frac{1}{2}(7 + 5^3) \right|$ **26.** $\left| \left(-\frac{1}{2}\right)^3 \div \left(\frac{1}{4}\right)^2 \right|$

27. **MONEY** You have a part-time job. One day your boss offers to pay you either $2^h - 1$ or 2^{h-1} dollars for each hour h you work that day. Copy and complete the table. Which option should you choose? Explain.

h	1	2	3	4	5
$2^h - 1$					
2^{h-1}					

28. **CARBON-14 DATING** Scientists use carbon-14 dating to determine the age of a sample of organic material.

a. The amount C (in grams) of a 100-gram sample of carbon-14 remaining after t years is represented by the equation $C = 100(0.99988)^t$. Use a calculator to find the amount of carbon-14 remaining after 4 years.

b. What percent of the carbon-14 remains after 4 years?

29. Critical Thinking The frequency (in vibrations per second) of a note on a piano is represented by the equation $F = 440(1.0595)^n$, where n is the number of notes above A-440. Each black or white key represents one note.

a. How many notes do you take to travel from A-440 to A?

b. What is the frequency of A?

c. Describe the relationship between the number of notes between A-440 and A and the increase in frequency.

Fair Game Review What you learned in previous grades & lessons

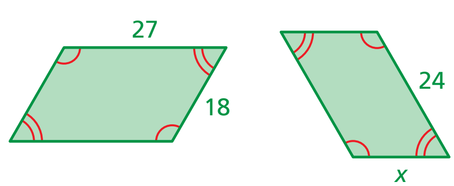
Tell which property is illustrated by the statement. *(Skills Review Handbook)*

30. $8 \cdot x = x \cdot 8$ **31.** $(2 \cdot 10)x = 2(10 \cdot x)$ **32.** $3(x \cdot 1) = 3x$

33. **MULTIPLE CHOICE** The polygons are similar. What is the value of x? *(Section 2.5)*

Ⓐ 15 Ⓑ 16

Ⓒ 17 Ⓓ 36



10.2 Product of Powers Property

Essential Question How can you use inductive reasoning to observe patterns and write general rules involving properties of exponents?

1 ACTIVITY: Finding Products of Powers

Work with a partner.

a. Copy and complete the table.

Product	Repeated Multiplication Form	Power
$2^2 \cdot 2^4$		
$(-3)^2 \cdot (-3)^4$		
$7^3 \cdot 7^2$		
$5.1^1 \cdot 5.1^6$		
$(-4)^2 \cdot (-4)^2$		
$10^3 \cdot 10^5$		
$\left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^5$		

b. **INDUCTIVE REASONING** Describe the pattern in the table. Then write a *general rule* for multiplying two powers that have the same base.

$$a^m \cdot a^n = a^{\square}$$

c. Use your rule to simplify the products in the first column of the table above. Does your rule give the results in the third column?

d. Most calculators have *exponent* keys that you can use to evaluate powers. Use a calculator with an exponent key to evaluate the products in part (a).

Common Core

Exponents

In this lesson, you will

- multiply powers with the same base.
- find a power of a power.
- find a power of a product.

Learning Standard 8.EE.1

2 ACTIVITY: Writing a Rule for Powers of Powers

Work with a partner. Write the expression as a single power. Then write a *general rule* for finding a power of a power.

a. $(3^2)^3 = (3 \cdot 3)(3 \cdot 3)(3 \cdot 3) = \square^{\square}$

b. $(2^2)^4 = \square$

c. $(7^3)^2 = \square$

d. $(y^3)^3 = \square$

e. $(x^4)^2 = \square$

3 ACTIVITY: Writing a Rule for Powers of Products

Work with a partner. Write the expression as the product of two powers. Then write a *general rule* for finding a power of a product.

a. $(2 \cdot 3)^3 = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = \square^{\square} \cdot \square^{\square}$

b. $(2 \cdot 5)^2 =$ ______ **c.** $(5 \cdot 4)^3 =$ ______

d. $(6a)^4 =$ ______ **e.** $(3x)^2 =$ ______

4 ACTIVITY: The Penny Puzzle

Work with a partner.

- The rows y and columns x of a chessboard are numbered as shown.
- Each position on the chessboard has a stack of pennies. (Only the first row is shown.)
- The number of pennies in each stack is $2^x \cdot 2^y$.

Math Practice 7

Look for Patterns

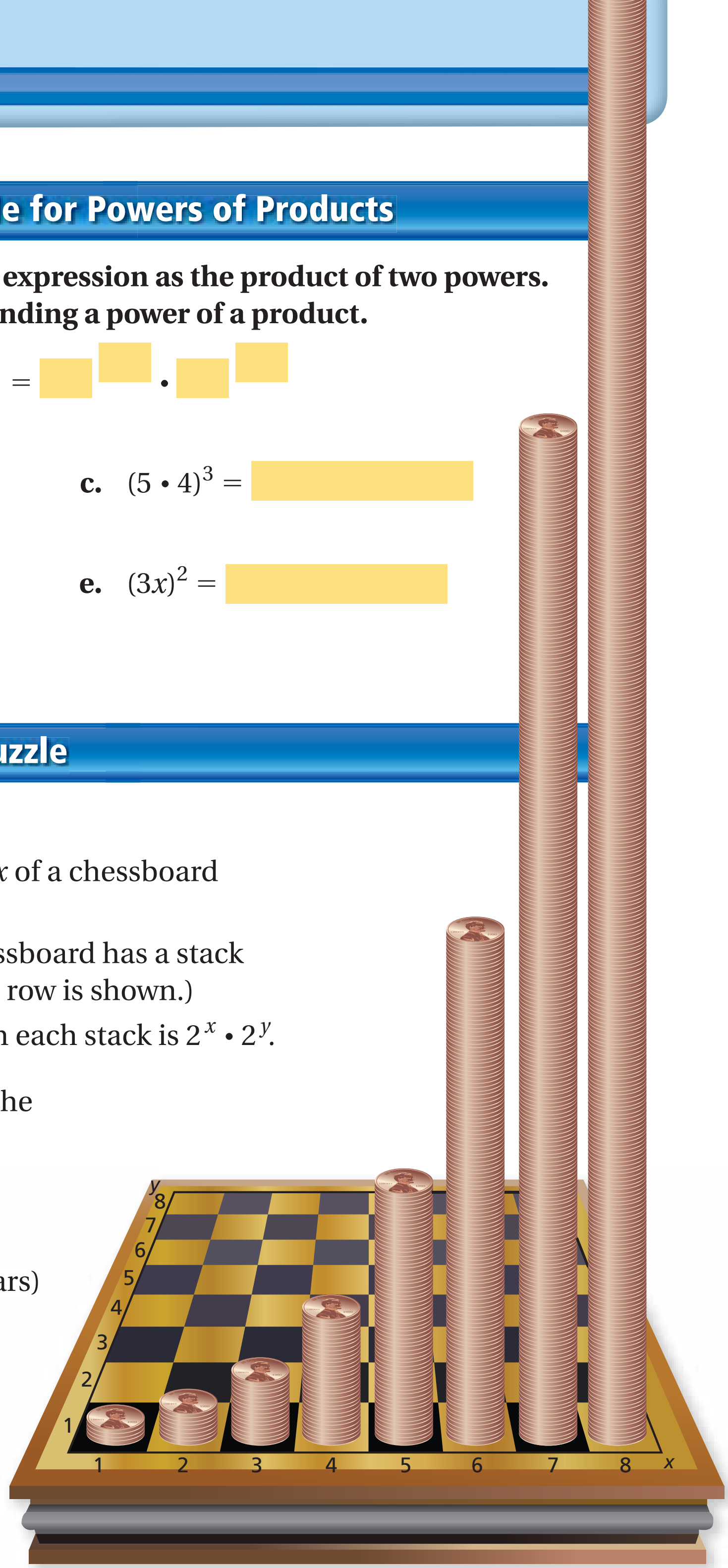
What patterns do you notice? How does this help you determine which stack is the tallest?

a. How many pennies are in the stack in location (3, 5)?

b. Which locations have 32 pennies in their stacks?

c. How much money (in dollars) is in the location with the tallest stack?

d. A penny is about 0.06 inch thick. About how tall (in inches) is the tallest stack?



What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you use inductive reasoning to observe patterns and write general rules involving properties of exponents?

Practice Use what you learned about properties of exponents to complete Exercises 3–5 on page 420.

10.2 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Ideas

Product of Powers Property

Words To multiply powers with the same base, add their exponents.

Numbers $4^2 \cdot 4^3 = 4^{2+3} = 4^5$ **Algebra** $a^m \cdot a^n = a^{m+n}$

Power of a Power Property

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^{6 \cdot 3} = 4^{18}$ **Algebra** $(a^m)^n = a^{mn}$

Power of a Product Property

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$ **Algebra** $(ab)^m = a^m b^m$

EXAMPLE 1 Multiplying Powers with the Same Base

a. $2^4 \cdot 2^5 = 2^{4+5}$ Product of Powers Property

$= 2^9$ Simplify.

b. $-5 \cdot (-5)^6 = (-5)^1 \cdot (-5)^6$ Rewrite -5 as $(-5)^1$.

$= (-5)^{1+6}$ Product of Powers Property

$= (-5)^7$ Simplify.

c. $x^3 \cdot x^7 = x^{3+7}$ Product of Powers Property

$= x^{10}$ Simplify.

Study Tip

When a number is written without an exponent, its exponent is 1.

EXAMPLE 2 Finding a Power of a Power

a. $(3^4)^3 = 3^{4 \cdot 3}$ Power of a Power Property

$= 3^{12}$ Simplify.

b. $(w^5)^4 = w^{5 \cdot 4}$ Power of a Power Property

$= w^{20}$ Simplify.

EXAMPLE 3 Finding a Power of a Product

a. $(2x)^3 = 2^3 \cdot x^3$ — Power of a Product Property

$= 8x^3$ — Simplify.

b. $(3xy)^2 = 3^2 \cdot x^2 \cdot y^2$ — Power of a Product Property

$= 9x^2y^2$ — Simplify.

On Your Own

Now You're Ready
Exercises 3–14 and 17–22

Simplify the expression.

1. $6^2 \cdot 6^4$

2. $\left(-\frac{1}{2}\right)^3 \cdot \left(-\frac{1}{2}\right)^6$

3. $z \cdot z^{12}$

4. $(4^4)^3$

5. $(y^2)^4$

6. $((-4)^3)^2$

7. $(5y)^4$

8. $(ab)^5$

9. $(0.5mn)^2$

EXAMPLE 4 Simplifying an Expression

Details

Local Disk (C:)
Local Disk

Free Space: 16GB

Total Space: 64GB

A gigabyte (GB) of computer storage space is 2^{30} bytes. The details of a computer are shown. How many bytes of total storage space does the computer have?

Ⓐ 2^{34} Ⓑ 2^{36} Ⓒ 2^{180} Ⓓ 128^{30}

The computer has 64 gigabytes of total storage space. Notice that you can write 64 as a power, 2^6. Use a model to solve the problem.

Total number of bytes = Number of bytes in a gigabyte • Number of gigabytes

$= 2^{30} \cdot 2^6$ — Substitute.

$= 2^{30+6}$ — Product of Powers Property

$= 2^{36}$ — Simplify.

The computer has 2^{36} bytes of total storage space. The correct answer is Ⓑ.

On Your Own

10. How many bytes of free storage space does the computer have?

10.2 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **REASONING** When should you use the Product of Powers Property?
2. **CRITICAL THINKING** Can you use the Product of Powers Property to multiply $5^2 \cdot 6^4$? Explain.

Practice and Problem Solving

Simplify the expression. Write your answer as a power.

1 2 **3.** $3^2 \cdot 3^2$ **4.** $8^{10} \cdot 8^4$ **5.** $(-4)^5 \cdot (-4)^7$

6. $a^3 \cdot a^3$ **7.** $h^6 \cdot h$ **8.** $\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^6$

9. $\left(-\frac{5}{7}\right)^8 \cdot \left(-\frac{5}{7}\right)^9$ **10.** $(-2.9) \cdot (-2.9)^7$ **11.** $(5^4)^3$

12. $(b^{12})^3$ **13.** $(3.8^3)^4$ **14.** $\left(\left(-\frac{3}{4}\right)^5\right)^2$

ERROR ANALYSIS **Describe and correct the error in simplifying the expression.**

15.
$$5^2 \cdot 5^9 = (5 \cdot 5)^{2+9} = 25^{11}$$

16.
$$(r^6)^4 = r^{6+4} = r^{10}$$

Simplify the expression.

3 **17.** $(6g)^3$ **18.** $(-3v)^5$ **19.** $\left(\frac{1}{5}k\right)^2$

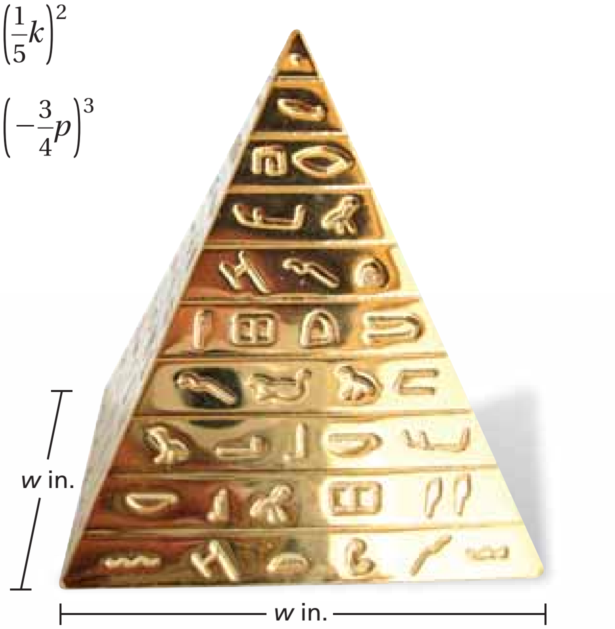
20. $(1.2m)^4$ **21.** $(rt)^{12}$ **22.** $\left(-\frac{3}{4}p\right)^3$

23. **PRECISION** Is $3^2 + 3^3$ equal to 3^5? Explain.

24. **ARTIFACT** A display case for the artifact is in the shape of a cube. Each side of the display case is three times longer than the width of the artifact.

a. Write an expression for the volume of the case. Write your answer as a power.

b. Simplify the expression.



Simplify the expression.

25. $2^4 \cdot 2^5 - (2^2)^2$

26. $16\left(\frac{1}{2}x\right)^4$

27. $5^2(5^3 \cdot 5^2)$

28. **CLOUDS** The lowest altitude of an altocumulus cloud is about 3^8 feet. The highest altitude of an altocumulus cloud is about 3 times the lowest altitude. What is the highest altitude of an altocumulus cloud? Write your answer as a power.

29. **PYTHON EGG** The volume V of a python egg is given by the formula $V = \frac{4}{3}\pi abc$. For the python eggs shown, $a = 2$ inches, $b = 2$ inches, and $c = 3$ inches.

a. Find the volume of a python egg.

b. Square the dimensions of the python egg. Then evaluate the formula. How does this volume compare to your answer in part (a)?

30. **PYRAMID** A square pyramid has a height h and a base with side length b. The side lengths of the base increase by 50%. Write a formula for the volume of the new pyramid in terms of b and h.

31. **MAIL** The United States Postal Service delivers about $2^8 \cdot 5^2$ pieces of mail each second. There are $2^8 \cdot 3^4 \cdot 5^2$ seconds in 6 days. How many pieces of mail does the United States Postal Service deliver in 6 days? Write your answer as an expression involving powers.

32. **Critical Thinking** Find the value of x in the equation without evaluating the power.

a. $2^5 \cdot 2^x = 256$

b. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^x = \frac{1}{729}$

Fair Game Review What you learned in previous grades & lessons

Simplify. *(Skills Review Handbook)*

33. $\frac{4 \cdot 4}{4}$

34. $\frac{5 \cdot 5 \cdot 5}{5}$

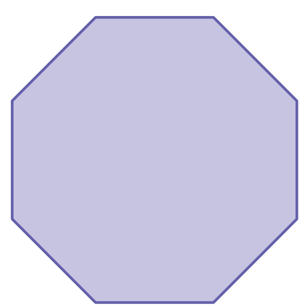
35. $\frac{2 \cdot 3}{2}$

36. $\frac{8 \cdot 6 \cdot 6}{6 \cdot 8}$

37. **MULTIPLE CHOICE** What is the measure of each interior angle of the regular polygon? *(Section 3.3)*

Ⓐ 45° Ⓑ 135°

Ⓒ 1080° Ⓓ 1440°



10.3 Quotient of Powers Property

Essential Question How can you divide two powers that have the same base?

1 ACTIVITY: Finding Quotients of Powers

Work with a partner.

a. Copy and complete the table.

Quotient	Repeated Multiplication Form	Power
$\frac{2^4}{2^2}$		
$\frac{(-4)^5}{(-4)^2}$		
$\frac{7^7}{7^3}$		
$\frac{8.5^9}{8.5^6}$		
$\frac{10^8}{10^5}$		
$\frac{3^{12}}{3^4}$		
$\frac{(-5)^7}{(-5)^5}$		
$\frac{11^4}{11^1}$		

b. **INDUCTIVE REASONING** Describe the pattern in the table. Then write a rule for dividing two powers that have the same base.

$$\frac{a^m}{a^n} = a^{\square}$$

c. Use your rule to simplify the quotients in the first column of the table above. Does your rule give the results in the third column?

COMMON CORE

Exponents

In this lesson, you will

- divide powers with the same base.
- simplify expressions involving the quotient of powers.

Learning Standard
8.EE.1

2 ACTIVITY: Comparing Volumes

Math Practice 8

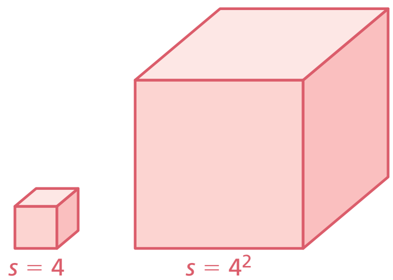
Repeat Calculations

What calculations are repeated in the table?

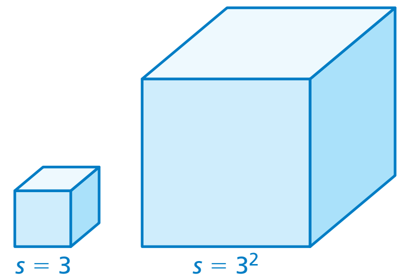
Work with a partner.

How many of the smaller cubes will fit inside the larger cube? Record your results in the table. Describe the pattern in the table.

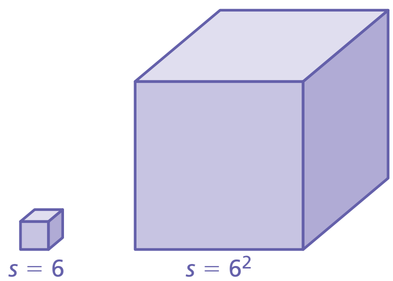
a.



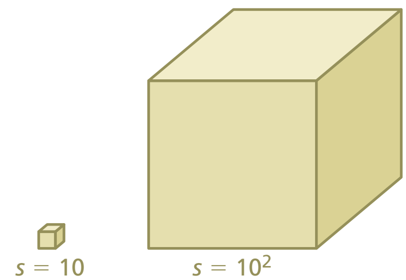
b.



c.



d.



	Volume of Smaller Cube	Volume of Larger Cube	$\dfrac{\text{Larger Volume}}{\text{Smaller Volume}}$	Answer
a.				
b.				
c.				
d.				

What Is Your Answer?

3. **IN YOUR OWN WORDS** How can you divide two powers that have the same base? Give two examples of your rule.

Practice Use what you learned about dividing powers with the same base to complete Exercises 3–6 on page 426.

10.3 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

Key Idea

Quotient of Powers Property

Words To divide powers with the same base, subtract their exponents.

Numbers $\dfrac{4^5}{4^2} = 4^{5-2} = 4^3$ **Algebra** $\dfrac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

EXAMPLE 1 Dividing Powers with the Same Base

a. $\dfrac{2^6}{2^4} = 2^{6-4}$ Quotient of Powers Property

$= 2^2$ Simplify.

b. $\dfrac{(-7)^9}{(-7)^3} = (-7)^{9-3}$ Quotient of Powers Property

$= (-7)^6$ Simplify.

c. $\dfrac{h^7}{h^6} = h^{7-6}$ Quotient of Powers Property

$= h^1 = h$ Simplify.

Common Error

When dividing powers, do not divide the bases.

$\dfrac{2^6}{2^4} = 2^2$, not 1^2.

On Your Own

Now You're Ready
Exercises 7–14

Simplify the expression. Write your answer as a power.

1. $\dfrac{9^7}{9^4}$ **2.** $\dfrac{4.2^6}{4.2^5}$ **3.** $\dfrac{(-8)^8}{(-8)^4}$ **4.** $\dfrac{x^8}{x^3}$

EXAMPLE 2 Simplifying an Expression

Simplify $\dfrac{3^4 \cdot 3^2}{3^3}$. Write your answer as a power.

The numerator is a product of powers. Add the exponents in the numerator.

$\dfrac{3^4 \cdot 3^2}{3^3} = \dfrac{3^{4+2}}{3^3}$ Product of Powers Property

$= \dfrac{3^6}{3^3}$ Simplify.

$= 3^{6-3}$ Quotient of Powers Property

$= 3^3$ Simplify.

EXAMPLE 3 Simplifying an Expression

Simplify $\dfrac{a^{10}}{a^6} \cdot \dfrac{a^7}{a^4}$. Write your answer as a power.

$$\begin{aligned}
\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4} &= a^{10-6} \cdot a^{7-4} && \text{Quotient of Powers Property} \\
&= a^4 \cdot a^3 && \text{Simplify.} \\
&= a^{4+3} && \text{Product of Powers Property} \\
&= a^7 && \text{Simplify.}
\end{aligned}$$

Study Tip

You can also simplify the expression in Example 3 as follows.

$$\begin{aligned}
\frac{a^{10}}{a^6} \cdot \frac{a^7}{a^4} &= \frac{a^{10} \cdot a^7}{a^6 \cdot a^4} \\
&= \frac{a^{17}}{a^{10}} \\
&= a^{17-10} \\
&= a^7
\end{aligned}$$

On Your Own

Now You're Ready
Exercises 16–21

Simplify the expression. Write your answer as a power.

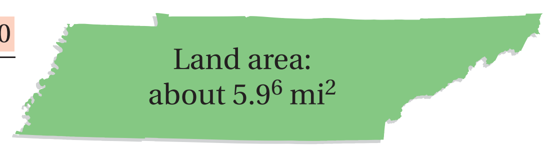
5. $\dfrac{2^{15}}{2^3 \cdot 2^5}$ **6.** $\dfrac{d^5}{d} \cdot \dfrac{d^9}{d^8}$ **7.** $\dfrac{5^9}{5^4} \cdot \dfrac{5^5}{5^2}$

EXAMPLE 4 Real-Life Application

The projected population of Tennessee in 2030 is about $5 \cdot 5.9^8$. Predict the average number of people per square mile in 2030.

Use a model to solve the problem.

$$\text{People per square mile} = \frac{\text{Population in 2030}}{\text{Land area}}$$



$$\begin{aligned}
&= \frac{5 \cdot 5.9^8}{5.9^6} && \text{Substitute.} \\
&= 5 \cdot \frac{5.9^8}{5.9^6} && \text{Rewrite.} \\
&= 5 \cdot 5.9^2 && \text{Quotient of Powers Property} \\
&= 174.05 && \text{Evaluate.}
\end{aligned}$$

So, there will be about 174 people per square mile in Tennessee in 2030.

On Your Own

Now You're Ready
Exercises 23–28

8. The projected population of Alabama in 2030 is about $2.25 \cdot 2^{21}$. The land area of Alabama is about 2^{17} square kilometers. Predict the average number of people per square kilometer in 2030.

10.3 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

1. **WRITING** Describe in your own words how to divide powers.

2. **WHICH ONE DOESN'T BELONG?** Which quotient does *not* belong with the other three? Explain your reasoning.

$\frac{(-10)^7}{(-10)^2}$ $\frac{6^3}{6^2}$ $\frac{(-4)^8}{(-3)^4}$ $\frac{5^6}{5^3}$

Practice and Problem Solving

Simplify the expression. Write your answer as a power.

3. $\frac{6^{10}}{6^4}$
4. $\frac{8^9}{8^7}$
5. $\frac{(-3)^4}{(-3)^1}$
6. $\frac{4.5^5}{4.5^3}$

1 7. $\frac{5^9}{5^3}$
8. $\frac{64^4}{64^3}$
9. $\frac{(-17)^5}{(-17)^2}$
10. $\frac{(-7.9)^{10}}{(-7.9)^4}$

11. $\frac{(-6.4)^8}{(-6.4)^6}$
12. $\frac{\pi^{11}}{\pi^7}$
13. $\frac{b^{24}}{b^{11}}$
14. $\frac{n^{18}}{n^7}$

15. **ERROR ANALYSIS** Describe and correct the error in simplifying the quotient.

$$\frac{6^{15}}{6^5} = 6^{\frac{15}{5}} = 6^3$$

Simplify the expression. Write your answer as a power.

2 3 16. $\frac{7^5 \cdot 7^3}{7^2}$
17. $\frac{2^{19} \cdot 2^5}{2^{12} \cdot 2^3}$
18. $\frac{(-8.3)^8}{(-8.3)^7} \cdot \frac{(-8.3)^4}{(-8.3)^3}$

19. $\frac{\pi^{30}}{\pi^{18} \cdot \pi^4}$
20. $\frac{c^{22}}{c^8 \cdot c^9}$
21. $\frac{k^{13}}{k^5} \cdot \frac{k^{17}}{k^{11}}$

22. **SOUND INTENSITY** The sound intensity of a normal conversation is 10^6 times greater than the quietest noise a person can hear. The sound intensity of a jet at takeoff is 10^{14} times greater than the quietest noise a person can hear. How many times more intense is the sound of a jet at takeoff than the sound of a normal conversation?

Simplify the expression.

4 **23.** $\dfrac{x \cdot 4^8}{4^5}$ **24.** $\dfrac{6^3 \cdot w}{6^2}$ **25.** $\dfrac{a^3 \cdot b^4 \cdot 5^4}{b^2 \cdot 5}$

26. $\dfrac{5^{12} \cdot c^{10} \cdot d^2}{5^9 \cdot c^9}$ **27.** $\dfrac{x^{15}y^9}{x^8y^3}$ **28.** $\dfrac{m^{10}n^7}{m^1n^6}$

29. MEMORY The memory capacities and prices of five MP3 players are shown in the table.

MP3 Player	Memory (GB)	Price
A	2^1	\$70
B	2^2	\$120
C	2^3	\$170
D	2^4	\$220
E	2^5	\$270

a. How many times more memory does MP3 Player D have than MP3 Player B?

b. Do memory and price show a linear relationship? Explain.

30. CRITICAL THINKING Consider the equation $\dfrac{9^m}{9^n} = 9^2$.

a. Find two numbers m and n that satisfy the equation.

b. Describe the number of solutions that satisfy the equation. Explain your reasoning.

Milky Way galaxy
$10 \cdot 10^{10}$ stars

31. STARS There are about 10^{24} stars in the universe. Each galaxy has approximately the same number of stars as the Milky Way galaxy. About how many galaxies are in the universe?

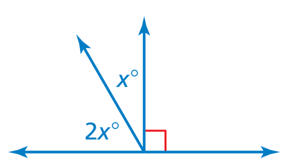
32. Number Sense Find the value of x that makes $\dfrac{8^{3x}}{8^{2x+1}} = 8^9$ true. Explain how you found your answer.

Fair Game Review What you learned in previous grades & lessons

Subtract. *(Skills Review Handbook)*

33. $-4 - 5$ **34.** $-23 - (-15)$ **35.** $33 - (-28)$ **36.** $18 - 22$

37. MULTIPLE CHOICE What is the value of x?
(Skills Review Handbook)



Ⓐ 20 Ⓑ 30

Ⓒ 45 Ⓓ 60

10.4 Zero and Negative Exponents

Essential Question How can you evaluate a nonzero number with an exponent of zero? How can you evaluate a nonzero number with a negative integer exponent?

1 ACTIVITY: Using the Quotient of Powers Property

Work with a partner.

a. Copy and complete the table.

Quotient	Quotient of Powers Property	Power
$\frac{5^3}{5^3}$		
$\frac{6^2}{6^2}$		
$\frac{(-3)^4}{(-3)^4}$		
$\frac{(-4)^5}{(-4)^5}$		

b. **REPEATED REASONING** Evaluate each expression in the first column of the table. What do you notice?

c. How can you use these results to define a^0 where $a \neq 0$?

2 ACTIVITY: Using the Product of Powers Property

Work with a partner.

a. Copy and complete the table.

Product	Product of Powers Property	Power
$3^0 \cdot 3^4$		
$8^2 \cdot 8^0$		
$(-2)^3 \cdot (-2)^0$		
$\left(-\frac{1}{3}\right)^0 \cdot \left(-\frac{1}{3}\right)^5$		

b. Do these results support your definition in Activity 1(c)?

Common Core

Exponents

In this lesson, you will

- evaluate expressions involving numbers with zero as an exponent.
- evaluate expressions involving negative integer exponents.

Learning Standard
8.EE.1

3 ACTIVITY: Using the Product of Powers Property

Work with a partner.

a. Copy and complete the table.

Product	Product of Powers Property	Power
$5^{-3} \cdot 5^{3}$		
$6^{2} \cdot 6^{-2}$		
$(-3)^{4} \cdot (-3)^{-4}$		
$(-4)^{-5} \cdot (-4)^{5}$		

b. According to your results from Activities 1 and 2, the products in the first column are equal to what value?

c. **REASONING** How does the Multiplicative Inverse Property help you rewrite the numbers with negative exponents?

d. **STRUCTURE** Use these results to define a^{-n} where $a \neq 0$ and n is an integer.

4 ACTIVITY: Using a Place Value Chart

Math Practice 2

Use Operations

What operations are used when writing the expanded form?

Work with a partner. Use the place value chart that shows the number 3452.867.

Place Value Chart

thousands	hundreds	tens	ones	and	tenths	hundredths	thousandths
10^{3}	10^{2}	10^{1}	$10^{\square}$		$10^{\square}$	$10^{\square}$	$10^{\square}$
3	4	5	2	.	8	6	7

a. **REPEATED REASONING** What pattern do you see in the exponents? Continue the pattern to find the other exponents.

b. **STRUCTURE** Show how to write the expanded form of 3452.867.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you evaluate a nonzero number with an exponent of zero? How can you evaluate a nonzero number with a negative integer exponent?

Practice

Use what you learned about zero and negative exponents to complete Exercises 5–8 on page 432.

10.4 Lesson

Check It Out
Lesson Tutorials
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Key Ideas

Zero Exponents

Words For any nonzero number a, $a^0 = 1$. The power 0^0 is *undefined*.

Numbers $4^0 = 1$

Algebra $a^0 = 1$, where $a \neq 0$

Negative Exponents

Words For any integer n and any nonzero number a, a^{-n} is the reciprocal of a^n.

Numbers $4^{-2} = \frac{1}{4^2}$

Algebra $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

EXAMPLE 1 Evaluating Expressions

a. $3^{-4} = \frac{1}{3^4}$ Definition of negative exponent

$= \frac{1}{81}$ Evaluate power.

b. $(-8.5)^{-4} \cdot (-8.5)^4 = (-8.5)^{-4+4}$ Product of Powers Property

$= (-8.5)^0$ Simplify.

$= 1$ Definition of zero exponent

c. $\frac{2^6}{2^8} = 2^{6-8}$ Quotient of Powers Property

$= 2^{-2}$ Simplify.

$= \frac{1}{2^2}$ Definition of negative exponent

$= \frac{1}{4}$ Evaluate power.

On Your Own

Now You're Ready
Exercises 5–16

Evaluate the expression.

1. 4^{-2}
2. $(-2)^{-5}$
3. $6^{-8} \cdot 6^8$
4. $\frac{(-3)^5}{(-3)^6}$
5. $\frac{1}{5^7} \cdot \frac{1}{5^{-4}}$
6. $\frac{4^5 \cdot 4^{-3}}{4^2}$

EXAMPLE 2 Simplifying Expressions

a. $-5x^0 = -5(1)$ — Definition of zero exponent

$= -5$ — Multiply.

b. $\dfrac{9y^{-3}}{y^5} = 9y^{-3-5}$ — Quotient of Powers Property

$= 9y^{-8}$ — Simplify.

$= \dfrac{9}{y^8}$ — Definition of negative exponent

On Your Own

Now You're Ready
Exercises 20–27

Simplify. Write the expression using only positive exponents.

7. $8x^{-2}$ **8.** $b^0 \cdot b^{-10}$ **9.** $\dfrac{z^6}{15z^9}$

EXAMPLE 3 Real-Life Application

A drop of water leaks from a faucet every second. How many liters of water leak from the faucet in 1 hour?

Drop of water: 50^{-2} liter

Convert 1 hour to seconds.

$$1\,\cancel{h} \times \frac{60\,\cancel{\text{min}}}{1\,\cancel{h}} \times \frac{60\text{ sec}}{1\,\cancel{\text{min}}} = 3600\text{ sec}$$

Water leaks from the faucet at a rate of 50^{-2} liter per second. Multiply the time by the rate.

$3600\text{ sec} \cdot 50^{-2}\,\dfrac{\text{L}}{\text{sec}} = 3600 \cdot \dfrac{1}{50^2}$ — Definition of negative exponent

$= 3600 \cdot \dfrac{1}{2500}$ — Evaluate power.

$= \dfrac{3600}{2500}$ — Multiply.

$= 1\dfrac{11}{25} = 1.44\text{ L}$ — Simplify.

So, 1.44 liters of water leak from the faucet in 1 hour.

On Your Own

10. WHAT IF? The faucet leaks water at a rate of 5^{-5} liter per second. How many liters of water leak from the faucet in 1 hour?

10.4 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **VOCABULARY** If a is a nonzero number, does the value of a^0 depend on the value of a? Explain.
2. **WRITING** Explain how to evaluate 10^{-3}.
3. **NUMBER SENSE** Without evaluating, order 5^0, 5^4, and 5^{-5} from least to greatest.
4. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Rewrite $\dfrac{1}{3 \cdot 3 \cdot 3}$ using a negative exponent.	Write 3 to the negative third.
Write $\dfrac{1}{3}$ cubed as a power.	Write $(-3) \cdot (-3) \cdot (-3)$ as a power.

Practice and Problem Solving

Evaluate the expression.

1 5. $\dfrac{8^7}{8^7}$ 6. $5^0 \cdot 5^3$ 7. $(-2)^{-8} \cdot (-2)^8$ 8. $9^4 \cdot 9^{-4}$

9. 6^{-2} 10. 158^0 11. $\dfrac{4^3}{4^5}$ 12. $\dfrac{-3}{(-3)^2}$

13. $4 \cdot 2^{-4} + 5$ 14. $3^{-3} \cdot 3^{-2}$ 15. $\dfrac{1}{5^{-3}} \cdot \dfrac{1}{5^6}$ 16. $\dfrac{(1.5)^2}{(1.5)^{-2} \cdot (1.5)^4}$

17. **ERROR ANALYSIS** Describe and correct the error in evaluating the expression.

$$(4)^{-3} = (-4)(-4)(-4) \\ = -64$$

18. **SAND** The mass of a grain of sand is about 10^{-3} gram. About how many grains of sand are in the bag of sand?

19. **CRITICAL THINKING** How can you write the number 1 as 2 to a power? 10 to a power?

Simplify. Write the expression using only positive exponents.

2 20. $6y^{-4}$ 21. $8^{-2} \cdot a^7$ 22. $\dfrac{9c^3}{c^{-4}}$ 23. $\dfrac{5b^{-2}}{b^{-3}}$

24. $\dfrac{8x^3}{2x^9}$ 25. $3d^{-4} \cdot 4d^4$ 26. $m^{-2} \cdot n^3$ 27. $\dfrac{3^{-2} \cdot k^0 \cdot w^0}{w^{-6}}$

28. **OPEN-ENDED** Write two different powers with negative exponents that have the same value.

METRIC UNITS **In Exercises 29–32, use the table.**

Unit of Length	Length (meter)
Decimeter	10^{-1}
Centimeter	10^{-2}
Millimeter	10^{-3}
Micrometer	10^{-6}
Nanometer	10^{-9}

29. How many millimeters are in a decimeter?

30. How many micrometers are in a centimeter?

31. How many nanometers are in a millimeter?

32. How many micrometers are in a meter?

33. **BACTERIA** A species of bacteria is 10 micrometers long. A virus is 10,000 times smaller than the bacteria.

   a. Using the table above, find the length of the virus in meters.

   b. Is the answer to part (a) *less than*, *greater than*, or *equal to* one nanometer?

34. **BLOOD DONATION** Every 2 seconds, someone in the United States needs blood. A sample blood donation is shown. ($1 \text{ mm}^3 = 10^{-3}$ mL)

   a. One cubic millimeter of blood contains about 10^4 white blood cells. How many white blood cells are in the donation? Write your answer in words.

   b. One cubic millimeter of blood contains about 5×10^6 red blood cells. How many red blood cells are in the donation? Write your answer in words.

   c. Compare your answers for parts (a) and (b).

35. **PRECISION** Describe how to rewrite a power with a positive exponent so that the exponent is in the denominator. Use the definition of negative exponents to justify your reasoning.

36. **Reasoning** The rule for negative exponents states that $a^{-n} = \frac{1}{a^n}$. Explain why this rule does not apply when $a = 0$.

Fair Game Review What you learned in previous grades & lessons

Simplify the expression. Write your answer as a power. *(Section 10.2 and Section 10.3)*

37. $10^3 \cdot 10^6$

38. $10^2 \cdot 10$

39. $\frac{10^8}{10^4}$

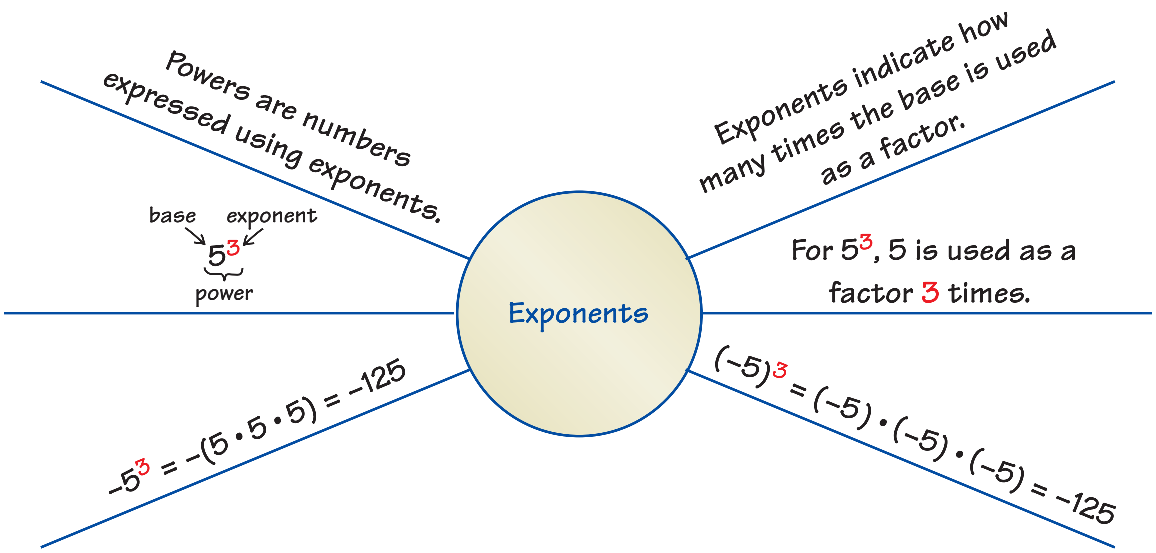
40. **MULTIPLE CHOICE** Which data display best orders numerical data and shows how they are distributed? *(Section 9.4)*

   Ⓐ bar graph Ⓑ line graph

   Ⓒ scatter plot Ⓓ stem-and-leaf plot

Check It Out
Graphic Organizer
BigIdeasMath.com

You can use an **information wheel** to organize information about a topic. Here is an example of an information wheel for exponents.



On Your Own

Make information wheels to help you study these topics.

1. Product of Powers Property
2. Quotient of Powers Property
3. zero and negative exponents

After you complete this chapter, make information wheels for the following topics.

4. writing numbers in scientific notation
5. writing numbers in standard form
6. adding and subtracting numbers in scientific notation
7. multiplying and dividing numbers in scientific notation
8. Choose three other topics you studied earlier in this course. Make an information wheel for each topic to summarize what you know about them.

"I decided to color code the different flavors in my **information wheel**."

10.1–10.4 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Write the product using exponents. *(Section 10.1)*

1. $(-5) \cdot (-5) \cdot (-5) \cdot (-5)$

2. $7 \cdot 7 \cdot m \cdot m \cdot m$

Evaluate the expression. *(Section 10.1 and Section 10.4)*

3. 5^4

4. $(-2)^6$

5. $(-4.8)^{-9} \cdot (-4.8)^9$

6. $\dfrac{5^4}{5^7}$

Simplify the expression. Write your answer as a power. *(Section 10.2)*

7. $3^8 \cdot 3$

8. $(a^5)^3$

Simplify the expression. *(Section 10.2)*

9. $(3c)^4$

10. $\left(-\dfrac{2}{7}p\right)^2$

Simplify the expression. Write your answer as a power. *(Section 10.3)*

11. $\dfrac{8^7}{8^4}$

12. $\dfrac{6^3 \cdot 6^7}{6^2}$

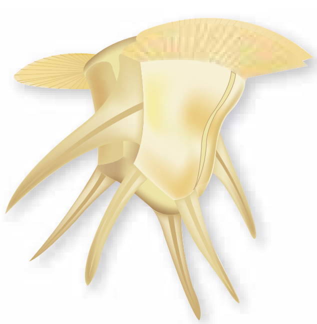
13. $\dfrac{\pi^{15}}{\pi^3 \cdot \pi^9}$

14. $\dfrac{t^{13}}{t^5} \cdot \dfrac{t^8}{t^6}$

Simplify. Write the expression using only positive exponents. *(Section 10.4)*

15. $8d^{-6}$

16. $\dfrac{12x^5}{4x^7}$



17. ORGANISM A one-celled, aquatic organism called a dinoflagellate is 1000 micrometers long. *(Section 10.4)*

a. One micrometer is 10^{-6} meter. What is the length of the dinoflagellate in meters?

b. Is the length of the dinoflagellate equal to 1 millimeter or 1 kilometer? Explain.

18. EARTHQUAKES An earthquake of magnitude 3.0 is 10^2 times stronger than an earthquake of magnitude 1.0. An earthquake of magnitude 8.0 is 10^7 times stronger than an earthquake of magnitude 1.0. How many times stronger is an earthquake of magnitude 8.0 than an earthquake of magnitude 3.0? *(Section 10.3)*

10.5 Reading Scientific Notation

Essential Question How can you read numbers that are written in scientific notation?

1 ACTIVITY: Very Large Numbers

Work with a partner.

- Use a calculator. Experiment with multiplying large numbers until your calculator displays an answer that is *not* in standard form.
- When the calculator at the right was used to multiply 2 billion by 3 billion, it listed the result as

  6.0E+18.

- Multiply 2 billion by 3 billion by hand. Use the result to explain what 6.0E+18 means.
- Check your explanation by calculating the products of other large numbers.
- Why didn't the calculator show the answer in standard form?
- Experiment to find the maximum number of digits your calculator displays. For instance, if you multiply 1000 by 1000 and your calculator shows 1,000,000, then it can display seven digits.

2 ACTIVITY: Very Small Numbers

Work with a partner.

- Use a calculator. Experiment with multiplying very small numbers until your calculator displays an answer that is *not* in standard form.
- When the calculator at the right was used to multiply 2 billionths by 3 billionths, it listed the result as

  6.0E–18.

- Multiply 2 billionths by 3 billionths by hand. Use the result to explain what 6.0E–18 means.
- Check your explanation by calculating the products of other very small numbers.

Common Core

Scientific Notation

In this lesson, you will

- identify numbers written in scientific notation.
- write numbers in standard form.
- compare numbers in scientific notation.

Learning Standards
8.EE.3
8.EE.4

3 ACTIVITY: Powers of 10 Matching Game

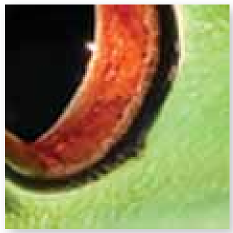
Math Practice 4

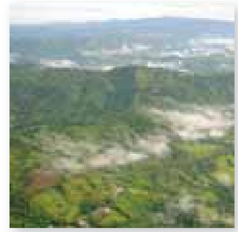
Analyze Relationships

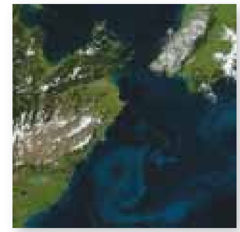
How are the pictures related? How can you order the pictures to find the correct power of 10?

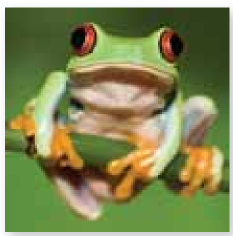
Work with a partner. Match each picture with its power of 10. Explain your reasoning.

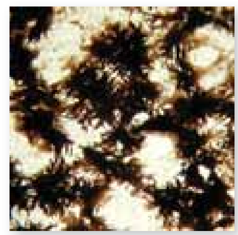
10^5 m $\quad$ 10^2 m $\quad$ 10^0 m $\quad$ 10^{-1} m $\quad$ 10^{-2} m $\quad$ 10^{-5} m

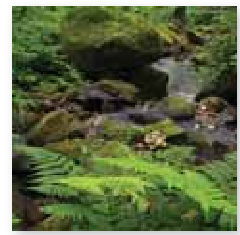
A. 

B. 

C. 

D. 

E. 

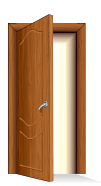
F. 

4 ACTIVITY: Choosing Appropriate Units

Work with a partner. Match each unit with its most appropriate measurement.

inches $\quad$ centimeters $\quad$ feet $\quad$ millimeters $\quad$ meters

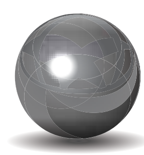
A. Height of a door: 2×10^0



B. Height of a volcano: 1.6×10^4

C. Length of a pen: 1.4×10^2

D. Diameter of a steel ball bearing: 6.3×10^{-1}



E. Circumference of a beach ball: 7.5×10^1

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you read numbers that are written in scientific notation? Why do you think this type of notation is called *scientific notation*? Why is scientific notation important?

Practice Use what you learned about reading scientific notation to complete Exercises 3–5 on page 440.

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Key Vocabulary
scientific notation, *p. 438*

Study Tip
Scientific notation is used to write very small and very large numbers.

Key Idea

Scientific Notation

A number is written in **scientific notation** when it is represented as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.

The factor is greater than or equal to 1 and less than 10. → 8.3×10^{-7} ← The power of 10 has an integer exponent.

EXAMPLE 1 Identifying Numbers Written in Scientific Notation

Tell whether the number is written in scientific notation. Explain.

a. 5.9×10^{-6}

The factor is greater than or equal to 1 and less than 10. The power of 10 has an integer exponent. So, the number is written in scientific notation.

b. 0.9×10^{8}

The factor is less than 1. So, the number is not written in scientific notation.

Key Idea

Writing Numbers in Standard Form

The absolute value of the exponent indicates how many places to move the decimal point.

- If the exponent is negative, move the decimal point to the left.
- If the exponent is positive, move the decimal point to the right.

EXAMPLE 2 Writing Numbers in Standard Form

a. Write 3.22×10^{-4} in standard form.

$3.22 \times 10^{-4} = \underbrace{0.000322}_{4}$ Move decimal point $|-4| = 4$ places to the left.

b. Write 7.9×10^{5} in standard form.

$7.9 \times 10^{5} = \underbrace{790{,}000}_{5}$ Move decimal point $|5| = 5$ places to the right.

On Your Own

Now You're Ready
Exercises 6–23

1. Is 12×10^4 written in scientific notation? Explain.

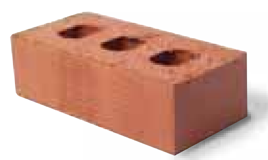
Write the number in standard form.

2. 6×10^7 3. 9.9×10^{-5} 4. 1.285×10^4

EXAMPLE 3 Comparing Numbers in Scientific Notation

An object with a lesser density than water will float. An object with a greater density than water will sink. Use each given density (in kilograms per cubic meter) to explain what happens when you place a brick and an apple in water.

Water: 1.0×10^3 **Brick:** 1.84×10^3 **Apple:** 6.41×10^2



You can compare the densities by writing each in standard form.

Water	Brick	Apple
$1.0 \times 10^3 = 1000$	$1.84 \times 10^3 = 1840$	$6.41 \times 10^2 = 641$

The apple is less dense than water, so it will float. The brick is denser than water, so it will sink.

EXAMPLE 4 Real-Life Application

A female flea consumes about 1.4×10^{-5} liter of blood per day.

A dog has 100 female fleas. How much blood do the fleas consume per day?

$$1.4 \times 10^{-5} \cdot 100 = 0.000014 \cdot 100 \qquad \text{Write in standard form.}$$

$$= 0.0014 \qquad \text{Multiply.}$$

The fleas consume about 0.0014 liter, or 1.4 milliliters of blood per day.

On Your Own

Now You're Ready
Exercise 27

5. **WHAT IF?** In Example 3, the density of lead is 1.14×10^4 kilograms per cubic meter. What happens when you place lead in water?

6. **WHAT IF?** In Example 4, a dog has 75 female fleas. How much blood do the fleas consume per day?

10.5 Exercises

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Vocabulary and Concept Check

1. **WRITING** Describe the difference between scientific notation and standard form.

2. **WHICH ONE DOESN'T BELONG?** Which number does *not* belong with the other three? Explain.

2.8×10^{15} | 4.3×10^{-30} | 1.05×10^{28} | 10×9.2^{-13}

Practice and Problem Solving

Write the number shown on the calculator display in standard form.

3. 5.6E12

4. 2.1E-10

5. 8.73E16

Tell whether the number is written in scientific notation. Explain.

1

6. 1.8×10^{9}
7. 3.45×10^{14}
8. 0.26×10^{-25}
9. 10.5×10^{12}
10. 46×10^{-17}
11. 5×10^{-19}
12. 7.814×10^{-36}
13. 0.999×10^{42}
14. 6.022×10^{23}

Write the number in standard form.

2

15. 7×10^{7}
16. 8×10^{-3}
17. 5×10^{2}
18. 2.7×10^{-4}
19. 4.4×10^{-5}
20. 2.1×10^{3}
21. 1.66×10^{9}
22. 3.85×10^{-8}
23. 9.725×10^{6}

24. **ERROR ANALYSIS** Describe and correct the error in writing the number in standard form.

✗ $4.1 \times 10^{-6} = 4{,}100{,}000$

2.7×10^{8} platelets per milliliter

25. **PLATELETS** Platelets are cell-like particles in the blood that help form blood clots.

   a. How many platelets are in 3 milliliters of blood? Write your answer in standard form.

   b. An adult human body contains about 5 liters of blood. How many platelets are in an adult human body?

26. **REASONING** A googol is 1.0×10^{100}. How many zeros are in a googol?

3 27. **STARS** The table shows the surface temperatures of five stars.

a. Which star has the highest surface temperature?

b. Which star has the lowest surface temperature?

Star	Betelgeuse	Bellatrix	Sun	Aldebaran	Rigel
Surface Temperature (°F)	6.2×10^3	3.8×10^4	1.1×10^4	7.2×10^3	2.2×10^4

28. **NUMBER SENSE** Describe how the value of a number written in scientific notation changes when you increase the exponent by 1.

29. **CORAL REEF** The area of the Florida Keys National Marine Sanctuary is about 9.6×10^3 square kilometers. The area of the Florida Reef Tract is about 16.2% of the area of the sanctuary. What is the area of the Florida Reef Tract in square kilometers?

30. **REASONING** A gigameter is 1.0×10^6 kilometers. How many square kilometers are in 5 square gigameters?

31. **WATER** There are about 1.4×10^9 cubic kilometers of water on Earth. About 2.5% of the water is fresh water. How much fresh water is on Earth?

32. Critical Thinking The table shows the speed of light through five media.

a. In which medium does light travel the fastest?

b. In which medium does light travel the slowest?

Medium	Speed
Air	6.7×10^8 mi/h
Glass	6.6×10^8 ft/sec
Ice	2.3×10^5 km/sec
Vacuum	3.0×10^8 m/sec
Water	2.3×10^{10} cm/sec

Fair Game Review What you learned in previous grades & lessons

Write the product using exponents. *(Section 10.1)*

33. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

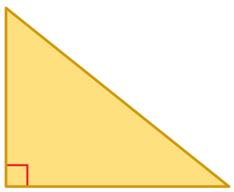
34. $3 \cdot 3 \cdot 3 \cdot y \cdot y \cdot y$

35. $(-2) \cdot (-2) \cdot (-2)$

36. **MULTIPLE CHOICE** What is the length of the hypotenuse of the right triangle? *(Section 7.3)*

(A) $\sqrt{18}$ in. (B) $\sqrt{41}$ in.

(C) 18 in. (D) 41 in.



10.6 Writing Scientific Notation

Essential Question How can you write a number in scientific notation?

1 ACTIVITY: Finding pH Levels

Work with a partner. In chemistry, pH is a measure of the activity of dissolved hydrogen ions (H^+). Liquids with low pH values are called *acids*. Liquids with high pH values are called *bases*.

Find the pH of each liquid. Is the liquid a base, neutral, or an acid?

a. Lime juice:
$[H^+] = 0.01$

b. Egg:
$[H^+] = 0.00000001$

c. Distilled water:
$[H^+] = 0.0000001$

d. Ammonia water:
$[H^+] = 0.00000000001$

e. Tomato juice:
$[H^+] = 0.0001$

f. Hydrochloric acid:
$[H^+] = 1$

pH	$[H^+]$	
14	1×10^{-14}	Bases
13	1×10^{-13}	
12	1×10^{-12}	
11	1×10^{-11}	
10	1×10^{-10}	
9	1×10^{-9}	
8	1×10^{-8}	
7	1×10^{-7}	Neutral
6	1×10^{-6}	Acids
5	1×10^{-5}	
4	1×10^{-4}	
3	1×10^{-3}	
2	1×10^{-2}	
1	1×10^{-1}	
0	1×10^{0}	

COMMON CORE

Scientific Notation

In this lesson, you will

- write large and small numbers in scientific notation.
- perform operations with numbers written in scientific notation.

Learning Standards
8.EE.3
8.EE.4

2 ACTIVITY: Writing Scientific Notation

Work with a partner. Match each planet with its distance from the Sun. Then write each distance in scientific notation. Do you think it is easier to match the distances when they are written in standard form or in scientific notation? Explain.

a. 1,800,000,000 miles

b. 67,000,000 miles

c. 890,000,000 miles

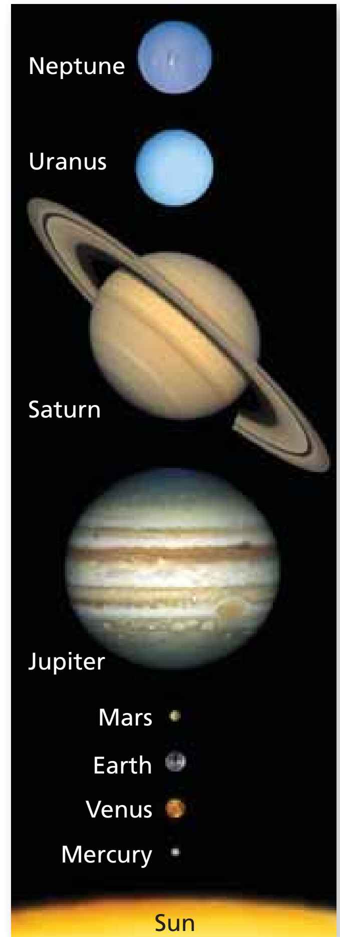
d. 93,000,000 miles

e. 140,000,000 miles

f. 2,800,000,000 miles

g. 480,000,000 miles

h. 36,000,000 miles



3 ACTIVITY: Making a Scale Drawing

Math Practice 6

Calculate Accurately

How can you verify that you have accurately written each distance in scientific notation?

Work with a partner. The illustration in Activity 2 is not drawn to scale. Use the instructions below to make a scale drawing of the distances in our solar system.

- **Cut a sheet of paper into three strips of equal width. Tape the strips together to make one long piece.**
- **Draw a long number line. Label the number line in hundreds of millions of miles.**
- **Locate each planet's position on the number line.**

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you write a number in scientific notation?

Practice Use what you learned about writing scientific notation to complete Exercises 3–5 on page 446.

10.6 Lesson

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Key Idea

Writing Numbers in Scientific Notation

Step 1: Move the decimal point so it is located to the right of the leading nonzero digit.

Step 2: Count the number of places you moved the decimal point. This indicates the exponent of the power of 10, as shown below.

Number Greater Than or Equal to 10

Use a positive exponent when you move the decimal point to the left.

$$\underbrace{8600}_{3} = 8.6 \times 10^3$$

Number Between 0 and 1

Use a negative exponent when you move the decimal point to the right.

$$\underbrace{0.0024}_{3} = 2.4 \times 10^{-3}$$

Study Tip

When you write a number greater than or equal to 1 and less than 10 in scientific notation, use zero as the exponent.

$6 = 6 \times 10^0$

EXAMPLE 1 Writing Large Numbers in Scientific Notation

A large corporation purchased a video-sharing website for $1,650,000,000. Write this number in scientific notation.

Move the decimal point 9 places to the left.

$$\underbrace{1,650,000,000}_{9} = 1.65 \times 10^9$$

The number is greater than 10. So, the exponent is positive.

EXAMPLE 2 Writing Small Numbers in Scientific Notation

The 2004 Indonesian earthquake slowed the rotation of Earth, making the length of a day 0.00000268 second shorter. Write this number in scientific notation.

Move the decimal point 6 places to the right.

$$\underbrace{0.00000268}_{6} = 2.68 \times 10^{-6}$$

The number is between 0 and 1. So, the exponent is negative.

On Your Own

Now You're Ready
Exercises 3–11

Write the number in scientific notation.

1. 50,000
2. 25,000,000
3. 683
4. 0.005
5. 0.00000033
6. 0.000506

EXAMPLE 3 Using Scientific Notation

An album receives an award when it sells 10,000,000 copies.

An album has sold 8,780,000 copies. How many more copies does it need to sell to receive the award?

Ⓐ 1.22×10^{-7} Ⓑ 1.22×10^{-6}

Ⓒ 1.22×10^{6} Ⓓ 1.22×10^{7}

Use a model to solve the problem.

Remaining sales needed for award = Sales required for award − Current sales total

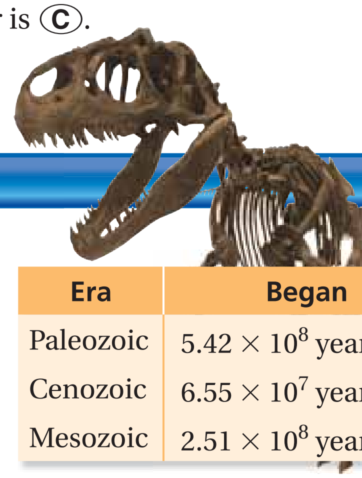
$= 10{,}000{,}000 - 8{,}780{,}000$

$= 1{,}220{,}000$

$= 1.22 \times 10^{6}$

The album must sell 1.22×10^{6} more copies to receive the award. So, the correct answer is Ⓒ.

EXAMPLE 4 Real-Life Application



The table shows when the last three geologic eras began. Order the eras from earliest to most recent.

Era	Began
Paleozoic	5.42×10^{8} years ago
Cenozoic	6.55×10^{7} years ago
Mesozoic	2.51×10^{8} years ago

Step 1: Compare the powers of 10.

Because $10^7 < 10^8$,
$6.55 \times 10^7 < 5.42 \times 10^8$ and
$6.55 \times 10^7 < 2.51 \times 10^8$.

Step 2: Compare the factors when the powers of 10 are the same.

Because $2.51 < 5.42$,
$2.51 \times 10^8 < 5.42 \times 10^8$.

From greatest to least, the order is 5.42×10^8, 2.51×10^8, and 6.55×10^7.

So, the eras in order from earliest to most recent are the Paleozoic era, Mesozoic era, and Cenozoic era.

Common Error

To use the method in Example 4, the numbers must be written in scientific notation.

On Your Own

Now You're Ready
Exercises 14–19

7. **WHAT IF?** In Example 3, an album has sold 955,000 copies. How many more copies does it need to sell to receive the award? Write your answer in scientific notation.

8. The *Tyrannosaurus rex* lived 7.0×10^{7} years ago. Consider the eras given in Example 4. During which era did the *Tyrannosaurus rex* live?

10.6 Exercises

Check It Out
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Vocabulary and Concept Check

1. **REASONING** How do you know whether a number written in standard form will have a positive or a negative exponent when written in scientific notation?
2. **WRITING** When is it appropriate to use scientific notation instead of standard form?

Practice and Problem Solving

Write the number in scientific notation.

1 2 **3.** 0.0021 **4.** 5,430,000 **5.** 321,000,000

6. 0.00000625 **7.** 0.00004 **8.** 10,700,000

9. 45,600,000,000 **10.** 0.000000000009256 **11.** 840,000

ERROR ANALYSIS **Describe and correct the error in writing the number in scientific notation.**

12. ✗ $\underbrace{0.000036}_{5}$

3.6×10^5

13. ✗ $\underbrace{72{,}500{,}000}_{6}$

72.5×10^6

Order the numbers from least to greatest.

4 **14.** $1.2 \times 10^8,\ 1.19 \times 10^8,\ 1.12 \times 10^8$

15. $6.8 \times 10^{-5},\ 6.09 \times 10^{-5},\ 6.78 \times 10^{-5}$

16. $5.76 \times 10^{12},\ 9.66 \times 10^{11},\ 5.7 \times 10^{10}$

17. $4.8 \times 10^{-6},\ 4.8 \times 10^{-5},\ 4.8 \times 10^{-8}$

18. $9.9 \times 10^{-15},\ 1.01 \times 10^{-14},\ 7.6 \times 10^{-15}$

19. $5.78 \times 10^{23},\ 6.88 \times 10^{-23},\ 5.82 \times 10^{23}$

20. **HAIR** What is the diameter of a human hair written in scientific notation?

Diameter: 0.000099 meter

21. **EARTH** What is the circumference of Earth written in scientific notation?

Circumference at the equator: about 40,100,000 meters

22. **CHOOSING UNITS** In Exercise 21, name a unit of measurement that would be more appropriate for the circumference. Explain.

Order the numbers from least to greatest.

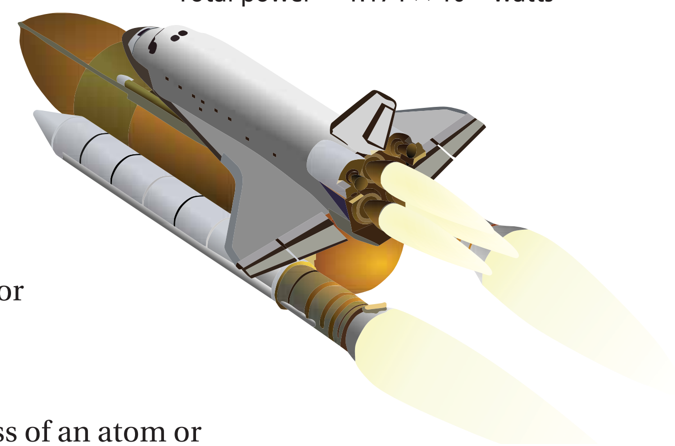
23. $\frac{68{,}500}{10}$, 680, 6.8×10^3

24. $\frac{5}{241}$, 0.02, 2.1×10^{-2}

25. 6.3%, 6.25×10^{-3}, $6\frac{1}{4}$, 0.625

26. 3033.4, 305%, $\frac{10{,}000}{3}$, 3.3×10^2

27. **SPACE SHUTTLE** The total power of a space shuttle during launch is the sum of the power from its solid rocket boosters and the power from its main engines. The power from the solid rocket boosters is 9,750,000,000 watts. What is the power from the main engines?

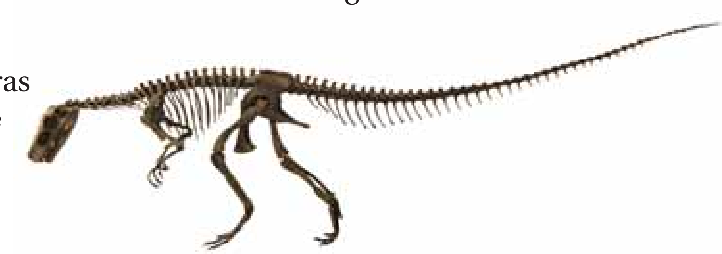


28. **CHOOSE TOOLS** Explain how to use a calculator to verify your answer to Exercise 27.

Equivalent to 1 Atomic Mass Unit
8.3×10^{-24} carat
1.66×10^{-21} milligram

29. **ATOMIC MASS** The mass of an atom or molecule is measured in atomic mass units. Which is greater, a *carat* or a *milligram*? Explain.

30. **Reasoning** In Example 4, the Paleozoic era ended when the Mesozoic era began. The Mesozoic era ended when the Cenozoic era began. The Cenozoic era is the current era.



a. Write the lengths of the three eras in scientific notation. Order the lengths from least to greatest.

b. Make a time line to show when the three eras occurred and how long each era lasted.

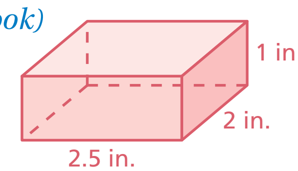
c. What do you notice about the lengths of the three eras? Use the Internet to determine whether your observation is true for *all* the geologic eras. Explain your results.

Fair Game Review What you learned in previous grades & lessons

Classify the real number. *(Section 7.4)*

31. 15 **32.** $\sqrt[3]{-8}$ **33.** $\sqrt{73}$

34. What is the surface area of the prism? *(Skills Review Handbook)*



Ⓐ 5 in.^{2} Ⓑ 5.5 in.^{2}

Ⓒ 10 in.^{2} Ⓓ 19 in.^{2}

10.7 Operations in Scientific Notation

Essential Question How can you perform operations with numbers written in scientific notation?

1 ACTIVITY: Adding Numbers in Scientific Notation

Work with a partner. Consider the numbers 2.4×10^3 and 7.1×10^3.

a. Explain how to use order of operations to find the sum of these numbers. Then find the sum.

$$2.4 \times 10^3 + 7.1 \times 10^3$$

b. The factor ____ is common to both numbers. How can you use the Distributive Property to rewrite the sum $(2.4 \times 10^3) + (7.1 \times 10^3)$?

$$(2.4 \times 10^3) + (7.1 \times 10^3) = \underline{\qquad\qquad}$$ Distributive Property

c. Use order of operations to evaluate the expression you wrote in part (b). Compare the result with your answer in part (a).

d. **STRUCTURE** Write a rule you can use to add numbers written in scientific notation where the powers of 10 are the same. Then test your rule using the sums below.

- $(4.9 \times 10^5) + (1.8 \times 10^5) = \underline{\qquad\qquad}$
- $(3.85 \times 10^4) + (5.72 \times 10^4) = \underline{\qquad\qquad}$

2 ACTIVITY: Adding Numbers in Scientific Notation

Work with a partner. Consider the numbers 2.4×10^3 and 7.1×10^4.

a. Explain how to use order of operations to find the sum of these numbers. Then find the sum.

$$2.4 \times 10^3 + 7.1 \times 10^4$$

b. How is this pair of numbers different from the pairs of numbers in Activity 1?

c. Explain why you cannot immediately use the rule you wrote in Activity 1(d) to find this sum.

d. **STRUCTURE** How can you rewrite one of the numbers so that you can use the rule you wrote in Activity 1(d)? Rewrite one of the numbers. Then find the sum using your rule and compare the result with your answer in part (a).

e. **REASONING** Do these procedures work when subtracting numbers written in scientific notation? Justify your answer by evaluating the differences below.

- $(8.2 \times 10^5) - (4.6 \times 10^5) = \underline{\qquad\qquad}$
- $(5.88 \times 10^5) - (1.5 \times 10^4) = \underline{\qquad\qquad}$

Common Core

Scientific Notation

In this lesson, you will

- add, subtract, multiply, and divide numbers written in scientific notation.

Learning Standards
8.EE.3
8.EE.4

3 ACTIVITY: Multiplying Numbers in Scientific Notation

Math Practice 3

Justify Conclusions

Which step of the procedure would be affected if the powers of 10 were different? Explain.

Work with a partner. Match each step with the correct description.

Step	**Description**
$(2.4 \times 10^3) \times (7.1 \times 10^3)$	Original expression
1. $= 2.4 \times 7.1 \times 10^3 \times 10^3$	**A.** Write in standard form.
2. $= (2.4 \times 7.1) \times (10^3 \times 10^3)$	**B.** Product of Powers Property
3. $= 17.04 \times 10^6$	**C.** Write in scientific notation.
4. $= 1.704 \times 10^1 \times 10^6$	**D.** Commutative Property of Multiplication
5. $= 1.704 \times 10^7$	**E.** Simplify.
6. $= 17{,}040{,}000$	**F.** Associative Property of Multiplication

Does this procedure work when the numbers have different powers of 10? Justify your answer by using this procedure to evaluate the products below.

- $(1.9 \times 10^2) \times (2.3 \times 10^5) =$ ______
- $(8.4 \times 10^6) \times (5.7 \times 10^{-4}) =$ ______

4 ACTIVITY: Using Scientific Notation to Estimate

Work with a partner. A person normally breathes about 6 liters of air per minute. The life expectancy of a person in the United States at birth is about 80 years. Use scientific notation to estimate the total amount of air a person born in the United States breathes over a lifetime.

What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you perform operations with numbers written in scientific notation?

6. Use a calculator to evaluate the expression. Write your answer in scientific notation and in standard form.

   a. $(1.5 \times 10^4) + (6.3 \times 10^4)$ **b.** $(7.2 \times 10^5) - (2.2 \times 10^3)$

   c. $(4.1 \times 10^{-3}) \times (4.3 \times 10^{-3})$ **d.** $(4.75 \times 10^{-6}) \times (1.34 \times 10^7)$

Practice Use what you learned about evaluating expressions involving scientific notation to complete Exercises 3–6 on page 452.

10.7 Lesson

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To add or subtract numbers written in scientific notation with the same power of 10, add or subtract the factors. When the numbers have different powers of 10, first rewrite the numbers so they have the same power of 10.

EXAMPLE 1 Adding and Subtracting Numbers in Scientific Notation

Find the sum or difference. Write your answer in scientific notation.

a. $(4.6 \times 10^3) + (8.72 \times 10^3)$

$= (4.6 + 8.72) \times 10^3$ — Distributive Property

$= 13.32 \times 10^3$ — Add.

$= (1.332 \times 10^1) \times 10^3$ — Write 13.32 in scientific notation.

$= 1.332 \times 10^4$ — Product of Powers Property

Study Tip

In Example 1(b), you will get the same answer when you start by rewriting 3.5×10^{-2} as 35×10^{-3}.

b. $(3.5 \times 10^{-2}) - (6.6 \times 10^{-3})$

Rewrite 6.6×10^{-3} so that it has the same power of 10 as 3.5×10^{-2}.

$6.6 \times 10^{-3} = 6.6 \times 10^{-1} \times 10^{-2}$ — Rewrite 10^{-3} as $10^{-1} \times 10^{-2}$.

$= 0.66 \times 10^{-2}$ — Rewrite 6.6×10^{-1} as 0.66.

Subtract the factors.

$(3.5 \times 10^{-2}) - (0.66 \times 10^{-2})$

$= (3.5 - 0.66) \times 10^{-2}$ — Distributive Property

$= 2.84 \times 10^{-2}$ — Subtract.

On Your Own

Now You're Ready Exercises 7–14

Find the sum or difference. Write your answer in scientific notation.

1. $(8.2 \times 10^2) + (3.41 \times 10^{-1})$
2. $(7.8 \times 10^{-5}) - (4.5 \times 10^{-5})$

To multiply or divide numbers written in scientific notation, multiply or divide the factors and powers of 10 separately.

EXAMPLE 2 Multiplying Numbers in Scientific Notation

Find $(3 \times 10^{-5}) \times (5 \times 10^{-2})$. Write your answer in scientific notation.

$(3 \times 10^{-5}) \times (5 \times 10^{-2})$

$= 3 \times 5 \times 10^{-5} \times 10^{-2}$ — Commutative Property of Multiplication

$= (3 \times 5) \times (10^{-5} \times 10^{-2})$ — Associative Property of Multiplication

$= 15 \times 10^{-7}$ — Simplify.

$= 1.5 \times 10^1 \times 10^{-7}$ — Write 15 in scientific notation.

$= 1.5 \times 10^{-6}$ — Product of Powers Property

Study Tip

You can check your answer using standard form.

$(3 \times 10^{-5}) \times (5 \times 10^{-2})$
$= 0.00003 \times 0.05$
$= 0.0000015$
$= 1.5 \times 10^{-6}$

EXAMPLE 3 Dividing Numbers in Scientific Notation

Find $\dfrac{1.5 \times 10^{-8}}{6 \times 10^{7}}$. Write your answer in scientific notation.

$\dfrac{1.5 \times 10^{-8}}{6 \times 10^{7}} = \dfrac{1.5}{6} \times \dfrac{10^{-8}}{10^{7}}$	Rewrite as a product of fractions.
$= 0.25 \times \dfrac{10^{-8}}{10^{7}}$	Divide 1.5 by 6.
$= 0.25 \times 10^{-15}$	Quotient of Powers Property
$= 2.5 \times 10^{-1} \times 10^{-15}$	Write 0.25 in scientific notation.
$= 2.5 \times 10^{-16}$	Product of Powers Property

On Your Own

Now You're Ready
Exercises 16–23

Find the product or quotient. Write your answer in scientific notation.

3. $6 \times (8 \times 10^{-5})$

4. $(7 \times 10^{2}) \times (3 \times 10^{5})$

5. $(9.2 \times 10^{12}) \div 4.6$

6. $(1.5 \times 10^{-3}) \div (7.5 \times 10^{2})$

EXAMPLE 4 Real-Life Application


Diameter = 1,400,000 km

How many times greater is the diameter of the Sun than the diameter of Earth?


Diameter = 1.28×10^4 km

Write the diameter of the Sun in scientific notation.

$$1{,}400{,}000 = 1.4 \times 10^{6}$$

6

Divide the diameter of the Sun by the diameter of Earth.

$\dfrac{1.4 \times 10^{6}}{1.28 \times 10^{4}} = \dfrac{1.4}{1.28} \times \dfrac{10^{6}}{10^{4}}$	Rewrite as a product of fractions.
$= 1.09375 \times \dfrac{10^{6}}{10^{4}}$	Divide 1.4 by 1.28.
$= 1.09375 \times 10^{2}$	Quotient of Powers Property
$= 109.375$	Write in standard form.

The diameter of the Sun is about 109 times greater than the diameter of Earth.

On Your Own

7. How many more kilometers is the radius of the Sun than the radius of Earth? Write your answer in standard form.

10.7 Exercises

Check It Out
Help with Homework
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Vocabulary and Concept Check

1. **WRITING** Describe how to subtract two numbers written in scientific notation with the same power of 10.

2. **NUMBER SENSE** You are multiplying two numbers written in scientific notation with different powers of 10. Do you have to rewrite the numbers so they have the same power of 10 before multiplying? Explain.

Practice and Problem Solving

Evaluate the expression using two different methods. Write your answer in scientific notation.

3. $(2.74 \times 10^7) + (5.6 \times 10^7)$
4. $(8.3 \times 10^6) + (3.4 \times 10^5)$
5. $(5.1 \times 10^5) \times (9.7 \times 10^5)$
6. $(4.5 \times 10^4) \times (6.2 \times 10^3)$

Find the sum or difference. Write your answer in scientific notation.

① 7. $(2 \times 10^5) + (3.8 \times 10^5)$
8. $(6.33 \times 10^{-9}) - (4.5 \times 10^{-9})$
9. $(9.2 \times 10^8) - (4 \times 10^8)$
10. $(7.2 \times 10^{-6}) + (5.44 \times 10^{-6})$
11. $(7.8 \times 10^7) - (2.45 \times 10^6)$
12. $(5 \times 10^{-5}) + (2.46 \times 10^{-3})$
13. $(9.7 \times 10^6) + (6.7 \times 10^5)$
14. $(2.4 \times 10^{-1}) - (5.5 \times 10^{-2})$

15. **ERROR ANALYSIS** Describe and correct the error in finding the sum of the numbers.

$$(2.5 \times 10^9) + (5.3 \times 10^8) = (2.5 + 5.3) \times (10^9 \times 10^8)$$
$$= 7.8 \times 10^{17}$$

Find the product or quotient. Write your answer in scientific notation.

② ③ 16. $5 \times (7 \times 10^7)$
17. $(5.8 \times 10^{-6}) \div (2 \times 10^{-3})$
18. $(1.2 \times 10^{-5}) \div 4$
19. $(5 \times 10^{-7}) \times (3 \times 10^6)$
20. $(3.6 \times 10^7) \div (7.2 \times 10^7)$
21. $(7.2 \times 10^{-1}) \times (4 \times 10^{-7})$
22. $(6.5 \times 10^8) \times (1.4 \times 10^{-5})$
23. $(2.8 \times 10^4) \div (2.5 \times 10^6)$

24. **MONEY** How many times greater is the thickness of a dime than the thickness of a dollar bill?

Thickness $= 0.135$ cm

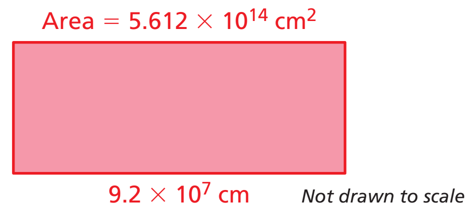
Thickness $= 1.0922 \times 10^{-2}$ cm

Evaluate the expression. Write your answer in scientific notation.

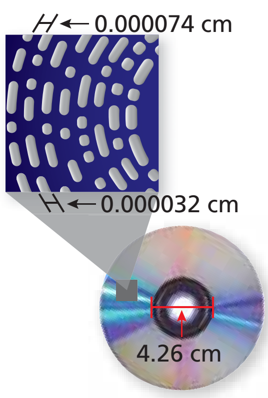
25. $5{,}200{,}000 \times (8.3 \times 10^2) - (3.1 \times 10^8)$

26. $(9 \times 10^{-3}) + (2.4 \times 10^{-5}) \div 0.0012$

27. **GEOMETRY** Find the perimeter of the rectangle.



28. **BLOOD SUPPLY** A human heart pumps about 7×10^{-2} liter of blood per heartbeat. The average human heart beats about 72 times per minute. How many liters of blood does a heart pump in 1 year? in 70 years? Write your answers in scientific notation. Then use estimation to justify your answers.



29. **DVDS** On a DVD, information is stored on bumps that spiral around the disk. There are 73,000 ridges (with bumps) and 73,000 valleys (without bumps) across the diameter of the DVD. What is the diameter of the DVD in centimeters?

30. **PROJECT** Use the Internet or some other reference to find the populations and areas (in square miles) of India, China, Argentina, the United States, and Egypt. Round each population to the nearest million and each area to the nearest thousand square miles.

a. Write each population and area in scientific notation.

b. Use your answers to part (a) to find and order the population densities (people per square mile) of each country from least to greatest.

31. Critical Thinking Albert Einstein's most famous equation is $E = mc^2$, where E is the energy of an object (in joules), m is the mass of an object (in kilograms), and c is the speed of light (in meters per second). A hydrogen atom has 15.066×10^{-11} joule of energy and a mass of 1.674×10^{-27} kilogram. What is the speed of light? Write your answer in scientific notation.

Fair Game Review What you learned in previous grades & lessons

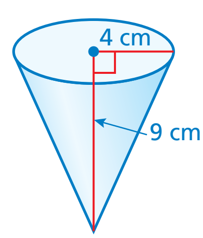
Find the cube root. *(Section 7.2)*

32. $\sqrt[3]{-729}$

33. $\sqrt[3]{\frac{1}{512}}$

34. $\sqrt[3]{-\frac{125}{343}}$

35. **MULTIPLE CHOICE** What is the volume of the cone? *(Section 8.2)*



Ⓐ 16π cm^3 Ⓑ 108π cm^3

Ⓒ 48π cm^3 Ⓓ 144π cm^3

10.5 – 10.7 Quiz

Check It Out
Progress Check
BigIdeasMath.com

Tell whether the number is written in scientific notation. Explain. *(Section 10.5)*

1. 23×10^9

2. 0.6×10^{-7}

Write the number in standard form. *(Section 10.5)*

3. 8×10^6

4. 1.6×10^{-2}

Write the number in scientific notation. *(Section 10.6)*

5. 0.00524

6. 892,000,000

Evaluate the expression. Write your answer in scientific notation. *(Section 10.7)*

7. $(7.26 \times 10^4) + (3.4 \times 10^4)$

8. $(2.8 \times 10^{-5}) - (1.6 \times 10^{-6})$

9. $(2.4 \times 10^4) \times (3.8 \times 10^{-6})$

10. $(5.2 \times 10^{-3}) \div (1.3 \times 10^{-12})$

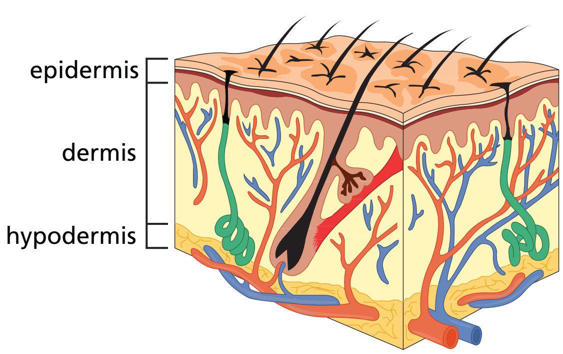
11. **PLANETS** The table shows the equatorial radii of the eight planets in our solar system. *(Section 10.5)*

a. Which planet has the second-smallest equatorial radius?

b. Which planet has the second-largest equatorial radius?

Planet	Equatorial Radius (km)
Mercury	2.44×10^3
Venus	6.05×10^3
Earth	6.38×10^3
Mars	3.4×10^3
Jupiter	7.15×10^4
Saturn	6.03×10^4
Uranus	2.56×10^4
Neptune	2.48×10^4

12. **OORT CLOUD** The Oort cloud is a spherical cloud that surrounds our solar system. It is about 2×10^5 astronomical units from the Sun. An astronomical unit is about 1.5×10^8 kilometers. How far is the Oort cloud from the Sun in kilometers? *(Section 10.6)*



13. **EPIDERMIS** The outer layer of skin is called the *epidermis*. On the palm of your hand, the epidermis is 0.0015 meter thick. Write this number in scientific notation. *(Section 10.6)*

14. **ORBITS** It takes the Sun about 2.3×10^8 years to orbit the center of the Milky Way. It takes Pluto about 2.5×10^2 years to orbit the Sun. How many times does Pluto orbit the Sun while the Sun completes one orbit around the Milky Way? Write your answer in standard form. *(Section 10.7)*

10 Chapter Review

Check It Out Vocabulary Help BigIdeasMath.com

Review Key Vocabulary

power, *p. 412*
base, *p. 412*
exponent, *p. 412*
scientific notation, *p. 438*

Review Examples and Exercises

10.1 Exponents (pp. 410–415)

Write $(-4) \cdot (-4) \cdot (-4) \cdot y \cdot y$ using exponents.

Because -4 is used as a factor 3 times, its exponent is 3. Because y is used as a factor 2 times, its exponent is 2.

So, $(-4) \cdot (-4) \cdot (-4) \cdot y \cdot y = (-4)^3 y^2$.

Exercises

Write the product using exponents.

1. $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$
2. $2 \cdot 2 \cdot 2 \cdot n \cdot n$

Evaluate the expression.

3. 6^3
4. $-\left(\frac{1}{2}\right)^4$
5. $\left|\frac{1}{2}(16 - 6^3)\right|$

10.2 Product of Powers Property (pp. 416–421)

a. $\left(-\frac{1}{8}\right)^7 \cdot \left(-\frac{1}{8}\right)^4 = \left(-\frac{1}{8}\right)^{7+4}$ — Product of Powers Property

$= \left(-\frac{1}{8}\right)^{11}$ — Simplify.

b. $(2.5^7)^2 = 2.5^{7 \cdot 2}$ — Power of a Power Property

$= 2.5^{14}$ — Simplify.

c. $(3m)^2 = 3^2 \cdot m^2$ — Power of a Product Property

$= 9m^2$ — Simplify.

Exercises

Simplify the expression.

6. $p^5 \cdot p^2$
7. $(n^{11})^2$
8. $(5y)^3$
9. $(-2k)^4$

10.3 Quotient of Powers Property (pp. 422–427)

a. $\dfrac{(-4)^9}{(-4)^6} = (-4)^{9-6}$ — Quotient of Powers Property

$= (-4)^3$ — Simplify.

b. $\dfrac{x^4}{x^3} = x^{4-3}$ — Quotient of Powers Property

$= x^1$

$= x$ — Simplify.

Exercises

Simplify the expression. Write your answer as a power.

10. $\dfrac{8^8}{8^3}$ **11.** $\dfrac{5^2 \cdot 5^9}{5}$ **12.** $\dfrac{w^8}{w^7} \cdot \dfrac{w^5}{w^2}$

Simplify the expression.

13. $\dfrac{2^2 \cdot 2^5}{2^3}$ **14.** $\dfrac{(6c)^3}{c}$ **15.** $\dfrac{m^8}{m^6} \cdot \dfrac{m^{10}}{m^9}$

10.4 Zero and Negative Exponents (pp. 428–433)

a. $10^{-3} = \dfrac{1}{10^3}$ — Definition of negative exponent

$= \dfrac{1}{1000}$ — Evaluate power.

b. $(-0.5)^{-5} \cdot (-0.5)^5 = (-0.5)^{-5+5}$ — Product of Powers Property

$= (-0.5)^0$ — Simplify.

$= 1$ — Definition of zero exponent

Exercises

Evaluate the expression.

16. 2^{-4} **17.** 95^0 **18.** $\dfrac{8^2}{8^4}$

19. $(-12)^{-7} \cdot (-12)^7$ **20.** $\dfrac{1}{7^9} \cdot \dfrac{1}{7^{-6}}$ **21.** $\dfrac{9^4 \cdot 9^{-2}}{9^2}$

10.5 Reading Scientific Notation *(pp. 436–441)*

Write (a) 5.9×10^4 and (b) 7.31×10^{-6} in standard notation.

a. $5.9 \times 10^4 = \underbrace{59,000}_{4}$

Move decimal point $|4| = 4$ places to the right.

b. $7.31 \times 10^{-6} = \underbrace{0.00000731}_{6}$

Move decimal point $|-6| = 6$ places to the left.

Exercises

Write the number in standard form.

22. 2×10^7

23. 3.4×10^{-2}

24. 1.5×10^{-9}

25. 5.9×10^{10}

26. 4.8×10^{-3}

27. 6.25×10^5

10.6 Writing Scientific Notation *(pp. 442–447)*

Write (a) 309,000,000 and (b) 0.00056 in scientific notation.

a. $\underbrace{309,000,000}_{8} = 3.09 \times 10^8$

The number is greater than 10. So, the exponent is positive.

b. $\underbrace{0.00056}_{4} = 5.6 \times 10^{-4}$

The number is between 0 and 1. So, the exponent is negative.

Exercises

Write the number in scientific notation.

28. 0.00036

29. 800,000

30. 79,200,000

10.7 Operations in Scientific Notation *(pp. 448–453)*

Find $(2.6 \times 10^5) + (3.1 \times 10^5)$.

$$(2.6 \times 10^5) + (3.1 \times 10^5) = (2.6 + 3.1) \times 10^5 \quad \text{Distributive Property}$$

$$= 5.7 \times 10^5 \quad \text{Add.}$$

Exercises

Evaluate the expression. Write your answer in scientific notation.

31. $(4.2 \times 10^8) + (5.9 \times 10^9)$

32. $(5.9 \times 10^{-4}) - (1.8 \times 10^{-4})$

33. $(7.7 \times 10^8) \times (4.9 \times 10^{-5})$

34. $(3.6 \times 10^5) \div (1.8 \times 10^9)$

10 Chapter Test

Check It Out
Test Practice
BigIdeasMath.com

Write the product using exponents.

1. $(-15) \cdot (-15) \cdot (-15)$

2. $\left(\frac{1}{12}\right) \cdot \left(\frac{1}{12}\right) \cdot \left(\frac{1}{12}\right) \cdot \left(\frac{1}{12}\right) \cdot \left(\frac{1}{12}\right)$

Evaluate the expression.

3. -2^3

4. $10 + 3^3 \div 9$

Simplify the expression. Write your answer as a power.

5. $9^{10} \cdot 9$

6. $(6^6)^5$

7. $(2 \cdot 10)^7$

8. $\dfrac{(-3.5)^{13}}{(-3.5)^9}$

Evaluate the expression.

9. $5^{-2} \cdot 5^2$

10. $\dfrac{-8}{(-8)^3}$

Write the number in standard form.

11. 3×10^7

12. 9.05×10^{-3}

Evaluate the expression. Write your answer in scientific notation.

13. $(7.8 \times 10^7) + (9.9 \times 10^7)$

14. $(6.4 \times 10^5) - (5.4 \times 10^4)$

15. $(3.1 \times 10^6) \times (2.7 \times 10^{-2})$

16. $(9.6 \times 10^7) \div (1.2 \times 10^{-4})$

17. **CRITICAL THINKING** Is $(xy^2)^3$ the same as $(xy^3)^2$? Explain.

18. **RICE** A grain of rice weighs about 3^3 milligrams. About how many grains of rice are in one scoop?

19. **TASTE BUDS** There are about 10,000 taste buds on a human tongue. Write this number in scientific notation.

One scoop of rice weighs about 3^9 milligrams.

20. **LEAD** From 1978 to 2008, the amount of lead allowed in the air in the United States was 1.5×10^{-6} gram per cubic meter. In 2008, the amount allowed was reduced by 90%. What is the new amount of lead allowed in the air?

10 Standards Assessment

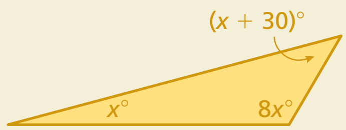
1. Mercury's distance from the Sun is approximately 5.79×10^7 kilometers. What is this distance in standard form? *(8.EE.4)*

   A. 5,790,000,000 km **C.** 57,900,000 km

   B. 579,000,000 km **D.** 5,790,000 km

2. The steps Jim took to answer the question are shown below. What should Jim change to correctly answer the question? *(8.G.5)*

   How many degrees are in the largest angle in the triangle below?

   $$x + 8x + x + 30 = 180$$
   $$10x = 150$$
   $$x = 15$$

   F. The left side of the equation should equal 360° instead of 180°.

   G. The sum of the acute angles should equal 90°.

   H. Evaluate the smallest angle when $x = 15$.

   I. Evaluate the largest angle when $x = 15$.

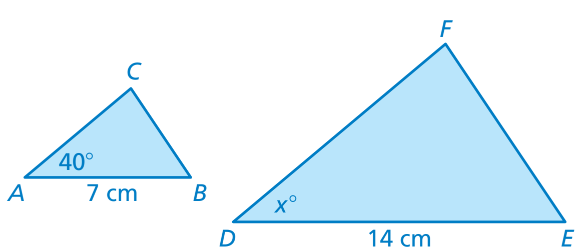
3. Which expression is equivalent to the expression below? *(8.EE.1)*

   $$2^4 2^3$$

   A. 2^{12} **C.** 48

   B. 4^7 **D.** 128

4. In the figure below, $\triangle ABC$ is a dilation of $\triangle DEF$.

   What is the value of x? *(8.G.4)*

Test-Taking Strategy
Use Intelligent Guessing

"It can't be 40 or 5000 because they aren't divisible by 3. So, you can intelligently guess between 3000 and 3072."

5. A bank account pays interest so that the amount in the account doubles every 10 years. The account started with \$5,000 in 1940. Which expression represents the amount (in dollars) in the account n decades later? *(8.EE.1)*

F. $2^n \cdot 5000$ **H.** 5000^n

G. $5000(n + 1)$ **I.** $2^n + 5000$

6. The formula for the volume V of a pyramid is $V = \frac{1}{3}Bh$. Solve the formula for the height h. *(8.EE.7b)*

A. $h = \frac{1}{3}VB$ **C.** $h = \frac{V}{3B}$

B. $h = \frac{3V}{B}$ **D.** $h = V - \frac{1}{3}B$

7. The gross domestic product (GDP) is a way to measure how much a country produces economically in a year. The table below shows the approximate population and GDP for the United States. *(8.EE.4)*

Think Solve Explain

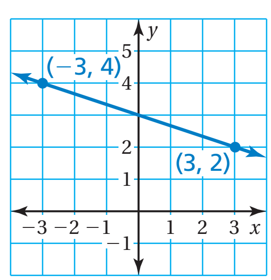
United States 2012	
Population	312 million (312,000,000)
GDP	15.1 trillion dollars (\$15,100,000,000,000)

Part A Find the GDP per person for the United States. Show your work and explain your reasoning.

Part B Write the population and the GDP using scientific notation.

Part C Find the GDP per person for the United States using your answers from Part B. Write your answer in scientific notation. Show your work and explain your reasoning.

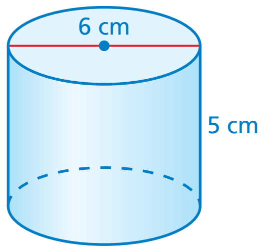
8. What is the equation of the line shown in the graph? *(8.EE.6)*



F. $y = -\frac{1}{3}x + 3$ **H.** $y = -3x + 3$

G. $y = \frac{1}{3}x + 1$ **I.** $y = 3x - \frac{1}{3}$

9. A cylinder and its dimensions are shown below.



What is the volume of the cylinder? (Use 3.14 for π.) *(8.G.9)*

A. $47.1\ \text{cm}^3$

B. $94.2\ \text{cm}^3$

C. $141.3\ \text{cm}^3$

D. $565.2\ \text{cm}^3$

10. Find $(-2.5)^{-2}$. *(8.EE.1)*

11. Two lines have the same y-intercept. The slope of one line is 1, and the slope of the other line is -1. What can you conclude? *(8.EE.6)*

F. The lines are parallel.

G. The lines meet at exactly one point.

H. The lines meet at more than one point.

I. The situation described is impossible.

12. The director of a research lab wants to present data to donors. The data show how the lab uses a great deal of donated money for research and only a small amount of money for other expenses. Which type of display is best suited for showing these data? *(8.SP.1)*

A. box-and-whisker plot

B. circle graph

C. line graph

D. scatter plot